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INTRANUCLEAR CASCADE CALCULATION (1964)
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Work done by:
R. L. Bivins
D. R. F. Cochran
S. L. Whetstone
J. K. Wooten

Report written by:
S. L. Whetstone

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ABSTRACT

This report contains the information presented by the writer at the Gordon Research Conference on Nuclear Chemistry on June 23, 1964.

Described briefly are the changes made from the original version of the LASL Monte Carlo intranuclear cascade calculation published in Phys. Rev. 110, 185, 204 (1958) by Metropolis et al. Changes include improved pion-production dynamics and the introduction of a Fermi-type non-uniform nuclear density distribution. A few intercomparisons between the results of the present and the original version are made. Some comparisons are also given between the present calculations and experiment.

ACKNOWLEDGMENTS

The present version of the calculation represents the work of R. L. Bivins and J. K. Wooten of T-7, and D. R. F. Cochran and the writer of P-11. A number of others have contributed in a consultant capacity. The administrative support of R. B. Lazarus of T-7 and D. E. Nagle of P-11 is acknowledged.
INTRODUCTION

This is a report of the status of the LASL Monte Carlo intranuclear cascade calculation as it existed during the latter two-thirds of 1964 and as it was used in a number of calculations of importance to the proposal\(^1\) for the Los Alamos Meson Production Facility (LAMPF). Described briefly are the changes made from the original version of the calculation that was coded for the Maniac I computer and reported in the publications of Metropolis et al. in 1958.\(^2\) Intercomparisons are also given in the present report between results calculated from the current code on the Maniac II computer and published results from the earlier Maniac I version; some differences are noted. Two examples of agreement of the present calculation with experimentally observed outgoing particle energy spectra are also given.

The earlier calculations\(^2\) have been revived and improved, in the several respects to be described, primarily to provide better input data for calculations of the shielding required for the LAMPF facility, and secondarily to provide more information on the production of pions from various nuclei incorporated in targets to be placed in the proton beam of the accelerator. Important information on the neutron production from such targets has also been of value for estimating backgrounds to be expected in certain experimental arrangements, and particularly for designing experiments that might take advantage of the unparalleled large intensities of high energy neutrons in the 600 to 800 MeV range.\(^3\) The calculation is also of importance to the design of moderating facilities for the production of very high fluxes of neutrons of much lower energies, in direct competition with nuclear reactors.\(^4\)
The maximum proton energy ($\sim 800 \text{ MeV}$) proposed for the LAMPF accelerator permitted an upper bound of about $1 \text{ GeV}$ to be placed on the calculation. This allows multiple pion production in individual nucleon-nucleon collisions to be ignored, as well as the production of additional pions in subsequent pion-nucleon interactions. The urgency of the shielding design problem, and the consequent need for information on the fast neutron production, has confined efforts thus far to the calculation of the results of the initial, high-energy-cascade part of the nuclear reaction.

At the conclusion of the 1958 (II) paper the following improvements were suggested:

1. a pion-nucleus potential,
2. a diffuse nuclear boundary,
3. a better approximation to the kinematics of pion production events, and
4. changes in the assumptions about the pion absorption mechanism.

Another obvious improvement would be better cross-section data.

In the current revision of the calculation, the greatest effort has been devoted to the improvement of the pion-production dynamics. A diffuse nuclear boundary for the nuclear density distribution has also been introduced. The only changes in the cross-section tables made thus far are in the n-$p$ (and $p-n$) cross sections $\sigma_{ij}(N \pm N)$ above $E_N = 350 \text{ MeV}$. We have been advised to assume the pion-nucleus potential to be zero for lack of better knowledge; and, as yet, suggested changes in the pion absorption mechanism have not been made.

**PION PRODUCTION**

In the Maniac I calculation the dynamics of single pion production from nucleon-nucleon collisions were specified in terms of a highly simplified model: The three final particles were assumed to have equal momenta in the center-of-mass system and the pion direction was randomly selected to give, statistically, isotropic emission.
A slightly more complicated model, easily adaptable to Monte Carlo techniques and giving much better agreement with experimental data, is the so-called "isobar (or isobaric nucleon) model" originated by Lindenbaum and Sternheimer.6 This model describes the reaction as a sequence of two-body interactions, the first producing an "excited" nucleon (isobar), the second giving the pion in the rapid decay of the isobar. In the energy region within about 600 MeV of the pion production threshold, this isobar is identified with the state of the pion-nucleon system, characterized by total isotopic spin $T = \frac{3}{2}$, and total angular spin $J = \frac{3}{2}$, to which is attributed the strong resonance in the pion-proton scattering cross section observed at about 160 MeV pion center-of-mass kinetic energy. The arguments of Lindenbaum and Sternheimer then lead to a production cross section for this isobar which is proportional to the $\sigma(n^+p)$ scattering cross section times a two-body phase space factor. The model is summarized in Fig. 1, which includes the analytical expression for $\sigma(n^+p)$ used in the calculation (note that a factor $\chi^2$ has been neglected) as well as the derivation of the branching ratios obtained from the assumption of the conservation of isotopic spin (charge independence).

