COMPOSITE LINEAR DESIGN TO MAXIMIZE THE SHOCK PRESSURE BEYOND MEGABARS

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COMPOSITE LINER DESIGN TO MAXIMIZE THE SHOCK PRESSURE BEYOND MEGABARS

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Among the solid liners made of a single material which are imploded onto a target under the same driving condition, the aluminum liner produces the highest shock pressure. We propose the composite liner design which can increase the shock pressure several times over the best performance obtainable from an aluminum liner. We have also developed a general formulation to optimize the composite liner design for any driving current, and derived a set of very useful scaling relations. Finally, we present some 1-D simulations of the optimal composite liners to be fielded at Pegasus and Prargon in the upcoming megabar experiments.

I. Introduction

Using pulsed power to implode the liner onto a target is a convenient way to produce high shock pressure. Two years ago a solid aluminum liner which could produce shock pressures in the hundreds of megabars was designed [1] and tested [2] at the LANL Pegasus facility. This liner design has since been used successfully for a variety of application experiments. Recently, there have been several table experiments, interest to produce shock pressures in the megabar regime and more importantly, to measure what is the practical pressure limit obtainable for a given pulsed power. It will be made clear later that among the hundreds of shots of a single material (for a material called superlithium, see the argument in the introduction) or generally the best to produce the highest shock pressure, but if we scale up the radiations and scale down the driving current in the Pegasus to best pressure we can get a perfect hydraulic shock wave.

The general form of a composite liner design, which we propose to use, is the shock pressure and performance. These relations provide us a quick benchmark estimate on the maximum pressure attainable from any scalable driving current. For example, we can scale our results obtained for Pegasus to the Atlas parameter regime, since the driving currents for both are approximately identical.

In the next section, we will present the liner implosion equations to lay the groundwork for our design. We next discuss the general behavior of the shock pressure through the Hugoniot in Sect. III. We establish a scheme for matching the best available materials which maximize the shock pressure. In Sect. IV, we give a detailed discussion on the laser heating, which is an ultimate limit on the laser intensity. The physical demonstration leading to the composite liner and general procedures to optimize the liner parameters are presented in Sect. V. Followed by discussions on the scaling relations in Sect. VI. Finally, in Sect. VII, we discuss the optimal composite liners to be fielded in the upcoming megabar implosion experiments at both Pegasus and Prargon and we present some 1-D simulation results.

II. Implosion Equation for Thin Liner

The most important equation is

\[ \frac{dF}{dt} = A - B \]

\[ \frac{dA}{dt} = C - D \]

\[ \frac{dC}{dt} = E - F \]

\[ \frac{dF}{dt} = G - H \]

\[ \frac{dG}{dt} = I - J \]

\[ \frac{dH}{dt} = K - L \]

\[ \frac{dI}{dt} = M - N \]

\[ \frac{dJ}{dt} = O - P \]

\[ \frac{dK}{dt} = Q - R \]

\[ \frac{dL}{dt} = S - T \]

\[ \frac{dM}{dt} = U - V \]

\[ \frac{dN}{dt} = W - X \]

\[ \frac{dO}{dt} = Y - Z \]

\[ \frac{dP}{dt} = A - B \]

\[ \frac{dQ}{dt} = C - D \]

\[ \frac{dR}{dt} = E - F \]

\[ \frac{dS}{dt} = G - H \]

\[ \frac{dT}{dt} = I - J \]

\[ \frac{dU}{dt} = K - L \]

\[ \frac{dV}{dt} = M - N \]

\[ \frac{dW}{dt} = O - P \]

\[ \frac{dX}{dt} = Q - R \]

\[ \frac{dY}{dt} = S - T \]

\[ \frac{dZ}{dt} = U - V \]
Rather, we will take advantage of any good approximation which helps to simplify the implosion equation and render the scaling possible. The thin-liner approximation will be assumed in this paper, it is justified if the thickness of the liner is much smaller than the radius.

