Potential Fatigue Problems in First-Wall Laser-Controlled Fusion Reactors

by

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ABSTRACT

In this report an estimate is made of possible fatigue problems that are likely to be encountered in the metal first wall of laser-controlled thermonuclear (fusion) reactors (LCTRs). A review is given of different sources that produce fluctuating stresses. The largest stresses occur in the form of short wavelength, thermoeelastic stress pulses that start at the inner wall and move into the interior of the metal shell. It is concluded that it is likely (as suggested by Zukas) that these short wavelength stress pulses are attenuated rapidly with distance. If they are indeed attenuated strongly, high amplitude stress pulses should not cause fatigue failure of first wall metal shells. (Other sources produce cyclic stresses that are of too low an amplitude to cause fatigue failure.) Measurements are needed of the internal friction of possible first wall materials taken under conditions of high stress amplitude, and radiation damage will indicate which materials can attenuate rapidly high amplitude, short wavelength stress pulses and thus be resistant to fatigue failure. Experiments also are needed to determine if fatigue crack growth can be produced by 14-MeV neutron collision cascades. Whether such crack growth can occur is only a theoretical speculation at the present time. If it does occur after a year's operation first-wall material may contain a high density of millimeter length cracks. (Stresses are calculated for a spherical niobium shell of radius 2 m and thickness 1 cm. These stresses generally can be converted to the values appropriate to a niobium shell of radius R by multiplying by the factor \((R/R_0)^2\), where \(R_0 = 2\) m. Since the elastic moduli of other metals considered for a first-wall (molybdenum, vanadium, stainless steel) are of the same order as the modulus of niobium, the calculated stress values for niobium will be approximately correct for all these materials.)

I. INTRODUCTION

In proposed designs of laser controlled thermonuclear reactors (LCTRs) deuterium-tritium micropellets are fusion burned at a rate between 1 and 10 Hz. Each pellet produces about 100 MJ of energy in its explosive burn. It is inevitable that the material out of which an LCTR is constructed will be subjected to fluctuating stresses because of these periodic bursts of fusion-created energy. Since fluctuating stresses can cause fatigue failure of material, potential fatigue problems may exist for LCTRs. It is the purpose of this informal report to examine the potential fatigue problems of the first-wall material of LCTRs. A review is given first of the sources of the fluctuating stresses and the estimated magnitude of these stresses. This review is followed by an estimate of the seriousness of the fatigue damage likely to be suffered by the first-wall material.

II. SOURCES OF FLUCTUATING STRESSES

An estimated energy release spectrum from a 99-MJ microexplosion of a DT pellet (taken from Refs. 1 and 5) is given in Table I. Suppose that, periodically, DT pellets are fusion burned at the center...
TABLE I

TYPICAL ENERGY-RELEASE MECHANISMS FROM A 99-MJ DT PELLET MICROEXPLOSION

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Fraction of Total Energy Release</th>
<th>Particles per Pulse</th>
<th>Average Energy per Particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>X rays</td>
<td>0.01</td>
<td></td>
<td>~ 4 keV peak</td>
</tr>
<tr>
<td>Alpha particles that escape plasma</td>
<td>0.07</td>
<td>2.2 x 10^19</td>
<td>2 MeV</td>
</tr>
<tr>
<td>Plasma kinetic energy</td>
<td>0.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>alpha particles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>deuterons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>tritons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neutrons</td>
<td>0.77</td>
<td>3.3 x 10^19</td>
<td>14.1 MeV</td>
</tr>
</tbody>
</table>

of a spherical first-wall metal shell of radius R and thickness ΔR. What cyclic stresses are produced within the metal shell by microexplosions of the pellets?

A. Stress Pulses Produced by Momentum Transfer

An upper limit can be set to the magnitude of the stress pulse produced by the transfer to the shell of the momentum of the neutrons produced in a DT pellet burn if it is assumed that all the neutrons are absorbed uniformly within the metal shell. (Actually most of the neutrons will be absorbed in the liquid lithium blanket that surrounds the first wall.) In an incremental solid angle δΩ the momentum carried by the neutrons is equal to \( N_n m_n v_n \delta \Omega / 4 \pi \), where \( N_n \) is total number of neutrons created in a microexplosion, \( m_n \) is the mass of a neutron \((m_n = 1.7 \times 10^{-27} \text{ kg})\), and \( v_n \) is the velocity of a neutron. The neutrons are produced in a time interval \((< 10 \text{ ps})\) that is short compared with the time required for a sound wave to traverse the thickness of the metal shell \((\sim \Delta R (\rho / \mu)^{1/2} \sim 5 \mu \text{m} \text{ for } \Delta R = 1 \text{ cm} \text{ where } \rho \text{ is the density of the material of the shell and } \mu \text{ is an appropriate elastic constant})\). Immediately after the neutrons are absorbed in the metal a segment of solid angle \( \delta \Omega \) of the shell has a total momentum given by \( \rho \delta \Omega R^2 \Delta R v_n \), where \( v \) is the radial velocity of the shell. It is assumed that \( \Delta R \ll R \).