To complete the description of the production process it is necessary to make assumptions concerning the angular distributions $P(\theta_p^i)$, for the production of the isobar, and $P(\theta_p^d)$, for the decay of the isobar. These distributions are not specified by the Lindenbaum and Sternheimer model, but can be chosen to give agreement with experiment. The angular distribution of the isobar production can be studied independently to the extent that the branching ratio (see Fig. 1) for the number of neutrons produced in the first step of the reaction $p+p$ (isobar production) to the number produced in the subsequent isobar decay is 9:1. Comparisons of the results calculated from the model, assuming center-of-mass angular distributions of the form $P(\theta_p^i) \sim a + b \cos^2 \theta_p^i$, with the observed neutron energy spectra7 are shown in Fig. 2. These indicate that a reasonable fit to the experimental observations is given by $a = 1$ and
\( b = 3 \) for an incident proton energy of about 1 GeV. Since this distribution proved to be as good as any other tried for \( P(\theta_p) \) in other comparisons, it was adopted for the present calculations for all incident energies. The simplest one-pion-exchange field theory would predict a dependence of \( P(\theta_p) \) on incident energy, and this may be used in future refinements of the model.

The other angular distribution \( P(\theta_D) \) for the decay of a particle with spin \( 3/2 \) and even parity should be \( \sim 1 + 3 \cos^2 \theta_D \), with \( \theta_D \) the angle between the spin of the isobar and the final momentum of the resulting nucleon. Since the polarization of the isobar is not known, the simplest assumption would be to assume no polarization, and hence an isotropic emission. This appears to be borne out by the results of comparisons with experimentally determined pion energy spectra. A number of comparisons are shown in Figs. 3 to 8. Shown are all of the experimental data on pion energy spectra useful to us; the difficulty in obtaining reliable fits is obvious. The data at about 1 GeV even appear to be inconsistent, although the Yuan and Lindenbaum measurements\(^9\) (at the largest angle) suffer from large statistical uncertainty and the Gailbraith et al. measurements,\(^{10}\) from a lack of information in the low-energy part of the spectra, which makes it difficult even to identify the energy of the peak of the distribution.

In the intranuclear cascade calculation the isobars are assumed always to decay before having moved any appreciable distance in the nucleus, and therefore they are never allowed to interact with any of the nuclear constituents or to have their decay properties affected in any way by the presence of the nuclear environment. Z. Fraenkel\(^{11}\) has called attention to the probable appreciable probability for interactions of the isobars and to some important consequences (particularly in the pion absorption mechanism).
THE DIFFUSE NUCLEAR BOUNDARY

The earlier calculation has been improved by the use of a diffuse-edge nuclear density. A Fermi-type distribution is used with the parameters obtained\(^{12}\) from the nuclear charge density distributions determined by the electron scattering experiments:\(^{13}\)

\[
\rho(n) = \rho_1 \left[ 1 + \exp \left( \frac{r - c}{Z_1} \right) \right]^{-1},
\]

\[c = 1.07 \, A^{1/3} f,\]
\[Z_1 = 0.55 \, f.\]

\(\rho_1\) is from normalization, and spherical nuclei are assumed. The tail of the distribution is neglected for \(r > 1.5R_c\), \(R_c = 1.3 \, A^{1/3} f\). The same distribution was used in the intranuclear cascade calculations of Bertini,\(^{12}\) except that the continuous function was approximated by a three-step function. In the present calculation the continuous distribution is used to define the density at a given radius, but the density is then assumed to be constant as a particle is advanced a small distance (typically 0.1 \(R_c\) or less) in the Monte Carlo procedure.

The nuclear density distribution is expected to have a particularly strong effect on the escape of pions produced in complex nuclei. This effect has not yet been studied in any detail.

