Parameter Study
of a Long, Separated-Shock Ω Pinch
with Superconducting
Inductive-Energy Storage
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by

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PARAMETER STUDY OF A LONG, SEPARATED-SHOCK Q PINCH
WITH SUPERCONDUCTING INDUCTIVE-ENERGY STORAGE

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ABSTRACT

Present Q-pinch experiments involve high-$\beta$ plasmas with area compression ratios of the order of 40, using high-voltage capacitors to provide both the early shock heating (during the first few tenths $\mu$sec) and the later adiabatic compression, in a single-turn coil. In the separated-shock concept, the shock heating is done by high-voltage circuits whose energy content is only a few percent of that of the total system. The compression magnetic energy, which is preponderant, is contained in a multiturn coil with slow rise time (of the order of milliseconds) appropriate to adiabatic compression of the plasma in long systems. The area compression ratios are less than 10. Physical parameters and costs of a large system, appropriate to a scientific feasibility experiment at $kT_e \approx 10$ keV and $n_e \approx 10^{24}$ cm$^{-3}$ sec are derived in this report. The shock heating is calculated on the basis of plasma sheaths driven by Blumlein transmission lines; although other electrical circuits might be used. The adiabatic compression field is assumed to be energized by a superconducting inductive energy source, switched by normal-going superconductors.

I. PLASMA HEATING

A. Shock Heating

In the model taken here (Fig. 1) a sheath separating a magnetic field $B_s$ and the plasma advances radially inward at a speed $v_s$, projecting ions ahead of it at a speed $2v_s$. The plasma filling density inside this "magnetic piston" is $n_0$, and the plasma ahead of the piston is assumed to be cold ($\approx 1$ eV in practice). Only ions are assumed to gain energy, and the electrons remain cold. (In practice there is some joule heating of the electrons in the sheath; however, this heating is much less than that of the ions.) Plasma-simulation studies show that the actual process whereby the ions are given a spread of energies from an ideal sheath is one of repeatedly bouncing off the sheath wall as they pass through the center of the discharge tube, leading to oscillations which finally (on a scale of 0.1 $\mu$sec) settle down to some sheath radius $b_{SH}$ at the end of the shock phase. Another likely physical process

Fig. 1. Illustrating the quantities involved in a magnetically driven plasma sheath.
for imparting a spread of ion velocities is that the sheath becomes corrugated by Taylor or other instabilities so that ions bouncing off it are reflected in random directions, spreading energy into two or three degrees of freedom.

Simple dynamical considerations show that all of the ions projected from the sheath will hit it again after passing through the axis of the discharge when the sheath has advanced to \( x_0 = 1/3 \). As an average approximation to the actual train of events we assume that all of the ions inside the sheath at this instant thermalize and then push the sheath back to \( x_{SH} \), doing an amount of work \( (B_s^2/8\pi)n_o^2[x_{SH}^2 - (1/3)^2] \) against the constant field \( B_s \). Hence

\[
\frac{n_o^2}{x_{SH}^2} [1 - (1/3)^2] \frac{1}{1/2} m_i\left(2v_s^2\right)^2 = \frac{3}{2} \frac{n_o^2 k T_{SH}}{x_{SH}^2} + \frac{1}{8} \frac{B_s^2}{x_{SH}^2} \left(1 - (1/3)^2\right),
\]

where \( m_i \) is the ion mass. The momentum of the projected ions is related to \( B_s \) by

\[
b_s^2/\pi = 2m_i n_o v_s^2.
\]

After the piston-heated plasma becomes quiescent, pressure balance requires

\[
\frac{n_o}{x_{SH}} k T_{SH} = B_s^2/8\pi.
\]

Note that \( v_s = (k T_{SH}/2m_i)^{1/2}/x_{SH} = 2.20 \times 10^7 \text{ [kT}_{SH}\text{ (keV)]}^{1/2} \). Substituting (3) and (2) into (1), we find

\[
x_{SH} = (2/5)^{1/2} = 0.632.
\]

The initial back emf around the inside of the discharge tube is given by

\[
V_s = 2\pi \times 10^{-8} \text{ b v_s B_s}, \quad \text{(Volts)}
\]

Substituting Eqs. (2) and (3),

\[
V_s = 10^{-8} \left(\frac{10}{n_o x_{SH}}\right)^{1/2} B_s^2 = 4 \times 10^{-8} \left(\frac{3}{4}\right)^{1/2} \frac{n_o}{m_i} \frac{k T_{SH}}{x_{SH}^2}.
\]

In more convenient units (for \( m_i = 4.15 \times 10^{-24} \text{ g} \),

an "average" D-T ion),

\[
V_s (\text{kv}) = 0.517 \times 10^{-6} \text{ b } B_s^2/\sqrt{n_o} = 3.68 \text{ b } k T_{SH}\left(\text{keV}\right), \quad \left(x_{SH} = 0.632\right)
\]

where \( P_0 \) is the filling pressure in mT, \( b \) is in cm and \( B_s \) is in gauss.