By conservation of momentum

\[
v = \frac{N_n m_n v_n \delta \Omega}{4 \pi \rho R^2 \Delta R}, \tag{1}\]

As the metal shell expands it becomes stressed elastically and the radial velocity decreases. The shell is set into a ringing motion. In dynamic elasticity problems the relationship \((1/2)\rho v^2 = (1/2) \sigma^2 / \mu\) holds for any volume element because of the principle of conservation of energy. Here \( v \) is the maximum velocity of the volume element, \( \sigma \) is the stress amplitude to which the volume element is subjected, and \( \mu \) is an appropriate elastic constant. Thus the maximum stress that can be produced by the transfer of all the momentum of the neutrons to the shell is of the order of

\[
\sigma \approx \frac{N_n m_n v_n (\mu / \rho)^{1/2}}{4 \pi R^2 \Delta R} \tag{2}\]

For a shell made out of niobium \((\rho = 8.6 \times 10^3 \text{ kg m}^{-3} \text{ and } \mu = 100 \text{ GPa}^{-2})\) of radius \( R = 2 \text{ m} \) and thickness \( \Delta R = 1 \text{ cm} \) the stress given by Eq. (2) for \( N_n = 3.3 \times 10^{19} \) neutrons per pellet of 14-MeV energy \((v_n = 5.2 \times 10^7 \text{ ms}^{-1})\) is only 32 kN m\(^{-2}\) (0.32 bar or 4.5 psi), a stress amplitude of trivial magnitude. The alpha particles that strike the inner wall of the shell will penetrate a distance \( d_\alpha \) that is small compared with the shell thickness \((d_\alpha \sim 2 \mu \text{m} \text{ for } 2-\text{MeV alpha particles in niobium})\). The transfer of momentum of the alpha particles will occur in a thickness of material of order of \( d_\alpha \). A stress wave will be set up. The stress amplitude of this wave will be of the order of

\[
\sigma \approx \frac{N_n m_\alpha v_\alpha (\mu / \rho)^{1/2}}{4 \pi R^2 d_\alpha}, \tag{3}\]

where \( m_\alpha \) and \( v_\alpha \) are the mass and velocity of the alpha particle, respectively.
where $N_a$ is the number of alpha particles per pellet that penetrate the first wall, $m_a$ is the mass of an alpha particle ($m_a \approx 4 \times 10^{-21}$ kg), and $v_a$ is the velocity of the alpha particles ($v_a = 9.8 \times 10^6$ m/s for 2-MeV alpha particles). For $N_a = 2.2 \times 10^{19}$ particles per pellet, the stress given by Eq. (3) is $\sigma = 80$ MN/m$^2$ (800 bars or 12 ksi).

The wavelength of the stress wave is of the order of $d_a$. The amplitude of the stress wave will decrease as it moves through the metal shell because of energy losses caused by internal friction (damping). If the logarithmic decrement is represented by $\delta$, the stress amplitude will equal $\sigma \exp(-\delta \alpha x)$ where $x$ is the distance traveled by the stress wave and $\sigma$ is given by Eq. (3). For the rather large value for the decrement of $\delta = 0.1$, the stress wave would die out in a distance of the order 0.02 mm. For a log dec. of $\delta = 10^{-3}$, only a moderately large value for the internal friction, the stress amplitude would not decrease appreciably in traversing a shell of thickness of 1 cm.

The x-ray photons of 4-keV energy would penetrate the shell to a distance $d_\gamma$ of the order of 10 $\mu$m. Their momentum, for a total energy of 1 MJ per pellet, would produce a stress pulse of only about $3.2 \times 10^6$ N/m$^2$ (0.32 bar or 4.5 psi).