Next we note that the liner radius affects the driving curve only through a logarithmic term in the induction, so the effect is negligible until the liner radius \( r \) becomes much smaller than its initial value \( r_0 \). In the region where \( r < r_0 \), the duration is so short that the liner velocity is affected only slightly by the error in current. The above reasoning justifies that we can decouple the driving current from the liner motion. This excellent approximation not only simplifies greatly the implosion equation but also makes the scaling of the optimal liner parameters possible. Using the above approximation the liner implosion equation is given by

\[
\frac{dr}{dt} = -\frac{\rho_0}{4} \left( \frac{L(t)}{r(t)} \right)
\]

with the initial conditions \( r(0) = r_0 \) and \( r(0) = 0 \), where \( r \) is the length \( L(t) \), the driving current \( I(t) \); the radius \( r \) and \( t \), the time of the liner.

A class of currents is said to be scalable to one another if we can represent them by a single function \( I(t) \) using two parameters \( \lambda \) and \( \rho_0 \). The current wave forms we usually see in many implosion experiments are approximately sinusoidal or like a step function each type forms a separate class. I see when we look for possible scaling relations of the optimal liner parameter and performance for scalable driving currents it is useful to express the implosion equations for the whole class in terms of the scaled distance traveled by the liner.

\[
\lambda \equiv \frac{r_0}{r}, \text{ and scaled time } \tau \equiv \frac{t}{\lambda}.
\]

The resulting implosion equation

\[
\frac{d\tau}{d\lambda} = \frac{I(\lambda)}{\lambda^{2} \tau^{2} \rho_0} \tag{12}
\]

now has an invariant set of initial conditions \( \lambda(0) = 0 \) where the \( I(t) \) stands for \( \lambda^{2} \rho_0 I(\lambda) \).

\[
\frac{d\tau}{d\lambda} = \frac{I(\lambda)}{\lambda^{2} \tau^{2} \rho_0} \tag{13}
\]

or \( I(\lambda) = \lambda^{2} \tau^{2} \rho_0 \frac{d\lambda}{d\tau} \).

III Behavior of Shock Pressures Generated from Implosions

When the imploding liner with the target at \( \tau = \tau_0 \), the shock pressure can be written

\[
P(t) = \rho t (\tau + \tau_0) \tag{14}
\]

and target (labeled by \( t \))

\[
P(t) = \rho t (\tau + \tau_0) \tag{15}
\]

by eliminating the particle velocity \( v \), where \( \rho \) is the density and \( \alpha \) and \( \sigma \) are material constants that relate the shock velocity to the particle velocity. From the above equations we see that higher collision velocity \( v \), and material densities will give rise to higher shock pressures, but the material with higher collision velocity \( v \) and \( \sigma \) also helps. While the above equations provide a precise guideline to find the best liner and target materials that will achieve the highest shock pressure under a given implosion condition, the process to examine all materials will be extremely time consuming. Fortunately we can take a shortcut by proving the following theorem: For any collision velocity let \( P_{AB} \) be the shock pressure generated from a collision between two materials A and B, its value is bounded in between \( P_{AA} \) and \( P_{BB} \). We can prove this statement as follows: First notice that the Hugoniot equation is parabolic and for \( 0 \leq t \leq \rho_0 \), the liner Hugoniot increases while the target Hugoniot decreases with increasing \( \tau \). Second, for the collision between identical material, the two Hugoniots always intersect at \( \tau = \rho_0 / 2 \) due to the inherent symmetry at the point we have the exact solution

\[
\frac{d^2\rho}{d\tau^2} = \frac{\rho(t)}{\rho_0 (\tau + \tau_0)^2} \tag{16}
\]

Without loss of generality we assume \( P_{AA} > P_{BB} \) and solve for \( P_{AB} \) at the intersection of the linear Hugoniot A and target Hugoniot B. Now \( P_{AA} \) and \( P_{BB} \) lie on the linear Hugoniot A which increases with \( \tau \) and \( P_{AB} \) is on the target Hugoniot B which decreases with \( \tau \). It follows that \( P_{AB} \) is always in between \( P_{AA} \) and \( P_{BB} \). Therefore we can use the Hugoniot equation to show that

\[
\frac{d^2\rho}{d\tau^2} = \frac{\rho(t)}{\rho_0 (\tau + \tau_0)^2} \tag{17}
\]