B. Equilibration of Ions and Electrons

After the shock, but before the slow adiabatic compression has progressed very far, the electrons and ions will come to the same temperature by means of collisions. We assume this process to begin at \( x = x_{SH}, T_i = T_{SH}, T_e = 0 \) and end at \( x = x_f, T_i = T_f, T_e \), while the magnetic field maintains the constant value \( B_s \). Thus \( T_f = 1/2 T_{SH}, x_f = x_{SH} \), and we write (3) as

\[
2n_o k T_f = x_{SH}^2 (B_s^2/3\pi).
\]

C. Isoenthalpic Expansion

Before considering the adiabatic compression to the final plasma state, we consider the option of suddenly expanding the shock-heated equilibrated plasma without its doing work or exchanging heat with its surroundings. To accomplish this the field \( B_s \) would suddenly be removed for a time \( \tau \approx 0.3 v_{th}^{-1} \). Then a field \( x_f B_s \) would be reapplied. Here \( v_{th} \) is the ion thermal velocity. The plasma temperature remains constant at \( T_f \), but the compression factor increases from \( x_{SH} \) to 1.

D. Adiabatic Compression

During this process the plasma behaves as a fluid, with specific-heat ratio \( \gamma = 5/3 \), for which \( k T x^2(\gamma - 1) = \text{ const.} \). Let subscript \( f \) refer to the final, compressed plasma state. Then

\[
T_f = \frac{T_{SH}^2(x_f)}{x_f^2(\gamma - 1)}
\]

\[
n_f = \frac{n_o}{x_f^2}.
\]

Pressure balance, \( 2n_o k T_f = B_f^2/8\pi \) then requires

\[
B_f/B_s = (x_{SH}/x_f)^{2\gamma/\gamma - 1} = \left(T_f/T_{SH}\right)^{2\gamma/\gamma - 1},
\]

where the final compression field is given by

\[
B_f (\text{kg}) = 2.386 \left[P_0 k T_f(\text{keV})\right]^{1/2} x_f.
\]

Substituting (6) and (8) into (9), we obtain (for \( \gamma = 5/3 \))

\[
k T_f = \frac{10^{8}}{8\pi^{3/2}} x_f^{2\gamma} \left[\frac{x_{SH}^2}{x_f^2(\gamma - 1)}\right]^{1/2} \frac{b^2 v_s}{n_o^2 B_s}.
\]
\( kT_f(\text{keV}) = 0.0737 \frac{V_s(\text{kV})}{b_0^2 x_f^{4/3}} \)

Figure 2 is a graph of the various plasma and field parameters for \( kT_f = 5 \text{ keV} \) and \( b = 8 \text{ cm} \).

For the case (primed) with added enthalpic expansion we set \( x = 1 \) after equilibration so that

\[
B_f/B_s = x_f^{2/3}
\]

\[
T_f/T_s = x_f^{2/3}(y - 1).
\]

Then

\[
kT_f'(\text{keV}) = 0.136 \frac{V_s(\text{kV})}{b_0^2 x_f^{4/3}}.
\]

E. Free Expansion of the Ions

In order to obtain lower compression (larger \( x_{SH} \)) for a given \( B_s \) and \( V_s \), a magnetic field having the time history shown in Fig. 3 may be used. An initial piston field of magnitude \( 2^{1/2} B_s \) is suddenly removed at time \( t_0 \) when the sheath has arrived at \( x_0 = 1/3 \). A larger field is reapplied at \( 3/2 t_0 \), when all ions have advanced to the opposite wall, and reflects them all at velocity \( 2v_s \). In contrast to expression (2), we now have

\[
B_s/8\pi = 4n_0 m_v v_s^2.
\]

We denote the temperature of the thermalized ions

\[
kT_{f}'(\text{keV}) = 0.259 x_f^{1/3} b_0^2 x_f^{4/3} kT_{f} (\text{keV})\]

(14)

\[
T_{f}'(\text{keV}) = x_f^{2/3}(y - 1).
\]

Figure 3. Time history of magnetic field for free expansion of the ions following the initial implosion.

\[
V_s' = 2 v_s \pi \times 10^{-5} b v_s B_s
\]

(20)

Note that \( v_s = (kT_{SH}/4\pi m_v)^{1/2} \). Corresponding to (1) we now have

\[
V_s'^2 = 1.275 \times 10^{-7} \left( kT_{SH}^2(\text{keV}) \right)^{2/3}.
\]

where the last term represents work done on the plasma after the reversing reflection. Solving (21), (20) and (19) yields

\[
x_{SH} = (26/45)^{1/2} = 0.76.
\]

Since the ions are accelerated by \( 2^{1/2} B_s \), we have

\[
v_s' = 2 v_s \pi \times 10^{-5} b v_s B_s
\]

(23)

\[
v_s'(\text{kV}) = 0.259 \times 10^{-5} b v_s B_s
\]

(24)

After adiabatic compression

\[
kT_f'^{2/3} = \frac{10^8}{4 \pi^2} \left( x_f^{2/3} \right)^{2} \frac{m_v v_s'}{b_0^2 x_f^{4/3}}.
\]

(25)

\[
kT_f'(\text{keV}) = 0.272 \frac{v_s'(\text{kV})}{b_0^2 x_f^{4/3}} \left( x_f' \right) = 0.76.
\]

(26)

Figure 4 is a graph of the quantities \( B_s, B_f, n_f \) and \( V_s' \) versus filling pressure \( P_0 \) for the final compression factor \( x_f = 0.3 \) and 0.4. A final D-T ion temperature \( kT_f' = 10 \text{ keV} \) is assumed.
II. A SHOCK-HEATING CIRCUIT

Figure 5 shows a shock-heating coil with a Blumlein transmission line connected at each of two feed slots. The charge voltage $V_{BL}$ is that between each half of each Blumlein line. The total voltage developed by the lines, considering the coil as an open circuit, is

$$V_0 = 2N_{BL}V_{BL},$$

where $N_{BL}$ is the total number of transmission lines. The characteristic impedance of each half of each line is

$$Z_{BL} = 0.4\pi \times 10^{-6} \frac{ct_{BL}}{e^{\frac{w}{2}}},$$

where $e$ is the dielectric constant of the insulator (for Mylar $e = 3$), $t_{BL}$ is the spacing of a half Blumlein line, $w$ is its width, and $c$ is the speed of light in cm/sec.