### B. Stress Pulses Produced by Temperature Gradients

The energy deposited by the neutrons, alpha particles, and photons raises the temperature of the metal shell and produces thermoelastic stresses. An exact analytical solution for this problem is being developed by Roy Axford. He has calculated the thermoelastic tensile stress that exists at the outer surface of the shell immediately after a micro-explosion has taken place. His results for four metals are given in Table II. The calculations are made assuming that $R = 2$ m, $\Delta R = 1$ cm, the 2-MeV alpha particles and the photons together deposit 8 MJ of energy at the inner surface of the shell, and 4 MJ of the 77 MJ of the total energy of the neutrons is deposited within the metal shell. The stresses are calculated for the case in which inertial effects are ignored. However, further development of the analysis will bring in these effects. The analysis predicts infinite (compressive) stresses to exist initially at the inner surface of the shell because of the assumption that all the energy of the alpha particles and the photons is deposited on this surface rather than in a layer of finite thickness.

An estimate can be made of the effect of the inertial term in the equations of thermoelasticity on the thermoelastic equations. For example, let $E_n$ represent the amount of energy of the neutrons that is deposited uniformly within the metal shell ($E_n = 4$ MJ). This sudden deposition of energy will raise the temperature of the shell by the amount $\Delta T$ given by

$$\Delta T = \frac{E_n}{4\pi R^2 \Delta R C},$$

where $C$ is the specific heat of the metal (for niobium $C = 290$ J/kg $K^{-1}$). For the values of $R = 2$ m and $\Delta R = 1$ cm, the instantaneous temperature rise $\Delta T$ in a niobium spherical shell is equal to 3 K. If the surfaces of the spherical shell were constrained to remain in their initial positions, this temperature rise would produce a thermoelastic (compressive) stress of the order of $\sigma = \mu \alpha T = 0.26$ MN/m$^2$ (2.6 bars or 38 psi). Here $\alpha$ is the linear coefficient of thermal expansion which for niobium is equal to $7 \times 10^{-6}$ K$^{-1}$.

When the surfaces of the shell are not constrained in a fixed position, the stress produced by the neutrons will be equal to $\mu \alpha T$ but an unloading stress wave will move from each free surface. Tensile stresses occur after the two unloading waves, which move from the inner and outer

<table>
<thead>
<tr>
<th>Metal</th>
<th>Stress $^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>niobium</td>
<td>1410 psi (10 MN/m$^2$)</td>
</tr>
<tr>
<td>stainless steel</td>
<td>3441 psi (24 MN/m$^2$)</td>
</tr>
<tr>
<td>molybdenum</td>
<td>2475 psi (17 MN/m$^2$)</td>
</tr>
<tr>
<td>vanadium</td>
<td>1580 psi (11 MN/m$^2$)</td>
</tr>
</tbody>
</table>

$^a$ If the energy of the cavity gas is added to the surface heating of the inner wall the stress level is approximately doubled.
surfaces, cross each other. The stress level in these waves, of the order of 0.26 MN m\(^{-2}\), is of trivial magnitude.

The 4-keV x-ray photons deposit their energy in a layer of thickness \(d_y = 10 \mu m\). The temperature rise that is produced in this layer by photons with a total energy \(E_\gamma\) is

\[
\Delta T = E_\gamma / 4\pi R d_y \rho C .
\]

For the values \(E_\gamma = 1\) MJ and \(R = 2\) m, the temperature rise for niobium calculated using Eq. (5) is \(\Delta T = 800\) K. This temperature rise corresponds to a stress of magnitude \(\sigma = \mu \Delta T = 0.56\) GN m\(^{-2}\) (5.6 kbars or 81 ksi). A stress pulse (an initial compressive component followed by a tensile component) of this order of magnitude will travel into the shell. In the Appendix is given a brief analysis of the stress pulse. The value of the log decrement undoubtedly is high at this stress level because the yield stress of niobium (13,000 psi) is greatly exceeded. Hence the stress amplitude of the wave should decrease significantly in value after the wave has traveled several wavelengths where the wavelength is approximately equal to \(d_y\).

The temperature rise produced by the alpha particles is given by

\[
\Delta T = E_\alpha / 4\pi R d_\alpha \rho C ,
\]

where \(E_\alpha\) is the total energy of the alpha particles. This expression is valid only if the melting temperature of the metal is not exceeded. However, for the values of \(E_\alpha = 7\) MJ, \(d_\alpha = 2 \mu m\), and \(R = 2\) m Eq. (6) predicts for niobium that \(\Delta T = 28,000\) K. Clearly this calculated temperature rise is so large that sufficient energy does exist to vaporize the metal in the surface layer of thickness \(d_\alpha\).

