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COMPOSITE LINER DESIGN TO MAXIMIZE THE SHOCK PRESSURE BEYOND MEGABARS

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Among the solid liners made of a single material which are imploded onto a target under the same driving condition, the aluminum liner produces the highest shock pressure. We propose the composite liner design which can increase the shock pressure several times over the the best performance obtainable from an aluminum liner. We have also developed a general formulation to optimize the composite liner design for any driving current, and derived a set of very useful scaling relations. Finally, we present some 1-D simulations of the optimal composite liners to be fielded at Pegasus and Procyon in the upcoming megabar experiments.

I. Introduction

Using pulsed power to implode the liner onto a target is a convenient way to produce high shock pressure. Two years ago, a solid aluminum liner which could produce shock pressures in the hundreds of kbar regime was designed [1] and fielded [2] at the LANL Pegasus facility. This liner design has since been used successfully for a variety of application experiments. Recently, there have been considerable experimental interests to produce shock pressures in the megabar regime and more importantly, to ascertain what is the practical pressure limit attainable for a given pulsed power. It will be made clear later that, among the liners made of a single material (for convenience called pure liners hereafter), the aluminum one is practically the best to produce the highest shock pressure. But if we scale up the solid aluminum liner design to the maximum driving current at Pegasus, the highest pressure we can get is just around 2 Mbar.

In this paper we propose a composite liner design which can increase the shock pressure several times over the best performance of the aluminum liner. The composite liner is made of aluminum on the outside and platinum inside. We obtained this particular combination through a systematic study of the shock pressure behavior inferred from the Hugoniot, and a general analysis of the Joule heating limitation on liner materials. We have also developed a general formulation to optimize the composite liner design for any given driving current. Using the thin-liner implosion equation, with the driving current decoupled from the liner motion, we have derived a set of very useful scaling relations for the optimal liner parameters and performance. These relations provide us a quick benchmark estimate on the maximum pressure attainable from any scalable driving current. For example, we can scale our results obtained for Pegasus to the Atlas parameter regime, since the driving currents for both are approximately sinusoidal.

In the next section we will present the liner implosion equation to lay the groundwork for liner design. We next discuss the general behavior of the shock pressure through the Hugoniots in Sect. III. We establish a shortcut for searching the best colliding materials which maximize the shock pressure. In Sect. IV we give a detailed discussion on the Joule heating, which sets an ultimate limit on the liner velocity. The physical considerations leading to the composite liner and general procedure to optimize the liner parameters are given in Sect. V, followed by the derivation of the scaling relations in Sect. VI. Finally, in Sect. VII we discuss the optimal composite liners to be fielded in the upcoming megabar liner experiments at both Pegasus and Procyon, and we present some 1-D simulation results.

II. Implosion Equation for Thin Liner

The liner implosion equation is rather complicated for the following two reasons: First, the driving current is not independent of the liner motion. This means that the equation of motion must be coupled to the circuit equation [xx] in a more refined treatment. Second, the liner thickness is time-dependent due to radial convergence, which complicates the force distribution. Since we only use the implosion equation to optimize the liner design, it is not crucial for the former to be exact.
Rather, we will take advantage of any good approximation which helps to simplify the implosion equation and render the scaling possible. The thin-liner approximation will be assumed in this paper; it is justified if the thickness of the liner is much smaller than the radius.

Next we note that the liner radius affects the driving current only through a logarithmic term in the inductance, so the effect is negligible until the liner radius \(r\) becomes much smaller than its initial value \(r_0\). In the region where \(r < r_0\), the duration is so short that the liner velocity is affected only slightly by the error in current. The above reasoning justifies that we can decouple the driving current from the liner motion. This excellent approximation not only simplifies greatly the implosion equation but also makes the scaling of the optimal liner parameters possible. Using the above approximations, the liner implosion equation is given by

\[
\ddot{r}(t) = -\left(\frac{\mu_0}{4\pi}\frac{\ell^2(t)}{mr(t)}\right),
\]

with the initial conditions \(r(0) = r_0\) and \(\dot{r}(0) = 0\), where \(\ell\) is the length; \(I(t)\), the driving current; \(r(t)\), the radius; and \(m\), the mass of the liner.