Using the above results we can simplify our task drastically in searching for the right material to maximize the shock pressure bound of \( \rho(t) \) for the maximum of \( P_{AB} \) we can just look for the maximum of \( P_{AB} \) provided that the highest attainment is only one of the many instants. We will show here that the highest attainment is only one of the many instants.
material always maximizes the pressure for all values of \( v \). But for multi-megabar pressures or higher, this is indeed the case. This follows from the fact that, for a wide variety of materials \([1]\), \( c \) is around a few \( \text{um/\mu s} \) and \( 1.2 < s < 2 \). At high megabar pressures, \( v \) is large enough so that the term \( sv \) dominates over \( 2c \) in Eq.(7) and consequently we have

\[
P \approx \frac{1}{4} \rho v^2.
\]

(9)

This ensures that \( P \) is the maximum for the material with the highest value in \( \rho v^2 \) at any \( v \). In the same approximation, the shock pressure between two different materials behaves like

\[
P_{AB} = \frac{v^2}{[(v_2/v_1)^{-1/2} + (v_1/v_2)^{-1/2}]^2}
\]

(10)

### IV. Joule Heating Limitation on Liner

The current passing through the liner has to diffuse into the interior from the outer surface, so calculating the resistive heating of the liner is quite complicated unless the diffusion time is faster than the implosion time. In general we do not expect the temperature distribution across the liner to be uniform, but rather to increase monotonically toward outside. To simplify the formulation, let us consider a pure liner and assume that the temperature is uniformly distributed. Since radiation loss is negligible, the time dependence of the liner temperature is given by the energy balance equation

\[
K(t)T^2(t)dt = mc(T)dt,
\]

(11)

where \( c \) is the specific heat of the liner material and \( R \) the resistance. In term of the resistivity \( \eta \) and density \( \rho \), we can integrate the above as

\[
\frac{L^2}{m} \int_0^{T_{in}} P^2(t)dt = \int_0^{T_{in}} \left( \frac{Q(T)}{\eta(T)} \right) dT,
\]

(12)

where \( T_{in} \) is the initial temperature. Notice that the right hand side is only a state function of the liner material. The left hand side is proportional to the electrical action integral defined as

\[
Q(T(t)) = \frac{1}{A} \int_0^{T_{in}} P^2(t)dt.
\]

(13)

where \( A \) is the liner cross section. The electrical action for any conductor can be measured by passing a current through a thin sample wire. Setting a limit on the action by requiring

\[
T(t_c) = T_c,
\]

we constrain the liner mass to be a function of the collision time \( t_c \) as

\[
\frac{L^2}{m} \int_0^{t_c} P^2(t)dt \sim \frac{Q(T_c)}{\rho^2}.
\]

(14)

For pure liners, a reasonable limit on \( T_c \) is the melting point \( T_{m} \), since the solid phase maintains a sharp shock front. The relation derived in Eq.(14) is still useful even when we deal with the realistic situations in which the temperature distribution is not uniform. In this case we should set the limit on the temperature of the inside liner surface, denoted by \( T(t) \), which is the coolest at any time since the current has to diffuse radially inward. It is easy to see that we can still write

\[
\frac{1}{m} \int_0^{t_c} P^2(t)dt \sim \beta(T(t), m),
\]

(15)

except that \( \beta \) now has a weak dependence on \( m \). Once we set \( T \) to a limit \( T_c \), \( \beta(T_c, m) \) can be determined by using the 1-D MHD code to compute the left hand side of Eq.(15). Later when we apply the above relation to optimize the liner mass, we only need to vary \( m \) in a narrow range around the optimal solution. We can therefore represent \( \beta(T_c, m) \) as a constant plus a small linear term in \( m \) and determine it by just two code simulations.