When the shock-heating switches are closed, a voltage $2V_{BL}$ is applied after one wave transit time over the length $l_{BL}$. It lasts for a time

$$\tau_{BL} = \frac{2l_{BL}}{v_{BL}},$$

where

$$v_{BL} = \frac{c}{e^{\frac{w}{2}}},$$

is the phase velocity in the line. The rise time of the current in the coil (when there is no plasma) is

$$\tau_r = \frac{I_{SH}}{Z_{BL}}.$$  

The coil current rises according to the relation

$$I = I_{BL}(1 - e^{-t/\tau_r})$$

to a final value

$$I_{BL} = \frac{V_{BL}}{Z_{BL}}.$$  

This must match the current necessary to produce the shock magnetic field $B_s$ in length $l_{SH}$ of the shock coil. If we specialize to the case of Mylar insulation whose dielectric strength is taken as $1.18 \times 10^6$ V/cm (3 kV/mil), we have

$$t_{BL} = 0.85 \times 10^{-6} v_{BL}.$$
Using (2), we derive the width of the transmission lines

\[ w = 1.85 \times 10^{-4} I_{BL}. \quad \text{(cm)} \quad (9) \]

### III. COUPLING OF THE PLASMA AND THE SHOCK-HEATING CIRCUIT

Having established the parameters of the shock-heating circuit, we now give an approximate treatment of the effect of the plasma back emf on the current driven by the Blumlein lines. We consider the situation at early times, \( x = t/\tau \lesssim 1 \), when the sheath has not moved appreciably from its initial position \( x = 1 \). Using (1.6), (1.7) and (IV.3), the plasma back emf is given by

\[ V_s = Al^2, \quad \text{(volts)} \quad (1) \]

where \( I \) is the shock-coil current in amperes, and

\[ A = (0.4\pi)^2 \times 10^{-5} \left( \frac{\pi R_0 p_0}{4N_{PL}} \right) \frac{b}{f_{SH}^2}. \quad (V/A^2) \quad (2) \]

A diagram of the shock circuit is given in Fig. 6. Thus

\[ 2N_{BL} Z_{BL} I_{SH} + Z_{SH} = 2N_{BL} V_{BL} = Al^2. \quad (3) \]

Setting \( \alpha = A_{PL}^2 / 2NV_{BL} = Al^2 / 2N_{PL} Z_{PL} \) and \( y = I/I_{BL} \), this reduces to

\[ dy/dx = 1 - y - \alpha y^2. \quad (4) \]

For \( y = 0 \) at \( x = 0 \) the solution is

\[ \tanh^{-1} \left( \frac{1 + 2\alpha y}{1 + 4\alpha y} \right) = \tanh^{-1} \left( \frac{1}{1 + 4\alpha y} \right). \]

\[ = (1 + 4\alpha)^{1/2} x, \quad (5) \]

With the limit for \( \alpha \to 0 \) given by \( \log (1 - y) = x \). Graphs of \( I/I_{BL} \) and \( \alpha y^2 = V_s / 2N_{BL} V_{BL} \) versus \( t/\tau \) are given in Fig. 7. It is seen that the back emf \( V_s \) reduces the attainable currents and coil voltages from their values \( I_{BL} \) and \( 2N_{BL} V_{BL} \) in the simple line-driven circuit.

### IV. THE MAGNETIC COMPRESSION CIRCUIT, LAYOUT AND NOTATION

A schematic arrangement of the magnetic compression circuit is shown in Fig. 8. The superconducting storage coil is assumed to consist of stabilized NbTi windings at \( 4\times K \), surrounded by a secondary at room temperature, or possibly \( 7\times K \). The superconducting switch \( R_{SC} \), on going normal, reaches a value \( R_{SW} \), much greater than that of the ballast resistor \( R_p \), which therefore dissipates most of the energy involved in the resistive magnetic energy transfer.

In the following the subscripts \( SH, C, P \) and \( S \) refer to the shock, compression, primary and secondary coils, respectively. The subscript \( O \) refers to conditions before switching the magnetic energy. A radial section through a coil is shown in Fig. 9:

- \( A_R = \) actual thickness of coil \( \text{(cm)} \)
- \( w = \) no. of coil layers in \( A_R \)
- \( \lambda = \) coil volume filling factor
- \( L = \) length of coil subdivision \( \text{(cm)} \)
- \( N = \) total number of turns in one coil subdivision*
- \( N_1 = N/L = \) no. of turns per cm \( \text{(cm}^{-1}) \)
- \( R_{SC} = \) thickness of superconducting material in \( A_R \) \( \text{(cm)} \)
- \( B = \) magnetic induction in a solenoid \( \text{(G)} \)
- \( R_0 = \) mean coil radius \( \text{(cm)} \)
- \( W = 1.25 \times 10^{-8} B^2 R_0^2 \times 10^{-8} = \) magnetic energy per unit length of solenoid \( \text{(J/cm)} \)
- \( W_1 = W = \) magnetic energy per coil subdivision \( \text{(J)} \)
- \( k = \) coupling coefficient from primary to secondary of storage coil