A class of currents is said to be scalable to one another if we can represent them by a single function as \(I_pF(\omega t)\) using two parameters \(I_p\) and \(\omega\). The current wave forms we usually see in many pulsed powers are approximately sinusoidal or like a step-function, each type forms a scalable class. Later when we look for possible scaling relations of the optimal liner parameters and performance for scalable driving currents, it is useful to express the implosion equations for the whole class in terms of the scaled distance traveled by the liner, \(z \equiv 1 - r/r_0\), and scaled time \(\tau \equiv \omega t\). The resulting implosion equation,

\[
\ddot{z} = \frac{\alpha F^2(\tau)}{(1 - z)},
\]

now has an invariant set of initial conditions \(z(0) = \dot{z}(0) = 0\), where the dot stands for \(d/d\tau\) and

\[
\alpha \equiv \left(\frac{\mu_0}{4\pi}\frac{\ell^2}{mr^3}\right)\omega^2.
\]

Let \(\tau_c\) be the scaled collision time, then the collision velocity is given by

\[
v_c = r_0\omega \ddot{z}(\tau_c).
\]

### III. Behavior of Shock Pressures Inferred from Hugoniots

When the liner collides with the target at a velocity \(v_c\), the shock pressure can be solved from the Hugoniots for the liner (labeled by \(\ell\))

\[
P_{\ell}(v) = \rho c_\ell(v + s_\ell v)
\]

and target (labeled by \(t\))

\[
P_t(v_c - v) = \rho_t(v_c - v)[c_t + s_t(v_c - v)]
\]

by eliminating the particle velocity \(v\), where \(\rho\) is the density, and \(c\) and \(s\) are material constants that relate the shock velocity to the particle velocity. From the above equations we see that higher collision velocity \(v_c\) and material densities will give rise to higher shock pressure, but the material with higher values in \(c\) and \(s\) also helps. While the above equations provide us a precise guideline to find the best liner and target materials that will achieve the highest collision shock under a given imploiding condition, the process to examine all material pairs will be extremely time consuming. Fortunately we can take a shortcut by proving the following theorem: For any collision velocity, let \(P_{AB}\) be the shock pressure generated from a collision between two materials A and B, its value is bounded between \(P_{AA}\) and \(P_{BB}\). We can prove this statement as follows: First, notice that the Hugoniots are parabolic functions of \(v\). In the physical region \(0 \leq v \leq v_c\), the liner Hugoniot increases while the target Hugoniot decreases with increasing \(v\). Second, for the collision between identical material, the two Hugoniots always intersect at \(v = v_c/2\) due to their reflection symmetry at the point, so we have the exact solution

\[
P = \frac{1}{4}\rho v c(2c + v_c).
\]

Without loss of generality, we assume \(P_{AA} > P_{BB}\) and solve for \(P_{AB}\) at the intersection of the liner Hugoniot A and target Hugoniot B. Now \(P_{AA}\) and \(P_{AB}\) lie on the liner Hugoniot A which increases with \(v\), and \(P_{BB}\) and \(P_{AB}\) lie on the target Hugoniot B which increases with \(v\). It follows that \(P_{AB}\) can only occur in the region \(v < v_c/2\) and therefore

\[
P_{AA} > P_{AB} > P_{BB}.
\]

Using the above rigorous result we can simplify our task considerably in searching for the right material to maximize the shock pressure. Instead of searching for the maximum of all \(P_{AB}\), we can just look for the maximum of \(P_{AA}\), provided that the highest attainable \(v_c\) is independent of the liner material. We will show later that the last condition is indeed valid for the composite liner.

Even though the inequalities in Eq.(8) are valid for any \(v_c\), Eq.(7) gives no assurance that the same
material always maximizes the pressure for all values of \( v_c \). But for multi-megabar pressures or higher, this is indeed the case. This follows from the fact that, for a wide variety of materials [4], \( c \) is around a few mm/\( \mu \text{s} \) and \( 1.2 < s < 2 \). At high megabar pressures, \( v_c \) is large enough so that the term \( sv_c \) dominates over \( 2c \) in Eq.(7) and consequently we have

\[
P \approx \frac{1}{4} \rho s v_c^2.
\]

This ensures that \( P \) is the maximum for the material with the highest value in \( \rho s \) at any \( v_c \). In the same approximation, the shock pressure between two different materials behaves like

\[
P_{AB} \approx \frac{v_c^2}{[(s_A \rho_A)^{-1/2} + (s_B \rho_B)^{-1/2}]^2}
\]