Among all metals, empirically aluminum has the highest value (only copper is in a close second) in the ratio \( Q(T_m)/\rho^2 \), where \( Q(T_m) \) is the action to the melting point. In terms of \( Q(T_m)/p^2 \), the aluminum is ahead of other heavier metals even more by an extra density factor. Using Eq.(14) the same can be said about the current integral on the left hand side. We therefore conclude that the aluminum liner can be driven with a longer \( t_c \), before reaching the melting point, than any other pure liner (of higher density) having the same mass \( m \) and length \( L \). But longer imploding time before melt implies higher attainable velocity since all these liners are governed by the same implosion equation. Using Eq.(10), we see that this \( 1/p \) advantage in attainable velocity for aluminum over materials of higher density is sufficient to ensure that the aluminum liner will also generate the highest shock pressure on any chosen target.

### V. Composite Liner and Optimization

With the physical insights gained from our discussions on shock Hugoniot And Joule heating, the composite liner seems to be an excellent idea to improve the attainable shock pressure substantially over the pure liners. Clearly we still want to use
aluminum on the outside for carrying most of the driving current to retain its highest attainable velocity. For the inner layer we look for a material with high value in as to enhance the shock pressure subject to some other criteria discussed below.

We find that platinum is the best impacting material for the composite layer, not just for its high density but also for its high melting point and electrical resistivity. Based on these criteria, other materials such as tungsten are equally satisfactory, but fact that platinum can be electrodeposited in a log for fabrication. The-plating layer on the inner layer dramatically since the current has to diffuse through it. The result is a composite of the platinum layer in two orders of magnitude higher than that of the aluminum, giving a much higher density and smaller cross section. This factor also helps to reduce the J-Side heating in Pt after the current is diffused in.

The high melting point is an extra advantage since we can now remove the Al layer beyond its melting point while still keeping the Pt layer solid. Consequently, the composite layer can take considerable more J-Side heating than a pure aluminum one with the same mass and thereby achieving a corresponding higher velocity. How much we can push this advantage depends on the ability of the solid Pt layer to withstand the magnetically driven Rayleigh-Taylor instabilities in the metal Al layer. No satisfactory answer has been known so far from the 2-D MHD simulations. Finally, we will get some valuable clue from the upcoming 3-D MHD experimental program.

In the composite layer, the J-Side heating changes slightly from the aluminum layer. In applying the J-Side heating, the relation is calculated by the aluminum mass flow and J-Side heating temperature. As time more surface, the aluminum mass flow should be kept the same as possible so that it will not reduce the velocity significantly. In the following layer, it is speeded up.

1. Estimates of the J-Side power means to find the heat input and data which determine the shock pressure at a given target radius. It is usually done by the exponential part of the magnetic field. Longitudinal heating is only a function of the pressure and along with the plasma flow at this point. In designing an optimum solution, we have to compare the J-Side heat input with the target pressure at this point. Optimum parameters are generally achieved by making the heat input m as the free parameter, we use Mathematica to solve Eq (1) iteratively to find the correct initial radius r₀(m) such that the solution for (2) satisfies r(0) = r₀(m). Here r(0) is given by the J-Side heating constraint Eq (15). The optimal mass is then the one which maximizes the expansion velocity r₀(m).

VI. Scaling Relations for Optimal Liners

We now proceed to derive a set of very useful scaling relations for the optimal liner parameters and performance. While these relations are derived under an idealized scaling condition, they nevertheless provide a useful tool to make an estimate on the maximum pressure achievable by an unoptimized pulsed power regime that is normally approximately a factor of two larger.

It is important to realize that the liners optimized by the procedure as described in Sect V do not scale in a simple way even though the driving currents are exactly similar. For one thing, in a realistic design the target radius is usually determined by experimental requirements, or at least, it will not stay the same from one driving condition to the other. Furthermore, the optimization requirements also complicate the scaling. Thus, some additional definitions are necessary for the definition of optimal liner and to derive their scaling relations. In the rest we have to assume that the input parameter is (r₀, r₀, r₀) i.e. it is the optimal liner radius. In terms of the scaled distance introduced in Eq 2.