Proposed designs of LCTRs avoid the problem of the vaporization of the surface of a bare wall metal shell either by coating the wall with liquid lithium\(^7\) or by deflecting the alpha particles with magnetic fields.\(^1\) In the latter method the alpha particles are forced to impinge at an oblique angle on the metal surface. The angle of impingement is such that the fluence is reduced by a factor of 10. The temperature rise given by Eq. (6) is thus reduced by an order of magnitude and the metal surface temperature is increased only to the melting temperature. The stress pulse produced by the sudden heating thus is of the order of 1.5 GN m\(^{-2}\) (15 kbars or 217,000 psi). Again, this stress amplitude will decay to a much lower level because of internal friction processes once the stress wave has traveled a number of wavelengths (say, \(10d_\alpha \sim 20\mu m\)).

In the method of avoiding the vaporization of the bare wall, which involves liquid lithium, the pressure wave set up in the liquid lithium would be transmitted to the metal wall.

C. Stress Pulse Produced by Cavity Gas Pressure

The remaining 15 MJ of energy from a pellet microexplosion listed in Table I resides in the deuterons and tritons that have not fused and in relatively low-energy alpha particles. These particles form a plasma or a gas that produces a gas pressure within the cavity. The shock wave pressure in this gas or plasma has been calculated\(^10\) to be of the order of, or less than, 1 MN m\(^{-2}\) (140 psi) for \(R = 2\) m. (The pressure is smaller the larger is \(R\).)

D. Localized Stress Pulses Produced in Neutron Collision Cascades

The collision cascades of neutrons produce localized, short-lived stress pulses in solids. For 14-MeV neutrons the stress pulse magnitude is of the order of the theoretical shear strength \((\sigma \sim \mu / 30)\) and extends over distance of the order of 10 \(\mu m\) in niobium.\(^11\) A theoretical treatment of the growth of fatigue cracks produced by such localized stress pulses is given in Ref. 12.

E. Stresses Transmitted From Liquid Lithium at Inner and Outer Surfaces

If the inner wall of the metal shell is coated with a liquid film of lithium, the vaporization of lithium in this film can produce stresses in the metal shell. In addition, energy deposition by neutrons in the liquid lithium blanket around the outer surface also can produce stresses within the metal shell. The report compiled by Booth\(^7\) presents the results of calculations of these stresses made for a shell of inner radius of 1 m and thickness of 0.7 cm for a pellet microexplosion of 200 MJ of energy. For a shell of 2 m radius and a microexplosion of 99 MJ of energy the magnitude of the maximum stresses should be only of the order of 10 MN m\(^{-2}\).
TABLE III
SUMMARY OF MAGNITUDE OF STRESS PULSES PRODUCED BY VARIOUS MECHANISMS

<table>
<thead>
<tr>
<th>Mechanism</th>
<th>Wavelength</th>
<th>Stress $\text{MN m}^{-2}$</th>
<th>Stress ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Momentum Transfer</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>neutrons</td>
<td>&gt; 1 cm</td>
<td>0.032</td>
<td>0.0045</td>
</tr>
<tr>
<td>alphas</td>
<td>2 $\mu$m</td>
<td>80</td>
<td>12</td>
</tr>
<tr>
<td>photons</td>
<td>10 $\mu$m</td>
<td>0.032</td>
<td>0.0045</td>
</tr>
<tr>
<td>Thermoelastic without inertial effects (at outer radius as calculated by Axford)</td>
<td>10</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Cavity gas pressure</td>
<td></td>
<td>1</td>
<td>0.14</td>
</tr>
<tr>
<td>Transmitted from liquid lithium layers</td>
<td>10</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Collision cascades</td>
<td>0.1 $\mu$m</td>
<td>3300 (theoretical strength)</td>
<td>480</td>
</tr>
</tbody>
</table>

a Calculated for niobium for spherical shell of radius of 2 m and thickness of 1 cm for a pellet microexplosion of 99 MJ of energy.

b Calculated for an oblique impingement at an angle that reduces the fluence by a factor of 10.

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F. Summary of Magnitude of Stress Pulses Estimated in Previous Sections

Table III summarizes the estimated magnitudes of the stress pulses produced in various ways and discussed in the previous sections.