IV. Joule Heating Limitation on Liner

The current passing through the liner has to diffuse into its interior from the outer surface, so calculating the resistive heating of the liner is quite complicated unless the diffusion time is faster than the implosion time. In general, we do not expect the temperature distribution across the liner to be uniform, but rather to increase monotonically toward outside. To simplify the formulation, let us consider a pure liner and assume that the temperature is uniformly distributed. Since radiation loss is negligible, the time dependence of the liner temperature is given by the energy balance equation

\[
R(t) I^2(t) dt = mc(T) dT,
\]

where \( c \) is the specific heat of the liner material and \( R \) the resistance. In term of the resistivity \( \eta \) and density \( \rho \), we can integrate the above as

\[
\frac{\ell^2}{m^2} \int_0^t I^2(t) dt = \int_{T_0}^{T(t)} \frac{\rho(T) c(T)}{\eta(T)} dT,
\]

where \( T_0 \) is the initial temperature. Notice that the right hand side is only a state function of the liner material. The left hand side is proportional to the electrical action integral defined as

\[
Q(T(t)) = \frac{1}{A^2} \int_0^t I^2(t) dt,
\]

where \( A \) is the liner cross section. The electrical action for any conductor can be measured by passing a current through a thin sample wire. Setting a limit on the action by requiring \( T(t_c) = T_c \), we constrain the liner mass to be a function of the collision time \( t_c \) as

\[
\frac{\ell^2}{m^2} \int_0^{t_c} I^2(t) dt = \frac{Q(T_c)}{\rho^2}.
\]

For pure liners, a reasonable limit on \( T_c \) is the melting point \( T_m \), since the solid phase maintains a sharp shock front. The relation derived in Eq.(14) is still useful even when we deal with the realistic situations in which the temperature distribution is not uniform. In this case, we should set the limit on the temperature of the inside liner surface, denoted by \( T(t) \), which is the coolest at any time since the current has to diffuse radially inward. It is easy to see that we can still write

\[
\frac{1}{m^2} \int_0^t I^2(t) dt = \beta(T(t), m),
\]

except that \( \beta \) now has a weak dependence on \( m \). Once we set \( T \) to a limit \( T_c \) at \( t = t_c \), \( \beta(T_c, m) \) can be determined by using the 1-D MHD code to compute the left-hand side of Eq.(15). Later when we apply the above relation to optimize the liner mass, we only need to vary \( m \) in a narrow range around the optimal solution. We can therefore represent \( \beta(T_c, m) \) as a constant plus a small linear term in \( m \) and determine it by just two code simulations.

Among all metals, empirically aluminum has the highest value (only copper is a close second) in the ratio \( Q(T_m)/\rho \), where \( Q(T_m) \) is the action to the melting point. In terms of \( Q(T_m)/\rho^2 \), the aluminum is ahead of other heavier metals even more by an extra density factor. Using Eq.(14) the same can be said about the current integral on the left-hand side. We therefore conclude that the aluminum liner can be driven with a longer \( t_c \), before reaching the melting point, than any other pure liner (of higher density) having the same mass \( m \) and length \( \ell \). But longer imploding time before melt implies higher attainable velocity since all these liners are governed by the same implosion equation. ***** Using Eq.(10), we see that this \( 1/\rho \) advantage in attainable velocity for aluminum over materials of higher density is sufficient to ensure that the aluminum liner will also generate the highest shock pressure on any chosen target.

V. Composite Liner and Optimization

With the physical insights gained from our discussions on shock Hugoniot and Joule heating, the composite liner seems to be an excellent idea to improve the attainable shock pressure substantially over the pure liners. Clearly we still want to use
aluminum on the outside for carrying most of the driving current to retain its highest attainable velocity. For the inner layer we look for a material with high value in $\rho s$ to enhance the shock pressure, subject to some other criteria discussed below.

We find that platinum is the best impacting material for the composite liner, not just for its high density but also for its high melting point and electrical resistivity. Based on these criteria, other materials such as tungsten are equally satisfactory; but the fact that platinum can be electroplated is a big plus for fabrication. The Joule heating in the platinum layer is reduced dramatically since the current has to diffuse in through the aluminum. The shunt resistance of the platinum layer is two orders of magnitude higher than that of the aluminum, owing to a much higher resistivity and smaller cross section. This factor also helps to reduce the Joule heating in Pt after the current is diffused in.