*In the case of a one-turn coil, as in a conventional Q pinch, \( N = 1 \) is associated with the length \( L \).
\[ \alpha = \frac{A}{I_{\text{BL}}/2N_{\text{BL}}} Z_{\text{BL}} \]

\[ y = \alpha \]

\[ \alpha^2 = \frac{y}{2N_{\text{BL}}} V_{\text{BL}} \]

**Fig. 7.** Shock-coil current and voltage as functions of time for various plasma-sheath back-emf parameters.

\[ I_1 = \frac{B}{0.4\pi} = \text{current per unit length of solenoid} \quad (\text{A/cm}) \quad (3) \]

\[ I = \frac{I_1}{N} = I_1/N_1 = \text{current per turn} \quad (\text{A}) \quad (4) \]

\[ j = \frac{\lambda B}{0.4\pi \Delta R} = \frac{\lambda I_1}{\partial R} = \text{average current density of solenoid} \quad (\text{A/cm}^2) \quad (5) \]

\[ j_{\text{SC}} = \frac{I_1}{\Delta R_{\text{SC}}} = \text{superconducting current density in primary solenoid} \quad (\text{A/cm}^2) \quad (6) \]

**Fig. 8.** Schematic diagram of the compression-field circuit of a separated-shock O pinch with cryogenic inductive energy storage.

**Fig. 9.** Radial section of compression coil.
The energy dissipated in \( R \) is
\[
W_R = \frac{1+x-k}{1+x} W_{P0} .
\]

The energy transferred to the load \( L_c \) is
\[
W_C = \frac{xk}{(1+x)^2} W_{P0} .
\]

The energy remaining in \( L_s \) (and filling \( L_p \)) is
\[
W_S = \frac{k}{(1+x)^2} W_{P0} .
\]

The transferred energy is a maximum when \( x = 1 \).
For \( k = 1 \) one half of the energy \( W_{P0} \) is dissipated and the secondary and load inductors each get one quarter.

VI. RELATIONS BETWEEN THE INDUCTANCES IN MAGNETIC ENERGY STORAGE

In the following we take \( L_c = L_s \) (\( x = 1 \)). Since \( W_c = (k/4)W_{P0} \) we have
\[
R_p = \frac{2/k^2}{R_c B_c/B_{P0}} ,
\]
where \( R_c \) and \( B_c \) are given. We also take \( R_p R_s \)
\((\approx 1.1)\) as given, thus determining \( k = (R_p/R_s) \).

The field \( B_{P0} \) will be taken as an independent variable. We shall also take the initial current \( I_{P0} \)
in a primary-coil subdivision as an independent variable. We then find
\[
N_p = B_{P0} I_p / 0.4\pi I_{P0} .
\]
From the condition \( I_{SC1} = I_{C1} \), we have
\[
N_{1C} = \frac{R_S}{R_C} \tag{3}
\]
We note that
\[
I_C = \frac{B_C I_C}{0.4 \pi N_C} \tag{4}
\]

**VII. RELATIONS IN THE SUPERCONDUCTING CIRCUIT**

A. The Superconducting Switch

From (V.7) we have, for \( x = 1 \),
\[
W_{PO} = \frac{1}{2 \pi} \left( 1 - \frac{k}{2} \right) \frac{B_C}{s} \tag{1}
\]
From (V.3), for \( x = 1 \),
\[
R = \frac{\frac{1}{s}}{s} \left( 1 - \frac{k}{2} \right) \tag{2}
\]
\[
R = \frac{1}{s} \left( 1 - \frac{k}{2} \right) W_{PO} \tag{5}
\]
The voltage across the resistors shortly after the instant of switching is
\[
E_0 = R T_{PO} = \frac{1}{s} \left( 1 - \frac{k}{2} \right) \frac{W_{PO}}{s} \tag{4}
\]
Referring to Fig. 8, \( R = \frac{R_B R_{SC}}{R_B + R_{SC}} \). We shall assume that \( R_{SC} \gg R_B \) so that \( R \approx R_B \). We also assume that only a fraction \( \varphi \) of the energy \( W_{PO} \) of the superconducting coil be dissipated in \( R_{SC} \), and hence appear as a load on the refrigerator, we set
\[
R_{SC} = \frac{2 - k}{2 \varphi} R = \frac{2 - k}{2 \varphi} \frac{W_{PO}}{s} \tag{5}
\]
Let \( A_{SC} \) be the cross sectional area of superconductor carrying \( I_{PO} \) in \( R_{SC} \) before the switching takes place and let \( l_{SC} \) be the length of superconductor. Let \( J_{SC} \) be the (near-critical) current density of the superconductor before switching. Then from (5)
\[
\eta_{com SC} = \frac{2 - k}{2 \varphi} \frac{W_{PO}}{s} \tag{5}
\]
where \( \eta \) is the area ratio of stabilizing matrix material plus superconductor to superconductor. It is estimated that values of \( \eta \) as small as 1.5 can be attained. The volume of superconductor in \( \text{cm}^3 \) is
\[
V_{SC} = l_{SC} A_{SC} = \frac{2(2 - k)^2}{k \varphi} \frac{W_{PO}}{s} \tag{6}
\]
In terms of the switching voltage \( E_0 \), the area of the superconductor is gotten from (4):
\[
A_{SC} = \frac{2(2 - k)}{k} \frac{W_{PO}}{s} \tag{7}
\]
The length of superconductor is \( V_{SC}/A_{SC} \):
\[
l_{SC} = \frac{2 - k}{2 \varphi} \frac{W_{PO}}{s} \tag{8}
\]
In order to express in terms of presently available, fabricated, stranded superconductor we note the empirical value \( J_{SC} = 5 \times 10^5 \text{A/cm}^2 \) for braided NbTi switches and estimate the value \( \eta_{com} = 5 \times 10^{-5} \text{ohm-cm} \) for NbTi stabilized in a CuNi or German-silver matrix.

