TRANSPORT CROSS SECTION
EXPRESSED IN TERMS OF PHASE SHIFT

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TRANSPORT CROSS SECTION EXPRESSED IN TERMS OF PHASE SHIFTS

The differential cross-section for elastic scattering is (see Mott and Massey Ch. II):

\[ d\sigma = 2\pi \left| f(\theta) \right|^2 \sin \theta \, d\theta \]

where:

\[ f(\theta) = \frac{1}{2ik} \sum_{n=0}^{\infty} \left( 2n + 1 \right) \left[ e^{2i\eta_n} - 1 \right] P_n(\cos \theta) \]

This leads to:

\[ \left| f(\theta) \right|^2 = \frac{1}{4k^2} \sum_{n,n'} \left( 2n + 1 \right) \left( 2n' + 1 \right) \left[ e^{2i\eta_n} - 1 \right] \left[ e^{-2i\eta_{n'}} - 1 \right] P_n P_{n'} \]

We can transform:

\[ \left[ e^{2i\eta_n} - 1 \right] \left[ e^{-2i\eta_{n'}} - 1 \right] = e^{i(\eta_n - \eta_{n'})} \left[ \left( e^{i\eta_n} - e^{-i\eta_n} \right) \left( e^{-i\eta_{n'}} - e^{i\eta_{n'}} \right) \right] \]

which leads to:

\[ \left| f(\theta) \right|^2 = \frac{1}{4k^2} \sum_{n,n'} \left( 2n + 1 \right) \left( 2n' + 1 \right) \cos (\eta_n - \eta_{n'}) \sin \eta_n \sin \eta_{n'} P_n P_{n'} \]

From this follows immediately the well-known formula for the total elastic cross section:

\[ \sigma_t = \int d\sigma = \frac{4\pi}{k^2} \sum_{n=0}^{\infty} \left( 2n + 1 \right) \sin^2 \eta_n \]

Another quantity of interest is the transport part of the elastic cross section:

\[ \sigma_{tr} = \int (1 - \cos \theta) d\sigma = \sigma_t - \int \cos \theta \, d\sigma \]

We need integrals:

\[ \int f_n P_n \, u \, du, \text{ when } u = \cos \theta \]
The only non-vanishing integrals are those for which \( n' = n \pm 1 \)

\[
\int P_n P_{n+1} u \, du = \frac{2n+2}{(2n+1)(2n+3)}
\]

Thus:

\[
\sigma_t - \sigma_{tr} = (3n/k^2) \sum (n+1) \cos (\eta_{n+1} - \eta_n) \sin \eta_{n+1} \sin \eta_n
\]
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