The high melting point is an extra advantage since we can now drive the Al layer beyond its melting point while still keeping the Pt layer solid. Consequently, the composite liner can take considerably more Joule heating than a pure aluminum one with the same mass, and thereby achieving a corresponding higher velocity. How much we can push this advantage depends on the ability of the solid Pt layer to withstand the magnetically driven Rayleigh-Taylor instabilities in the melted Al layer. No definitive answer has been known so far from the 2-D MHD simulations. Hopefully, we will get some valuable clue from the upcoming megabar liner experiment at Pegasus.

For the composite liner, clearly the Joule heating constraint should be applied to the aluminum layer. In applying Eq. (15), therefore, $m$ is replaced by the aluminum mass $m_{Al}$ and $T_e$ is the limiting temperature set on its inner surface. The platinum mass $m_{Pt}$ should be kept as low as practical so that it will not reduce the liner velocity significantly. In the following, $m_{Pt}$ is a specified quantity.

To optimize the liner design means to find the liner mass and radius which maximize the shock pressure at a given target radius $r_e$, which is usually determined by the experimental requirement or diagnostic limitation. For a step-function current, the optimization can be carried out analytically using a good approximation. Due to limitation of space, this result will be presented elsewhere. For a general current wave form, we optimize the liner parameters numerically as follows: Taking the liner mass $m$ as the free parameter, we use Mathematica to solve Eq. (1) iteratively to find the correct initial radius $r_0(m)$ such that the solution for $r(t)$ satisfies $r(t_e) = r_e$, where $t_e(m)$ is given by the Joule heating constraint Eq. (15). The optimal mass is then the one which maximizes the collision velocity $v_c(m)$. The result is then used in the 1-D MHD code to compute the more accurate liner motion and detailed shock pressure history. In spite of the thin-liner approximation and using a motion-independent current in our formulation, the code simulations (with coupled circuit model) have demonstrated that we hardly need to refine the optimal liner parameters.

VI. Scaling Relations for Optimal Liners

We now proceed to derive a set of very useful scaling relations for the optimal liner parameters and performance. While these relations are derived under some idealized scaling conditions, they nevertheless provide us a valuable benchmark to make a good estimate on the maximum pressure achievable by an unexplored pulsed power regime that is usually approximately scalable to a known one.

It is important to realize that the liners optimized by the procedures as described in Sect. V do not scale in a simple way even though the driving currents are exactly scalable. For one thing, in realistic design the target radius $r_e$ is usually determined by experiment requirement so the ratio $r_e/r_0$ will not stay the same from one driving condition to the other. Furthermore, the optimization requirement also complicates the scaling. Therefore, some idealized conditions are necessary for us to deduce a set of simplified but adequate scaling relations. To this end, we have to assume first that the implosion distance ($r_0 - r_e$) scales like the optimal liner radius, $r_0$. In terms of the scaled distance introduced in Eq. (2),

$$z_e = 1 - r_e/r_0$$

stays constant. However, $r_e/r_0 \ll 1$ is generally valid for any optimized liner design; therefore $z_e$ is in fact approximately constant.

Next we require that the scaled collision time defined by $t_c = \omega r_c$ must also stay constant. This is equivalent to the condition that the kinetic energy for the liner at collision remains a constant fraction of the total driving energy. While we are unable to prove rigorously that this condition ensures the scaled liner parameters to remain optimal, physically it seems very reasonable. More importantly, its validity is justified a posteriori by comparing the scaling results with the some chosen ones obtained from actual optimization. We note
that the solutions for Eq.(2) with different values of \( \alpha \) do not intersect except at \( \tau = 0 \), so there is only one solution which passes through \( z = z_c \) at \( \tau = \tau_c \) as required. Using Eq.(3), the unique value of \( \alpha \) implies
\[
mr_0^2 \propto I_p^2 \omega^{-2}.
\] (16)
The Joule heating constraint given by Eq. (15) can be written as
\[
\frac{I_p^2}{m^2 \omega} \int_0^{\tau_c} P^2(r)dr = \beta(T_c, m),
\] (17)
when we ignore the small amount of platinum mass. Neglecting the weak \( m \)-dependence in \( \beta \), we get the scaling for the liner mass as
\[
m \propto I_p \omega^{-1/2}.
\] (18)
From Eqs.(16) and (18) we obtain the scaling for the liner radius as
\[
r_0 \propto I_p^{1/2} \omega^{-3/4}.
\] (19)
Finally, \( \dot{z}(\tau_c) \) is constant and using Eq.(4) we get
\[
v_c \propto I_p^{1/2} \omega^{1/4}
\] (20)
for the collision velocity and
\[
P \propto I_p^{1/2}
\] (21)
for the shock pressure. As mentioned earlier, we have \( m \nu_c^2 \propto I_p^2 \) to verify that the liner kinetic energy at collision is indeed a constant fraction of the total driving energy.