B. The Superconducting Primary Solenoid

In order to express in terms of presently available, fabricated superconductor we note the following empirical relationship between \( J_{SC} \) (in \( \text{A/cm}^2 \)) and \( B_{PO} \) in G for \( B_{PO} \leq 50 \text{kG} \):
\[
B_{PO} I_{SC} = 6 \times 10^3 \text{G/\text{cm}^2} \tag{9}
\]
Let \( \Delta R_{SC} \) be the thickness of superconductor in the primary solenoid. Using (9):
\[
\Delta R_{SC} = \frac{B_{PO}}{0.4 \pi A_{SC}^2} = 1.53 \times 10^{-10} \text{cm} \tag{10}
\]
The volume of superconductor in length \( l_p \) is then
\[
V_{SC} = 2 \pi R l_p \Delta R_{SC} = 1.67 \times 10^{-9} \frac{l_p R}{\text{B}_{PO} R_{SC}^2} \tag{11}
\]
where we have used (VI.1).

VIII. PARAMETERS OF A 10-KEV SCIENTIFIC FEASIBILITY EXPERIMENT

A. Choice of Shock and Compression-Coil Dimensions and Fields

We start with a considerably larger discharge tube (about a factor of 2) than those in previous experiments and proceed outward.

- Inner radius of discharge tube = \( b = 12 \text{ cm} \)
- Inner radius of shock heating coil = \( x = 15 \text{ cm} \)
- Thickness of shock heating coil = \( 0.5 \text{ cm} \)
- Outer radius of shock heating coil = \( 13.5 \text{ cm} \)
In choosing the inner radius \( R_c = 16 \text{ cm} \) of the compression coil we leave space for shock-circuit connections and insulation around the shock-heating coil. We assume room-temperature copper \( (\eta = 1.73 \times 10^{-6} \text{ ohm-cm}) \) and a containment time of 42 ms or greater.

As a convenient value we choose the compression-coil thickness \( \Delta R_c = 2.5 \text{ cm} \). The \( L/R \) decay time of the compression coil is then

\[
\tau_c = \frac{2\pi R_c \Delta R_c \times 10^{-9}}{\eta} = 0.15 \text{ sec} \ ] (1)

The skin depth in the copper is approximately

\[
\delta_E = 1.1 \times 10^4 \eta^{-\frac{1}{2}} = 6 \text{ cm} \] (2)

We choose a rise time \( \tau_s = 2 \text{ ms} \). It is shown elsewhere that eddy-current loss \( W_{ed} \) represents approximately the following fraction of the "d.c." joule loss \( W_J \) during the pulse of compression field:

\[
\frac{W_{ed}}{W_J} = \frac{\frac{V^2}{n}}{\frac{V}{\eta}} = \frac{2}{\frac{8}{\eta}} = 0.15 \text{ sec} \] (3)

If we choose a 5-layer coil \( (\nu = 5) \) with a filling factor \( \lambda = 0.7 \), the winding thickness if

\[
d \approx \Delta R_c / 5^{\frac{1}{2}} = 0.6 \text{ cm} \] (4)

The plasma and magnetic-field parameters

From the curves of Fig. 4 we see that a D-T ion temperature \( kT_e = 10 \text{ keV} \) \((8.9 \text{ keV} \text{ for D-D})\) can be obtained at a filling pressure \( p_0 \) of about 3 mT and a final radial compression factor \( x_r = 0.3 \).

The corresponding final magnetic field at \( B = 1 \text{ is} \) \( 43 \text{ kG} \). Allowing for losses and experimental latitude, we choose

\[
\begin{align*}
B_f &= 50 \text{ kG} \\
B_s &= 9.2 \text{ kG} \\
n_e &= 3.0 \times 10^{15} \text{ cm}^{-3}
\end{align*}
\]

Correspondingly, the maximum plasma back emf \( V_s = 153 \text{ kV} \). The shock and pre-compression temperatures are given by

\[
kT_{SH} = 2kT_e = 5.8 \text{ keV},
\]

and the shock velocity is

\[
v_s = 3.11 \times 10^7 \text{ cm/sec}.
\]

We note that the shock-coil inductance outside the initial plasma sheath, for \( L_{SH} = 100 \text{ cm} \), is

\[
4\pi \times 10^{-9} (\frac{x_o^2}{c^2} - 1) = 9.8 \text{ nH}.
\]

This corresponds to \( 4.9 \text{ nH} \) for each of the two Blumlein feeds. We assume \( 10 \text{ nH} \) total parasitic inductance. Thus we take the initial inductance in series with the plasma sheath at one feed slot to be

\[
L_{SH} = 10 \text{ nH}.
\]

The transit time of the plasma sheath to the center of the shock coil is approximately \( 12 \text{ cm} / \frac{6 \times 10^7}{3.11 \times 10^7} \text{ cm/sec} = 0.39 \mu\text{sec} \). We assume that a sufficient approximation to ideal behavior can be achieved with a rise time \( \tau_r = 0.1 \mu\text{sec} \). From (III.5) the characteristic impedance is

\[
Z_{BL} = L_{SH} / 2\tau_r = 0.050 \text{ ohm}.
\]