III. ESTIMATED FATIGUE BEHAVIOR OF FIRST-WALL MATERIAL

From Table III it can be seen that the mechanisms that produce the largest stresses produce a stress pulse of rather short wavelength, of the order of 2 to 10 $\mu$m. The stress pulses of long wavelength have a rather low magnitude. I believe this difference in wavelength has an important and favorable implication for the fatigue resistance of first-wall material.

The fatigue resistance of a material is determined both by how long it takes to nucleate a fatigue crack and how long it takes a nucleated crack to grow to that critical length at which a static stress can cause catastrophic expansion of the crack. If pre-existing cracks are in the material the slow fatigue crack growth stage determines the fatigue life. Figure 1, taken from Ref. 12, shows schematically the functional relationship between the rate of growth, $\frac{da}{dN}$, where $a$ is the crack half length and $N$ is the number of stress cycles, of a fatigue crack as a function of the stress intensity factor $\Delta K$. The stress intensity factor is defined by the equation

$$\Delta K = \sigma (\pi a)^{1/2},$$

where $\sigma$ is the amplitude of the cyclic stress. (If the wavelength $\lambda$ of the stress pulse is smaller than the crack length, a stress intensity factor given by $\Delta K_\lambda = \sigma (\pi \lambda)^{1/2}$ should be used to calculate the rate of growth of a fatigue crack.) The crack growth rate decreases very rapidly with decreasing value of $\Delta K$ for values of growth rate $\frac{da}{dN}$ that are below about $10^{-10}$ to $10^{-9}$ m/cycle. In other words, a threshold effect seems to exist for
Fig. 1. Schematic plot of crack growth rate, $da/dN$, versus stress intensity factor $\Delta K$.

the growth of fatigue cracks. Above the threshold value the growth rate is given by the Paris equation

$$\frac{da}{dN} = C(\Delta K)^n,$$  \hspace{1cm} (8)

where $C$ and $n$ are constants. For smaller values of $\Delta K$ the value of $n$ is approximately equal to 2, and for larger values of $\Delta K$ the value of $n$ is of the order of 4 (see discussion in Ref. 12). When $n$ has the value of 2 the experimentally determined value of $C$ is approximately

$$C = \frac{8}{\pi} E^2,$$  \hspace{1cm} (9)

where $E$ is Young's modulus of the material. When $n$ has the value of 4, experimentally determined values of $C$ for many materials are of the order of $3 \times 10^{-37} \text{ N}^{-4} \text{ m}^2$.

To prevent appreciable crack growth and thus fatigue failure after $10^6$ to $10^7$ or more stress cycles the value of $\Delta K$ must be of the order of, or smaller than $\Delta K_c$, where $\Delta K_c$ is the value of $\Delta K$ at the threshold value of the growth rate $da/dN$.

This critical value $\Delta K_c$ is found by combining Eqs. (8) and (9) and setting $da/dN = 10^{-10}$ to $10^{-9} \text{ m/cycle}$.

Thus

$$\Delta K_c = cE,$$  \hspace{1cm} (10)

where the constant $c = 6.3 \times 10^{-6}$ to $2 \times 10^{-5} \text{ m}^{1/2}$. For niobium Young's modulus $E = 0.1 \text{ TN m}^{-2}$. Hence the critical value $\Delta K_c$ of the stress intensity factor for this metal is of the order of 0.6 to 2 MN m$^{-3/2}$. For a pre-existing crack of half length $a = 10 \mu m$ this value of $\Delta K_c$ corresponds to a cyclic stress amplitude of approximately 120 to 360 MN m$^{-2}$ (17 to 52 ksi). The experimentally determined value of the fatigue limit of niobium$^{15}$ that contains 0.001 wt% oxygen in the temperature range from room temperature to 600°C (873 K) is approximately 100 to 130 MN m$^{-2}$ (15 to 20 ksi). (The fatigue limit of niobium that contains 0.43% oxygen is as high as 42 ksi).

From Table III it can be seen that the stress levels of pulses of long wavelength are an order of magnitude smaller than the fatigue limit or the critical stress estimated from $\Delta K_c$ for a value of $a = 10 \mu m$. These long wave stress pulses should not produce any fatigue problem for the first-wall material.

Some of the stress pulses of short wavelength given in Table III do have a very high magnitude, much higher than the measured fatigue limit or the critical stress estimated with $\Delta K_c$. These stress pulses can cause fatigue cracks to grow. However, these short wavelength pulses (of wavelengths corresponding to frequencies of 0.5 to 2.5 MHz) may cause only cracks near the inner surface of the metal shell to grow and then only to a length no larger than about ten times the wavelength of the stress pulse. Cracks in the interior of the metal shell may not grow at all.