Next we are in fact the area function of a thin plate whose thickness is constant. This is equivalent to the condition that the energy for the heating layer a constant fraction of the total driving energy. While it is acceptable to prove rigorously that this assumption leads to the usual and most parameters to setting optimal physically of a situations in some of the calculations. However, it is not generally valid for an optimal liner design, and i.e. in fact fundamentally invalid.
that the solution for Eq. (2) with different values of \( r \) do not intersect except at \( r = 0 \), so there is only one solution which passes through \( r = 1 \) at \( r = r \), as required. Using Eq. (3), the unique value of \( a \\ n 

\[ a_n^2 \propto J_n^2 \mu_0^{-2} \]  

(16)

The Joule heating constraint given by Eq. (15) can be written as

\[ \frac{J^2}{\mu_0} \int_0^1 E^2(r) dr = \gamma T_n, n. \]  

(17)

when we ignore the small amount of platinum mass, \( L \), neglecting the weak dependence on \( t \). we get the scaling for the upper limit as

\[ m_n \propto L_n^{-1/2} \]  

(18)

From Eqs. (17) and (18) we obtain the scaling for the lower limit as

\[ \varepsilon \propto L_n^{1/2} \mu_0^{-1/2} \]  

(19)

Finally, \( \gamma = \) constant and using Eq. (16) we get

\[ \varepsilon \propto J_n^{1/2} \mu_0^{-3/2} \]  

(20)

for the collision velocities and

\[ P \propto J_n^{1/2} \mu_0^{-5/2} \]  

(21)

for the shock pressure. As mentioned earlier, we have used \( L_n^{-1/2} \) to verify that the lower limit of energy at collisions is well within the limit of the total driving energy.

In applying these scaling relations, we can always be necessary when the result by making the local correction due to the field and making use of the fact that the driving energy.

VII. Megabar Liners for Pegasus and Procyon

The process of shock wave has been applied on the upper and lower limits of the shock wave is the application. The lower limit for Pegasus consists of 99.4% aluminum and 1% gold. It has an upper limit of 3 cm and a length of 2 cm. The lower limit for Procyon consists of 99.4% aluminum and 1% gold, with a maximum driving gap of 0.1 cm. In both cases, the energy is well contained by a square cavity of 0.1 cm, with a total energy of 8 MJ. From Eq. (16) we expect a peak shock about 8 MJ in a platinum target in Fig. 2 as depicted in the lower limit for Pegasus. For Procyon, we expect a peak shock about 8 MJ in a platinum target in Fig. 2 as depicted in the lower limit for Procyon. Figure 1 shows the velocity of the shock wave in the plot.

In Fig. 3 we show the velocity of the shock wave in the plot. The shock waves are shown in Fig. 4. The dotted and solid dotted and solid dotted curves represent respectively the upper and lower limits of the shock wave in the plot. We note that the upper limit are about 78 ps but the platinum layer remains the shock wave in the plot. The shock wave in the plot. The shock wave in the plot. The shock wave in the plot. The shock wave in the plot.
I. Conclusion

We have proposed the aluminum-platinum composite heat design for the plasma-thermal materials, based on our study of the behavior of the Al-Pt system and its potential in high-pressure experiments. The composite heat design involves the use of a shock wave to stabilize the plasma and to limit the heat transfer. The best results were obtained with the heat pulse acting on the solid Al-Pt composite. The composite material was found to be effective in maintaining the temperature gradient and in slowing down the shock wave.

II. References

Figure 4: Temperature histories of the outer (dotted) and inner (dot-dash) and solid surface of the 
Al and Pt layers respectively.

Figure 5: Sketch of the fabricated OADM and 
Figure 6: Sketch of the fabricated OADM and
$T (eV)$