If the current \( I_{BL} = V_{BL} / Z_{BL} \) were completely effective in producing \( B_s \) we should require \( 0.73 \text{ MA/m} \). However Fig. 7 shows that the plasma back emf has the effect of raising this by a factor of about \( 2.2 \). Thus we have

\[
\begin{align*}
I_{BL} &= 1.6 \text{ MA} \\
V_{BL} &= 80 \text{ kV}.
\end{align*}
\]

From Fig. 7 (for \( \alpha = 2.2 \)) we find that at \( t = \tau_r \), the plasma back emf is

\[
V_s = 0.45 \text{ kV}, B_{BL}V_s = 145 \text{ kV}.
\]

This is approximately consistent with the value \((153 \text{ kV}) \) obtained from Fig. 4. The length of the line is given by

\[
L_{BL} = 2 \mu \frac{bc}{v_s} = 45 \text{ meters}.
\]

From (II.8) the thickness of mylar insulation must be

\[
t_{BL} = 0.68 \text{ mm}.
\]

From (II.9) the width of the Blumlein line must be

\[
w = 3.0 \text{ meters}.
\]
C. The Inductive Storage Secondary Circuit

This circuit consists of the compression coil and the single-turn secondary of the inductive energy store (Fig. 8). It must receive an amount of energy equal to that of the compression coil. From (IV.1) for \( B_C = B_f = 60 \text{kG} \)

\[ W_C = 1.15 \text{ MJ/m} \]

We take a magnetic field \( B_{PO} = 20 \text{kG} \)
in the primary, superconducting storage coil and assume a coupling coefficient \( k = 0.85 \). Using (VI.1), we derive for the primary and secondary coil radii:

\[ R_P = 104 \text{ cm} \]
\[ R_S = 122 \text{ cm} \]

The number of turns per unit length of compression coil is given by (VI.1):

\[ N_{IC} = 7.6 \text{ cm}^{-1} \]

For \( v = 5 \) layers, we arrive at

\[ n_C = 1.5 \text{ turns/cm/layer} \]

The current per turn is

\[ I_{IC} = \frac{B_C}{0.44 \mu_{IC}} = 6.24 \text{ kA} \]

As a check we note that the inductances of the secondary and compression coil per unit length are the same:

\[ L_{IC} = 4\pi R_C^2 X 10^{-9} = 0.58 \text{ mH} \]
\[ L_{IS} = 4\pi R_S^2 X 10^{-9} = 0.58 \text{ mH} \]

The linear current density in the secondary is 6.24 kA/cm, and the secondary magnetic field is

\[ B_S = 0.44 \pi X 6.24 = 7.84 \text{ kG} \]

From (IV.1) the secondary-coil magnetic energy is

\[ W_{MS} = 1.15 \text{ MJ/m} \]

(the same as \( W_C \)). The maximum voltage on the secondary is approximately

\[ V_S = 10^{-8} \mu_S^2 \frac{R_S}{\tau_S} = 1.8 \text{ kV} \]

D. The Superconducting Primary Circuit

From (VII.4) the maximum primary voltage over length \( l_P \) is related to \( I_P \) by the relation

\[ E_0 l_P / I_P = 3.11 \times 10^7 \]

For \( B_{PO} = 20 \text{kG} \) the ampere turns per cm are

\[ N_{LP} l_P = B_{PO}/0.44 \pi = 1.59 \times 10^4 \]

We assign as a maximum value, \( V_P = 40 \text{kV} \). The "charging" current \( I_P \) is assigned the value 25 kA. Thus the module length is \( l_P = 32 \text{ cm} \), and \( N_{LP} = 0.63 \text{ turns/cm} \). Thus a braided superconducting cable would have a width of about 1.6 cm. From (VII.9) and (VII.10) the superconductor thickness is

\[ d_{SC} = 0.053 \text{ cm} \]

and the volume per meter length is

\[ V_{SC} = 3.48 \times 10^3 \text{ cm}^3/\text{m} \]

E. The Superconducting Switches

From (VII.10) the volume of switch superconductor is

\[ V_{SW} = \frac{214}{\varphi} \text{ cm}^3/\text{cm} \]

where \( \varphi \) is taken as \( 5.5 \times 10^{-5} \) ohm-cm, \( a = 1.5 \) and \( \tau_s = 2 \times 10^{-3} \text{ sec} \). If we take the volume of switches to be 1.6 times that of the windings, we find

\[ \varphi = 214(1.5 \times 480) = 0.038 \]

The energy dissipated in the superconducting switch is

\[ W_{Diss} = (\varphi \varphi/k)w_C = 0.18w_C = 2.05 \times 10^5 \text{ J/m} \]

If there is one discharge every 15 minutes the power to be removed by the refrigerator at 4°K is

\[ P_{Diss} = 230 \text{ watts/m} \]

Using (VII.8), the length of superconductor in each switch is
\( f_{50} = 268 \) meters.

The superconductor volume is \( 5.6 \times 10^3 \text{ cm}^3/\text{m} \).

IX. COST ESTIMATE OF A 10-KEV SCIENTIFIC FEASIBILITY EXPERIMENT

A. Superconductor Circuit

For each meter of length we have found volume of coil and switch superconductor = \( 9.1 \times 10^3 \) cm\(^3\)/m.

Cost at $1.9/cm\(^3\) for fabricated coil wire $17.2 K/m

The coil form is backed by an outer shell of stainless steel with a working stress \( \sigma = 18.8 \) kpsi. Its thickness is \( t = 0.59 \times 10^{-6} \Omega R p / \sigma = 1.4 \text{ cm} \).