OTHER DETAILS OF THE PRESENT CALCULATION

The momentum distribution of the nucleons in the nucleus, as in the previous calculation,\(^2\) is assumed to be that appropriate to a zero-temperature Fermi distribution. Although a diffuse-edge nuclear density is used, the momentum distribution is still assumed to be constant throughout the nuclear volume and independent of nuclear mass. The Fermi energy used has been \(\gamma_F = 1.03 \, (28.2 \, \text{MeV})\). This over-simplification should be corrected in future revisions and a dependence on nucleon charge included.
The nuclear potential has thus far been ignored in the present calculation. Allowance for square-well type potentials (as assumed in the Maniac I calculation) can easily be made by subtracting an appropriate potential energy from the kinetic energy of the incident particle and from the kinetic energy of each escaping particle. To make the potential a function of the density at each point in the nucleus would slow the calculation excessively although some compromise might be useful for better accuracy particularly for the lower energy products.

The "cutoff energy," below which nucleons were no longer followed in the calculation and hence assumed not to escape from the nucleus in the cascade calculation, has been arbitrarily set at $\gamma_c = 1.05$ (47 MeV, inside the nucleus). There was no cutoff used for pions.

It should be noted that none of the following possibly important effects has been considered in the present version of the calculation: refraction and reflection of cascade particles, existence of nucleon clusters (except in the new standard treatment of pion absorption on nucleon pairs), and the depletion of the Fermi gas during the cascade.

**COMPARISONS WITH EXPERIMENTAL DATA ON PION PRODUCTION FROM COMPLEX NUCLEI**

To give a sample of the sort of agreement found with experimental data on pion production from complex nuclei, two figures are included. The first, Fig. 9, shows the calculated and observed energy spectra of neutrons produced at 0° and at 18° when Be⁹ is bombarded by approximately 680 MeV protons. The agreement is qualitatively good although there is a definite discrepancy in the width and position of the high-energy peak. Figure 10 shows the positive and negative pion energy spectra from the reaction p + C¹² at $\theta_L = 56°$, $E_p \sim 600$ MeV. The agreement between experiment and the calculation is astonishingly good.
COMPARISONS OF SOME RESULTS FROM THE MANIAC I AND MANIAC II CALCULATIONS

Figure 11 compares the average number of cascade nucleons per nonelastic event as a function of incident proton energy for the targets Cu and U. Here it should be noted that slightly different cutoff energies (2 MeV lower for Cu, 4 MeV higher for U) were used in the Maniac I calculation, so that small differences in absolute numbers of cascade particles are not significant. The energy dependences are seen to be very much the same. The striking difference appears in the neutron/proton ratio where the present calculation predicts somewhat smaller ratios from copper but very much smaller ratios (down by almost a factor of 1/2) from uranium. This result is of particular importance to the contemplated high-energy accelerator "neutron factories." Special note of the large cascade neutron emission from heavy nuclei was made in the published results of the Maniac I calculation where the effect was attributed to the higher n-p scattering cross sections compared to the p-p and n-n cross sections and the effect of choosing the same cutoff energy for both neutrons and protons. The present calculation used the same scattering cross sections (below 350 MeV) and also adopted the same cutoff energy for neutrons and protons. Figures 12 and 13 show the angular distributions and energy spectra of the cascade neutrons and protons for 910 MeV protons on $^{238}$U from the present calculations. In particular it is not obvious from the energy spectra that a change in the cutoff energy would substantially affect the overall neutron-proton ratio.

The pion production is compared in Figs. 14 and 15. Here the $\pi^+/\pi^-$ ratios are seen to be in substantial agreement. The production from very low A nuclei (not calculated in the earlier work) would be expected to reflect more strongly, in the ratios of the emitted pions, the somewhat different branching ratios used in the nucleon-nucleon production in the two calculations. The present calculations give a larger average number of pions per nonelastic event at about 450 MeV incident proton energy but nearly the same as the Maniac I calculation at ~ 925 MeV.

11
Lindenbaum-Sternheimer Isobaric Nucleon Model

(Pion production in nucleon-nucleon collisions)

Single pion production → Single excited nucleon, $N^*$: $T(N^*) = 3/2$, $T(N) = 1/2$. $\therefore T_{\text{iso}}(N^*+N) = 1$

From Conservation of Isotopic Spin, $T$:

![Diagram of pion production reactions](image)

Production of $N^*$:

Assumption: $dP_{N^*}(m) = F(E, m) \cdot \sigma_{N^*}(m) dm$

\[
\begin{align*}
\sigma_{N^*}(m) & = \text{intrinsic prob. of forming } N^* \\
& = \text{total c.s. for } \pi + N \text{ in } T = 3/2, J = 3/2 = \sigma_{\pi^0p}(m) \\
F(E, m) & \equiv \frac{\bar{E}_N \bar{E}_{N^*} E_{N^*}}{E} \propto \text{Two-body phase-space factor for product } N + N^* \\
m & = \text{total energy in c.m.s. for } \pi + N \text{ interaction} = \text{rest mass of } N^* \\
E & = \text{total energy in c.m.s. for } N + N^* \text{ interaction} = \bar{E}_N + \bar{E}_{N^*} \\
\end{align*}
\]