In applying these scaling relations, we can always, if necessary, refine the result by making the final correction due to the mild nonconstancy in \( \beta \) and minor deviation from strict scaling in either current or collision radius.

VII. Megabar Liners for Pegasus and Procyon

Our composite liner design has been adopted in the upcoming megabar liner experiments at both Pegasus and Procyon. The optimized liner to be fielded at Pegasus consists of 8 g of aluminum and 1 g of platinum, and it has an inner radius of 3 cm and a length of 2 cm. The liner design is based on the target radius set at 1 cm, using the maximum driving current as shown in Fig. 1. Notice that the current is well represented by a sine curve up to the collision time at 8.8 \( \mu \)s. From the 1-D simulation we expect a peak shock about 8 Mbar on a platinum target. In Fig. 2 we display the liner motion. The platinum layer is too thin to be resolved in the plot, so it merges with the inner radius of the aluminum layer as a single curve in the plot.

In Fig. 3 we show the velocity of the inner liner surface. The collision velocity is 8 mm/\( \mu \)s.

The temperature histories for the two liner layers are shown in Fig. 4. The dashed and solid (dotted and dot-dash) curves represent, respectively, the outer and inner surfaces of the platinum (aluminum) layer. We note that the outer aluminum surface begins to melt at 4 \( \mu \)s and so does the inner one about 7.5 \( \mu \)s, but the platinum layer remains way below its melting point before collision.

Finally, in Fig. 5 we plot the shock pressure profiles against the zone number at five different times. The platinum target covers the zones from 3 to 22, platinum in the liner from 23 to 42 and Al from 43 to 102. The solid curve is right after the collision at \( t = 8.883 \mu \)s, followed by the dotted and dot-dash ones at 2 ns interval and then the short and long dashed ones at one ns interval. The peak pressure is around 8 Mb and lasts slightly less than 6 ns. This peak shock duration is limited by the width of the Pt layer in the liner, which determines how long it takes the rarefaction wave coming from the Pt-Al interface to reach the collision interface.

Procyon is an explosive-driven pulsed power facility at LANL; its current wave form is approximately a step-function, with a sharp rise time about 1 \( \mu \)s and peak current of 23 MA. Based on this driving current and the target radius set at 1 cm, the optimized composite liner consists of 12 g
of Al and 1 g of Pt, and has a length of 2 cm and an inner radius of 3.3 cm. According to our 1-D simulation, this liner will generate a peak shock about 20 Mbar on a Pt target.

I Conclusions

We have proposed the aluminum-platinum composite liner design based on the physical insights obtained from our study of the behavior of the Hugoniots and electrical actions of various materials. The composite liner enables us to achieve a shock pressure several times over the best performance attainable from the solid aluminum liner. This improvement in peak shock pressures results mainly from the high density of the platinum layer, and to a less extent from the fact that we can keep the platinum layer in solid phase while driving the aluminum layer beyond its melting point and thereby achieving an even higher collision velocity than the pure aluminum liner. As we push this Joule heating limit, it is of practical interest to ascertain just how well the solid platinum layer is able to withstand the Rayleigh-Taylor instabilities developed in the melted aluminum layer. Hopefully the upcoming megabar experiments at both Pegasus and Procyon will shed some light on this question. We have developed a general formulation to optimize the composite liner design so that the shock pressure is maximized for any driving current. Using the thin-liner implosion equation decoupled from the liner motion, we have also derived a set of useful scaling relations for the optimal liner parameters and performance.

II Reference


3. Implosion for a thick liner coupled with the circuit model has been formulated by J. Parker: A Primer on Liner Implosions with Particular Application to the Pegasus II Capacitor Bank // Athena Technical Report, Los Alamos National Laboratory, November, 1993.


Figure 4. Temperature histories of the outer (dotted and dashed) and inner (dot-dash and solid) surfaces of the Al and Pt layers, respectively.

Elements // Sandia National Laboratory Report SAND-75-0041.

Figure 5. Shock pressure profile plotted over zone number right after the collision (solid), and at 2 (dotted), 4 (dot-dash), 5 (short dash), and 6 (long dash) ns later.