The reason that only cracks near the inner surface may grow, and then only to a limited extent, is that the thermoelastic stress pulse of short wavelength and high stress amplitude is likely to be strongly attenuated as it travels into the interior of the metal shell. (I am indebted to E. G. Zukas$^{16}$ for the suggestion that a short wavelength stress
pulse of high amplitude may be strongly attenuated after traveling only a short distance.) The only published information on the internal friction at high stress amplitude is that of Mason.\textsuperscript{17} He measured the internal friction in lead and aluminum and alloys of lead and of aluminum. He showed that in lead and pure aluminum the internal friction increases very rapidly as the stress amplitude approaches a value approximately equal to $E/200$. He found that the log decrement $\delta$ of lead reaches the value of 1 and of aluminum 0.1. (Note that the measure of internal friction used by Mason, $Q^{-1}$, is equal to $\delta / \pi$.) In the experiments on aluminum alloys the stress amplitude had to be increased before the region of a rapidly rising internal friction with increasing stress amplitude was reached. The maximum internal friction measured by Mason on aluminum alloys was an order of magnitude smaller than that measured on pure aluminum.

The mechanism that produces a high internal friction at a high stress amplitude in lead and aluminum clearly involves the motion of dislocations. It is reasonable to expect that high internal friction at high stress amplitudes will exist in other metals which are not alloyed to such an extent that dislocation motion is prevented under large stresses. There is, therefore, a high probability that the amplitude of the short wavelength thermoelastic stress pulses produced at the inner surface of a niobium shell will attenuate by at least an order of magnitude by the time the stress pulse has traveled 10 wavelengths. If this does happen fatigue cracks in the interior of the metal will not grow. Cracks near the inner surface will grow but only to a length of the order of, or less than, 10 wavelengths of the stress pulse (20 $\mu$m to 100 $\mu$m). Experiments are needed to confirm that the internal friction does indeed become very large at high stress amplitudes. Experiments also should be carried out under radiation damage conditions to make certain that irradiation does not decrease significantly the value of the internal friction at high stress amplitudes. The experiments of Mason on aluminum alloy do show that alloying increases the stress amplitude required to reach a region of a large increase in internal friction. Since the hardening produced by radiation damage is similar to that produced by alloying a similar effect on internal friction is to be expected.

Collision Cascades

Fatigue crack growth produced by neutron collision cascades may be a serious problem for first wall material.\textsuperscript{12} Fatigue crack growth by this mechanism is proportional to the neutron fluence. (It should be noted that this mechanism at present is only a theoretical speculation. No experimental evidence for or against this mechanism of fatigue crack growth exists at this time.) The growth of a crack by 14-MeV neutrons to a final length $a_f$ requires a neutron fluence $F$ given by\textsuperscript{12}

$$a_f = \beta F,$$

where $\beta = 1.1 \times 10^{-29} \text{ m}^3/\text{neutron}$. In a year’s time in a LCTR operating at 3 pellet microexplosions per second the fluence at a radius of 2 m is $6.2 \times 10^{25}$ neutrons $\text{m}^{-2}$ if each pellet produces $3.3 \times 10^{19}$ neutrons. This fluence will produce, according to Eq. (11), cracks of final half length a equal to $7 \times 10^{-4}$ m or a total length of 1.4 mm. In a year’s time, therefore, the metal shell may contain a high density of millimeter long cracks. Experiments are needed to decide whether fatigue crack growth produced by collision cascades does occur, and if it does the actual value of the constant $\beta$ in Eq. (11) should be determined.

High-Temperature Fatigue

The average temperature in the metal shell and outer liquid lithium blanket is expected\textsuperscript{7} to be of the order of 750°C (1024 K or 1400°F). This temperature is slightly higher than 1/3 of the melting temperature of niobium and thus can be considered to be in, or almost in, the high-temperature region as far as mechanical properties are considered. The Paris equation of fatigue crack growth generally is considered to apply to metals at low temperatures. No data on fatigue crack growth of niobium at high temperatures appear to exist. However, for stainless steel Shahinian et al.\textsuperscript{18} have shown that the Paris equation describes crack growth at a test temperature (593°C or 866 K) which is about 1/2 of the melting temperature. It is reasonable, therefore, to expect that the Paris equation is valid for
niobium at least up to the analogous temperature (1100°C or 1373 K).