Its volume for one meter length is \( V_{SS} = 200 \times 10^{-6} = 0.95 \times 10^5 \text{ cm}^3/\text{m} \).

Its weight is \( 7.8 \times 0.95 \times 10^5 / 4.54 \times 10^2 = 1632 \text{ lb/m} \).

Cost at $2/lb for fabricated coil backing $3.3 K/m

Cost of plastic inside coil form, 2 cm thick at $1.5/lb. $0.9 K/m

Labor for fabrication and assembly of one-layer coil $1.7 K/m

Total cost of primary coil plus switch $23.1 K/m

B. Primary-coil Dewar

The cost is scaled from an existing 13-foot Dewar, 30 feet long, assuming that the joints of a long system will cost the same as the ends of the present Dewar, which has a 1.25" Al inner vessel and a 3/8" S.S. outer vessel. The cost of the present Dewar is about $13 K/meter, including supports.

As a basic scaling we assume cost \( = (\text{volume})^{0.8} = 1550 D^{1.6} \), where \( D \) is the diameter in meters. Thus

Cost of 2.5-m diameter Dewar: $6.7 K/m

Installation at 4.7 man-day/m($100/man-day) $0.45K/m

Dewar cost with installation $7.2 K/m

C. Refrigeration

(1.) Radiation and support loss at \( 4^\circ \text{K} \). Experience indicates a figure of 0.026 watt/ft\(^2\), which corresponds to

\( Q = 0.9 D_{1n}^2 \) watts (\( 4^\circ \text{K} \))/m

where \( D_{1n} \) is the inner diameter of the Dewar in meters. In our case \( Q = 2.2 \) w/g and is negligible.

(2.) Lead-in loss at \( 4^\circ \text{K} \). Experience indicates a figure of 2 w/kA. The ballast resistors connected every 32 cm could cause an appreciable heat loss. However we propose to make this negligible by using pneumatically driven "banana-plug" switches (at 25 kA pulse rating) to break thermal contact during the times between pulses.

(3.) Dissipation in the superconducting switches. In Section (VIII.E) we found a refrigeration requirement of 230 watts/m, assuming 15 minutes between discharges. From Strobridge's\(^1\) and Fraas's\(^12\) curves for refrigeration capacities in the \( 10^5 \) to \( 10^6 \) watt (\( 30^\circ \text{K} \)) range we find

\( u = \text{fraction of Carnot efficiency} = 0.33 \).

Thus the input power to the refrigerator is given by

\[
\text{Input power} = \frac{1}{u} \times \frac{300 - 4}{4} = 222.
\]

In our case the refrigeration power is thus

\[ P_{1n} = 222 \times 230 = 51.1 \text{ kW/m} \]

For a 100-m length we are in the range of 5000 kW total. From Strobridge's Fig. 4 we find a unit cost of $380/kW. Thus we have:

Refrigerator cost at $380/kW $19.4 K/m

For a labor cost we assume 4 man-days per meter at $100/man-day. Thus

Refrigerator installation $0.4 K/m

Total refrigerator cost (15 min) $19.8 K/m

D. The Primary Charging Circuit

Figure 11 shows the charging circuit with its generator. Each primary module of length \( l \) has its superconducting switch in the normal state and the voltage \( V \) is increased until the current \( I_{1p} \) is built up. The superconducting switches are brought to their superconducting state, and the generator current \( I_{1p} \) is reduced to zero. The isolating switches are then opened, leaving the circulating currents \( I_{1p} \) in each module loop as shown. We assume that all primary modules are charged in series. Thus for a device 100 meters in length the total primary inductance, from (IV.7) with \( N_{1p} = 0.63 \) turns/cm and
SUPERCONDUCTING SWITCHES

![Diagram of Superconducting Switches]

**Fig. 11. Charging circuit for superconducting primary coils.**

\[ R_p = 104 \text{ cm}, \text{ would be} \]

\[ L_p = 1.7 \text{ H}. \]

Assuming a charge time of 300 sec to 25,000 A, we arrive at an average charging voltage of 71 volts. This indicates that rectifier supplies would be appropriate for this application. Their rating would be in the range of 1800 kVA. We take a unit cost of $0.21 K/kVA and arrive at

cost of charging generators at

\[ \$0.21 \text{ K/kVA} \]$ 3.6 K/m

To this we add the cost of the isolation switches (3 per meter), rated at 40 kV, 25 kA when closed. Each is estimated to cost $0.5 K. Thus

**Cost of isolating switches** $ 1.5 K/m

**Cost of primary charging circuit** $ 4 K/m

E. The Secondary Coil

We take a single turn of radius \( R_s = 122 \text{ cm} \), one cm thick for self support, since there is no appreciable magnetic pressure. At $0.56/lb the material cost for 250 lb is \( \$300/m \). Fabrication is taken as \( \$300 \). Installation at 2 man-days is \( \$200 \).

**Cost of secondary** $ 1 K/m

F. The Ballast Resistors

Each must carry a peak current of 25 kA and dissipate \( 2.3/4 \times 0.6 \text{ W} \). Thus, for a temperature rise of 20°C, electrolytic resistors, each with a volume of about 13 liters, are suitable. These are estimated to cost \$40 each including labor. With allowance for the banana contactors of \$70 per pair we arrive at

**Cost of ballast resistors and contactors with labor** $ 1 K/m

G. Unit Energy Cost of the Superconducting Inductive Energy Source

We neglect the cost of triggering, since pulses to drive the superconducting switches normal can probably be derived from the shock-heating circuit. Adding the above figures and allowing 10% for contingencies, we arrive at a total cost of \$58 K/m.