Use: $\sigma_{\pi^0p}(m) \propto \frac{\Gamma^2}{(E - E_0)^2 + (\Gamma^2/4)}$

\[
\begin{align*}
\bar{E}_0 & = 1240 \text{ Mev} \\
\Gamma & = 90 \text{ Mev}
\end{align*}
\]

Figure 1. Summary of the Lindenbaum-Sternheimer isobaric nucleon model for single pion production in nucleon-nucleon collisions.
Figure 2. Comparison of isobaric nucleon model calculations with the neutron energy spectra from the reaction $p+p\rightarrow n+p+\pi^+$ at 970 MeV. Data (heavy dashed line) are from Batson et al. Peripheral model calculation of Selleri is also shown.
Figure 3. Comparison of isobaric nucleon model calculation with observed pion energy spectra of Meshcheriakov et al.\textsuperscript{16} from $p^+p-n^+p+p^+$ at 657 MeV (not 670 MeV as shown above on Fig.) and $\theta_\pi = 24^\circ$ Lab.
Figure 4. Comparison of isobaric nucleon model calculation with observed pion energy spectra of Meshkovskii et al.\textsuperscript{17} from $p+p\rightarrow n+p+\pi^+$ at $670$ MeV and $\theta_{\pi} = 38^\circ$ Lab.
Figure 5. Comparison of isobaric nucleon model calculation with observed pion energy spectra of Meshkovskii et al. from $p+p\rightarrow n+p+p+\pi^+$ at $670$ MeV and $\theta_\pi = 56^\circ$ Lab.
Figure 6. Comparison of isobaric nucleon model calculation with observed energy spectra of Gailbraith et al. from $p+p\rightarrow n+p+\pi^+$ at 1.02 GeV and $\theta_\pi = 15^\circ$ Lab.
Figure 7. Comparison of isobaric nucleon model calculation with observed pion energy spectra of Gailbraith et al.\textsuperscript{10} from \( p+p-n+p+\pi^+ \) at 1.02 GeV and \( \theta_\pi = 25^\circ \) Lab.
Figure 8. Comparison of isobaric nucleon model calculation with observed energy spectra of Yuan and Lindenbaum from $p+p \to n+p+\pi^+$ at 1.0 GeV and $\theta_\pi = 32^\circ$ Lab.
Figure 9. Comparison of Maniac II intranuclear cascade calculation with observed neutron energy spectra of Kiselev and Fliagin\textsuperscript{14} from p+Be\textsuperscript{9} at 680 MeV and $\theta_n = 0^\circ$ and $18^\circ$ Lab.
Figure 10. Comparison of Maniac II intranuclear cascade calculation with observed positive and negative pion energy spectra of Azhgirey et al. from $p+C^{12}$ at 670 MeV and $\theta = 56^\circ$ Lab.
Figure 11. Comparison of Maniac I and Maniac II calculations: 
a) average number of cascade nucleons per nonelastic event and 
b) the neutron/proton ratio both as functions of incident 
proton energy $T_p$ and for Cu$^{64}$ and U$^{238}$ targets.

AVERAGE NUMBER OF CASCADE NUCLEONS
/ NONELASTIC EVENT

NEUTRON / PROTON RATIO

$T_p$ (MeV)

MANIAC I: Cu

MANIAC I: U

II: Cu

II: U
Figure 12. Maniac II calculation of neutron and proton angular distributions from 910 MeV protons incident on $^{238}$U (assumes a square-well potential of 30 MeV; protons just inside nuclear boundary have 940 MeV).
Figure 13. Maniac II calculation of neutron and proton energy spectra from 910 MeV protons incident on $^{238}$U. "E" is the energy of the particles just before escaping from the nuclear boundary; shift distribution by -30 MeV to obtain outgoing energy spectra.
Figure 14. Comparison of Maniac I and Maniac II calculations:

a) average number of pions per nonelastic event and

b) $\pi^+ / \pi^-$ ratio, both as functions of incident proton energy $T_p$ and for $^{64}$Cu and $^{238}$U targets.
Figure 15. Comparison of Maniac I and Maniac II calculations:
a) average number of pions per nonelastic event and
b) $\pi^+/\pi^-$ ratio, both as functions of target mass number $A$, and for incident proton energies of about 445 and 925 MeV.
REFERENCES


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5. Z. Fraenkel, private communication.


8. A. S. Goldhaber, private communication.