If the temperature is relatively high other fracture modes, in particular growth and coalescence of grain boundary voids, become active. Are they important for a niobium shell in an LCTR? I don't believe so. Because the fluctuating stresses applied to the shell are of such short duration, certainly less than 1 ms for one pellet microexplosion, the total time the stress is present during a year's operation of an LCTR is less than 9 hours. For niobium, a rupture time at 1800°F (980°C or 1253 K) of 9 hours corresponds to a static stress of 17 ksi (120 MN m⁻²). In fatigue the stress amplitude required to produce rupture in the same length of time will be much greater. It does not appear, therefore, that high temperature fracture in fatigue will present any serious problem to the first-wall material.

IV. SUMMARY

The periodic bursts of energy created in an LCTR will produce cyclic stresses in the metal first-wall shell. The largest stresses occur in short wavelength, thermoelastic stress pulses that start at the inner wall and move into the interior of the metal. The magnitude of the stress in these pulses is sufficiently large to cause rapid growth of fatigue cracks and consequently early failure by fatigue fracture of the metal shell. However, it is likely that these short wavelength stress pulses are attenuated rapidly with distance traveled by the wave. If the pulses are so attenuated they will not present a serious fatigue problem to the metal shell.

It is strongly recommended that internal friction measurements be made at a high stress amplitude of possible first-wall material to find suitable materials or coatings that do have high damping properties. Internal friction experiments also should be carried out under radiation damage conditions to check the extent to which irradiation reduces the internal friction of the material.

Experiments also are needed to determine if fatigue crack growth can be produced by 14-MeV neutron collision cascades. At the present time the possibility of fatigue crack growth produced by this mechanism is only a theoretical speculation. If neutron collision cascades can cause crack growth, first-wall material, after a year's operation, may contain a very high density of millimeter-long cracks. Further growth of these cracks will occur under the relatively low magnitude cyclic stresses produced by the mechanisms listed in Table III.

APPENDIX

STRESS PULSE PRODUCED BY SUDDENLY HEATING A SURFACE LAYER OF AN INFINITE HALF SPACE

The moving stress pulse that is produced by a sudden heating of a thin surface layer of an infinite elastic half space is easily determined using the approximate theory of uncoupled dynamic thermoelasticity if the thickness d of the surface layer that is heated is large compared with the distance 

\[
D_t/c_\lambda = 4 \text{ mm},
\]

a length orders of magnitude smaller than the value of d(d = 2 to 10 μm) considered in this report. If d >> D_t/c_\lambda, the characteristic distance (D_t/c_\lambda)^{1/2} for thermal diffusion is small compared with d for the time interval t = d/c_\lambda that is required for a sound wave to travel a distance d. Thus time variation effects produced by thermal diffusion are insignificant compared with ordinary time variation effects in an elastic wave. The problem of a stress pulse moving into a spherical shell whose inner radius and thickness are large compared with the wavelength of the stress pulse is essentially the same problem for an elastic half space.

Use of coupled dynamic thermoelasticity theory permits the calculation of the thermoelastic damping (internal friction) of the stress pulse as it travels through the half space. The maximum value of the log decrement is approximately equal to the coupling constant (Ref. 22, page 393; Ref. 23, page 501). The thermoelastic coupling constant for plane longitudinal waves is equal to the dimensionless expression \( (3\lambda + 2\mu)^2 \alpha T_0 / (\lambda + 2\mu) \rho C \) where \( \lambda \) and \( \mu \) are the Lame elastic constants, \( T_0 \) is the
reference temperature, \( p \) is the density, \( C \) is the specific heat, and \( \alpha \) is the linear coefficient of thermal expansion. For niobium the coupling constant is equal to 0.02. Hence the maximum possible value of the log decrement is only 0.02 and this damping constant is attained only for stress pulses of wavelength that are orders of magnitude smaller than those considered in this report.