Since we deliver \( 1.15 \times 10^6 \) joules to the compression coil, this gives a unit cost of \( 5 \text{ kJ/J} \).

H. The Compression Coil

We assume water-cooled square copper stock wound into pancakes in a conventional manner. The cost of material, fabrication and potting and miscellaneous fittings is taken as \$2.0 K/m. Installation at 4 man days per meter is \$400. Thus

**Cost of compression-coil backing** $ 3 K/m

I. Compression-Coil Backing

We consider of high strength stainless-steel backing of both the 37-cm coil and its horizontal feed. Material cost of 1550 lbs at \$0.60/lb = \$930/m. Fabrication and miscellaneous fitting = \$1700/m. Installation (2 man days/m) = \$200. Thus

**Cost of compression-coil backing** $ 3 K/m

J. Shock-Heating Apparatus

We estimate this item at the total cost per joule of the Scyllac device, exclusive of the building. The figure \$0.6/J includes materials, parts and labor.

In the present experiment the shock magnetic field is 9.3 kG (Fig. 4 at 3 mT with \( x_T = 0.3 \)). The energy in the 12-cm radius coil is thus 16 kJ/m. Assuming equal energies in the Blumlein-line and the capacitive crow bar circuit (52 kJ/m in all), we arrive at

**Cost of shock-heating apparatus** $ 19 K/m

K. Unit cost of the overall system

<table>
<thead>
<tr>
<th>Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superconducting coil and switch</td>
<td>$23 K/m</td>
</tr>
<tr>
<td>Dewar</td>
<td>$7 K/m</td>
</tr>
<tr>
<td>Refrigeration</td>
<td>$20 K/m</td>
</tr>
<tr>
<td>Primary charging circuit</td>
<td>$4 K/m</td>
</tr>
<tr>
<td>Secondary coil</td>
<td>$1 K/m</td>
</tr>
<tr>
<td>Ballast resistors and contactors</td>
<td>$1 K/m</td>
</tr>
<tr>
<td>Compression coil and backing</td>
<td>$5 K/m</td>
</tr>
<tr>
<td>Shock-heating apparatus</td>
<td>$19 K/m</td>
</tr>
<tr>
<td>Contingency for miscellaneous items (20%)</td>
<td>$16 K/m</td>
</tr>
<tr>
<td><strong>Total cost</strong></td>
<td>$55 K/m</td>
</tr>
</tbody>
</table>
Total energy in compression and shock systems
1180 KJ/m

Unit energy cost of Ω pinch
8 $/J

L. Total Cost of the Ω Pinch System

For a toroidal system we take an estimated circumference of 180 meters (see Section X below), arriving at a total Ω pinch cost of $17 M. The present low compression design is not suitable for a linear experiment, since the low density and correspondingly long containment time would require an excessive length (Section X below).

M. Building Costs

For the toroidal experiment we consider a building roughly 230-ft square at $105/ft^2$, giving a building cost of about $5.6 M.

X. THE OVERALL SYSTEM

A conceptual view of a portion of a separated-shock Ω pinch, based on the foregoing parameter study, is shown in Fig. 12. Here the Blumlein lines are shown folded to accommodate their length. In addition each shock heating coil is shown as fed from its two ends by two Blumlein lines. This arrangement presents insulation and parasitic inductance difficulties which may be better met by other geometrical configurations. In any case the high voltage pulse from the shock-heating circuit must not be allowed to couple to the slow compression circuit. This can be accomplished by surrounding the shock-heating coil by thin cylinders of (for example) stainless steel which return the magnetic flux of these coils inside the compression coils.

The rise times of the shock and compression circuits are greatly different. Therefore one can choose the thickness of these cylinders to lie between the skin depths corresponding to these times in order to provide this isolation with little Joule-heating loss as the compression field penetrates the cylinders.

The length of a toroidal system is determined by considerations of toroidal equilibrium and MHD stability. We assume the l = 1, 0 helical system presently being tested in the Scyllac experiment. The minimum aspect ratio is set by considerations of the strength and wavelength of the helical fields and the plasma radius. The ratio of shock-coil radius to plasma radius, for wall stabilization of the m = 1 MHD mode, will be important. On the basis of present theoretical and experimental indications a rough estimate of the diameter a toroidal device is from 50 to 70 meters. A conceptual drawing of the main components of a toroidal device is shown in Fig. 13.

The length $l_{Th}$ of a linear system is determined by loss of the ions at their thermal speed through ends constricted by mirrors and is given by

$$l_{Th} = \left(\frac{2}{v_{Th}^2}\right)\frac{\tau_0 \nu_{Th}}{\gamma M}$$

where $v_{Th} = (2kT/m)^{1/2} = 0.9 \times 10^8$ cm/sec and $\tau \approx 0.042$ sec is the burning time of the D-T plasma.

The factor $\gamma M$ is the high-β mirror ratio. We assume $\beta = 0.9$ and vacuum mirrors of 2.5, leading to $\gamma M = 9$. Thus a device about 5-km long is indicated. A conceptual diagram is given in Fig. 14.
Fig. 13. Conceptual view of a toroidal separated-shock Ω pinch.

Fig. 14. Conceptual view of a linear separated-shock Ω pinch.

REFERENCES


5. To the author's knowledge this idea was first put forward by R. L. Morse, 1971.


9. F. L. Ribe, Los Alamos Scientific Laboratory Report (to be published). Here the eddy current loss occurs only on the rise of the compression pulse.

10. We are indebted to F. J. Edeskuty (1971) for these cost estimates.


