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THE TRANSPORT CROSS SECTION OF HELIUM FOR FAST NEUTRONS

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ABSTRACT

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On the basis of experimental work by Staub and others on the back-scattering of neutrons by helium between 0.5 and 2 Mev, which shows resonance peaks at about 1 and 1.2 Mev, Bloch has given a Breit-Wigner formula for the cross section in this region. After generalization of this formula to energy regions outside the immediate resonance, there is here derived from it an expression for the transport cross section of helium, which is also shown graphically for reasonable values of the experimental constants.
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The scattering of neutrons by helium has been investigated by Staub and Stephens\(^1\)) and Staub and Tatel\(^2\)). They measured the back scattering in a region between 0.5 and 2 Mev and they found two resonance peaks at about 1 Mev and 1.2 Mev.

Bloch has interpreted these results as due to resonance with a \(P_{3/2}\) and a \(P_{1/2}\) level of \(\text{He}_\text{g}\). The Breit-Wigner formula (formula (1) in ref. 2) describing this resonance can be adjusted to fit the experimental values by a suitable choice of the constants, which are the width of the levels \(\Gamma\), the resonance energies, \(E_{3/2}\) and \(E_{1/2}\) and the residual \(s\)-scattering cross section \(\sigma_0\) as a function of the energy. The latter contribution is only a small correction in the resonance region. The experimental curves do not permit a decision between an inverted or a normal doublet but theoretical investigations by Dancoff\(^3\)) make a normal doublet \((E_{3/2} > E_{1/2})\) very probable. Under these conditions, the following values have been found by Staub and Tatel to fit the experiments on back scattering: \(\Gamma = 0.4\) Mev, \(E_{3/2} = 1.35\) Mev, \(E_{1/2} = 1.05\) Mev, \(\sigma_0 = 1.5 \times 10^{-24}\) cm\(^2\), and negative phase for the scattering.

From these data it is possible to calculate the transport average of the cross section. In order to obtain this cross section over a region larger than the immediate resonance the following generalizations have been made: The energy dependence of the width \(\Gamma\) has been taken into account. Since this is a \(p\)-level the neutron width was assumed to have the form:

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1) Staub and Stephens, Phys. Rev. 55, 131 (1939)
2) Staub and Tatel, Phys. Rev. 59, 820 (1940)
3) Dancoff, Phys. Rev. 56, 384 (1939)

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\[ \Gamma = \text{Const.} \sqrt{E} \frac{(kR)^2}{[1 + (kR)^2]} \]

The factor, \( (kR)^2/[1 + (kR)^2] \) is the reciprocal of the intensity, at the nuclear radius \( R \), of an outgoing p-wave with a wave number \( k \), the intensity being normalized to unity for \( r \to \infty \). With a nuclear radius of \( 3 \times 10^{-13} \text{ cm} \), one obtains

\[ \Gamma = \sqrt{E} \left( \frac{3}{1 - 2/\Gamma} \right) \Gamma', \]

where \( \Gamma' \) is the width at 1 Mev and \( E \) is measured in Mev.

In order to determine \( \sigma_0 \), two other experimental values are used, namely, the thermal cross section of 1.5 barns determined by Carrel and Dunning\(^4\) and the back scattering cross section at 2.8 Mev measured by Barschall and Kanner\(^5\) also of about 1.5 barns. At this energy there is still a considerable contribution from the resonance levels and an evaluation of the Breit-Wigner formula (1) in ref. (2) shows that the minimum cross section at 2.8 Mev with the above values of \( \Gamma' \), \( E_3/2 \) and \( E_1/2 \) is 1.9 barns. This would correspond to a \( \sigma_0 = 0.77 \) barns at that energy. The probable course of \( \sigma_0 \) is therefore: 1.5 barns at \( E = 0 \) and a slow decrease to 0.77 at 2.8 Mev.

The transport cross section at an energy \( E \) can be calculated from expression (1) in ref. (2) and is given by

\[
\sigma_t = 2 \pi \chi^2 \left\{ \frac{5}{3} \sin^2 \delta_0 - \frac{77}{60} \sin \delta_0 \cos \delta_0 \left[ \frac{2(\epsilon - x)}{(\epsilon - x)^2 + 1} + \frac{\epsilon + x}{(\epsilon + x)^2 + 1} \right] \right. \\
- \frac{77}{60} \sin^2 \delta_0 \left[ \frac{2}{(\epsilon - x)^2 + 1} + \frac{1}{(\epsilon + x)^2 + 1} \right] + \frac{5}{3} \frac{4x^2}{[(\epsilon - x)^2 + 1][(\epsilon + x)^2 + 1]} \\
+ \frac{251}{140} \left[ \frac{(\epsilon - x)^2 + 1 + 2[(\epsilon + x)^2 + 1] - 4x^2}{[(\epsilon - x)^2 + 1][(\epsilon + x)^2 + 1]} \right] \right\}
\]

\[^4\) Carrel and Dunning, Phys. Rev. 54, 541 (1938)\]
\[^5\) Barschall and Kanner, Phys. Rev. 58, 550 (1940)\]
where
\[ \epsilon = \frac{2}{\Gamma} \left( \frac{E_{3/2} + E_{1/2}}{2} - E \right) \]
\[ x = \frac{E_{1/2} - E_{3/2}}{\Gamma} \]
\[ \Gamma = \sqrt{E} \frac{3}{1 + 2/E} \]

and \( \lambda \) = the wave length of the neutron in cm in the center of gravity system
\[ = \sqrt{0.3235 \times 10^{-22}/\text{E}_{\text{meV}}} \]

The value of \( \sigma_0 \) is given by the heavy curve in Fig. 1 for \( \Gamma = 0.4 \)
and \( \sigma_0 = 0.77 \) barns throughout. Also \( E_{3/2} = 1.35 \) and \( E_{1/2} = 1.05 \). Some points
were computed for \( \Gamma = 0.3 \) and 0.5 and for \( \sigma_0 = 1.5 \) and 0.77 barns in order to
estimate the dependence on these magnitudes. The latter values represent about
the limits which are still possible on the basis of present evidence. The
circles in the figure represent the largest deviations from the heavy curve
obtainable within these limits. The transport cross sections of scattering
in deuterium and hydrogen are added for comparison.
Transport Cross Section for Neutron Scattering of He, H, and H₂. The points indicate reasonable upper and lower limits possible for the He curve, consistent with present experimental data.