Let \( x \) represent distance measured from the surface of the elastic half space in a direction perpendicular to surface of the half space. Suppose the surface layer \((0 \leq x \leq d)\) is heated suddenly at time \( t = 0 \) and its temperature is increased by the amount \( \Delta T \). The material in the surface layer initially will not have time to expand. Within the surface layer, but not on its boundaries (that is, at \( x = 0 \) and at \( x = d \)), the stress \( \sigma_{xx} \) that acts normally to any plane parallel to the half space surface must equal at \( t = 0 \)

\[
\sigma_{xx} = -\sigma_0 = \sigma_0(3\lambda + 2\mu) \Delta T \quad (A-1)
\]

In the interior of the half space \( \sigma_{xx} = 0 \). At the surface of the half space \((x = 0)\) the traction must vanish and thus \( \sigma_{xx} = 0 \). At the boundary between the heated surface layer and the unheated interior \((x = d)\) the stress \( \sigma_{xx} = -\sigma_0/2 \). Only if \( \sigma_{xx} \) has this latter value will the compressive stress pulse that originates at this boundary and travels in a positive \( x \) direction produce an elastic displacement at \( x = d \) that is identical to the elastic displacement produced in the tensile stress pulse that also originates at this boundary at the same time and travels in the negative \( x \) direction. See Eqs. (A-3) and (A-4).

A tensile stress wave starts at the free surface at \( t = 0 \). The elastic displacement \( u_x \) of this wave is given by (and obviously satisfies the wave equation \( \partial^2 u_x / \partial x^2 = c_\lambda^2 \partial^2 u_x / \partial t^2 \))

\[
u_x = \left[ \frac{\sigma_0}{(\lambda + 2\mu)} \right] (x - c_\lambda t) \quad (A-2)
\]

for \( x - c_\lambda t < 0 \) and \( u_x = 0 \) for \( x - c_\lambda t > 0 \). The stress in this tensile wave is constant and is equal to \( \sigma_{xx} = (\lambda + 2\mu) \left( \frac{\sigma_0}{\lambda} \right) \). A compressive wave starts at \( t = 0 \) from the boundary \( x = d \) and moves in the positive direction. Its stress is constant and is equal to \( \sigma_{xx} = -\sigma_0/2 \). The elastic displacement in this wave is given by

\[
u_x = \left[ -\frac{\sigma_0}{(\lambda + 2\mu)} \right] (x - d - c_\lambda t) \quad (A-3)
\]

for \( x - d - c_\lambda t < 0 \) and \( x > d \). For \( x - d - c_\lambda t > 0 \) the displacement \( u_x = 0 \). A tensile wave also starts at \( t = 0 \) at \( x = d \) and moves in the negative \( x \) direction. The stress in this wave is constant and is equal to \( \sigma_{xx} = \sigma_0/2 \). The elastic displacement is equal to

\[
u_x = \left[ \frac{\sigma_0}{(\lambda + 2\mu)} \right] (x - d + c_\lambda t) \quad (A-4)
\]

for \( x - d + c_\lambda t > 0 \) and \( x < d \). For \( x - d + c_\lambda t < 0 \) the displacement \( u_x = 0 \). The tensile wave given by Eq. (A-4) reaches the free surface at time \( t = d/c_\lambda \) and is reflected as a compressive wave of constant stress \( \sigma_{xx} = -\sigma_0/2 \). This reflected wave travels in the positive direction and has the elastic displacement (when \( t > d/c_\lambda \))

\[
u_x = \left[ -\frac{\sigma_0}{(\lambda + 2\mu)} \right] (x + d - c_\lambda t) \quad (A-5)
\]

for \( x + d - c_\lambda t < 0 \) and \( u_x = 0 \) for \( x + d - c_\lambda t > 0 \).

At a time \( t \) that is large compared with the expression \( 2d/c_\lambda \), the total stress \( \sigma_{xx} \) in the heated surface layer is equal to zero because the stress in the tensile wave given by Eq. (A-1) exactly cancels the thermal stress given by Eq. (A-1) and the tensile stress in the wave given by Eq. (A-4) is canceled by the compressive stress in the reflected wave given by Eq. (A-5). In the region \( d < x < [(t/c_\lambda) - 2d] \) the stress \( \sigma_{xx} \) also is equal to zero because the stress of the tensile wave given by Eq. (A-2) is canceled by the sum of the stresses in the compressive stress waves given by Eqs. (A-3) and (A-5). In the region \([(t/c_\lambda) - d < x < (t/c_\lambda)] \) a compressive stress \( \sigma_{xx} = -\sigma_0/2 \) exists and is followed in the region \( [(t/c_\lambda) - 2d] < x < [(t/c_\lambda) -] \) by a tensile stress \( \sigma_{xx} = \sigma_0/2 \). Thus the traveling stress wave produced by the sudden heating consists of an initial compressive stress pulse of length \( d \) followed by a tensile stress pulse of length \( d \).

REFERENCES


