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NUMBER OF NEUTRONS PER FISSION FOR 25 AND 49

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ABSTRACT

A direct measurement of the number of neutrons per fission has been made in the graphite block, using the cyclotron as a neutron source. Fissions were produced by the thermal flux which is available well back in the graphite block; the number of fast neutrons given off was measured by making a volume integral of the resonance activity acquired by indium foils and comparing this with a similar integral from a Ra-Be source of known output; the number of fissions was measured by counting the fissions from a thin foil on the face of a case containing the sample of fissionable material, and knowing the ratio of weights of material in the foil to material in the sample. The measurement gives a rather accurate value of the ratio $\mathcal{V}/Q$, where $Q$ is the neutron output of Ra-Be source #43; thus any improvement in the absolute calibration of a Ra-Be source can readily be applied to obtain an improved value of $\mathcal{V}$. The values found using source #43, were for 25, $\mathcal{V}/Q = (2.82 \pm 0.03) \times 10^{-7}$ sec, and for 49, $\mathcal{V}/Q = (3.30 \pm 0.05) \times 10^{-7}$ sec. This gives a ratio, independent of $Q$ and of any possible difference in the fission spectra, of $\mathcal{V}_{49}/\mathcal{V}_{25} = 1.17 \pm 0.02$. The current best value of $Q_{43}$ is $867 \times 10^7$ neutrons per second, which gives $\mathcal{V}_{25} = 2.44$, $\mathcal{V}_{49} = 2.86$.

A modification of the method was used to measure, in a manner independent of $Q$, the number of neutrons per neutron absorbed, $\tilde{\mathcal{V}} = \mathcal{V}/(1 + \alpha)$. These measurements gave $\tilde{\mathcal{V}}_{25} = 2.16$ and $\tilde{\mathcal{V}}_{49} = 1.99$ for thermal neutrons in the graphite block, using 644 barns and 1057 barns as the respective capture cross sections at 0.025 ev, and McDaniel's data on the variation of the 49 absorption cross section with energy. The method is less straightforward and presumably
much less accurate than the \( \sqrt{Q} \) measurement. Assuming \( Q \) as above, we get from these data the values \( \alpha_{25} = 0.13 \), \( \alpha_{49} = 0.14 \), for thermal neutrons.

There was no detectable difference in the shape of the slowing-down density curves from 25 and 49, indicating that the fission spectra for the two are similar.
Introduction

The determination of the number of neutrons emitted when fission occurs has been of the greatest interest since it became apparent that chain reactions might be sustained by fissionable material. In particular, the critical mass of a metal gadget depends on this quantity as \[ \frac{1}{\left(\sigma_f(\gamma - 1 - \delta)\right)^{1.7}} \], where \( \gamma \) is the number of neutrons per fission and \( \delta \) is the branching ratio, \( \delta = \sigma_r/\sigma_f \) for the pure material (\( \sigma_r \) refers to radiative capture, that is, the \((n,\gamma)\) process). The fission cross section, \( \sigma_f \), has been measured directly in the energy region which is of importance for the gadget (5 kev to 2 mev, depending on the amount of hydrogen present). The principal quantity measured by the experiment described here is \( \gamma \) for fission by thermal neutrons. An experiment has already been performed by Wilson, Woodward and DeWire \(^1\) to show that \( \gamma \) remains substantially constant as the energy of the fission-producing neutrons is raised from thermal energies to several hundred kilovolts. It is therefore important to have an accurate value of \( \gamma \) for thermal fission. The present paper also describes a measurement of \( (1 + \delta) \) for thermal fission, and summarizes the results of other measurements of this quantity. At present no method of measuring \( \delta \) at high energies has been found; it is expected theoretically to decrease with increasing neutron energy. The measurement of \( \gamma \) also completes the circle formed by the measurements in the thermal region of \( (1 + \delta) \) and \( \gamma = \sqrt{(1 + \delta)} \), the number

\(^1\) LAMS-95, p.10, experiment 15.
of neutrons emitted per thermal neutron absorbed.

Method of Measurement

To measure \( N \) directly one must count the number of fissions produced in a sample by a thermal flux, and count all the fast neutrons given off by the sample. The graphite block, used with the cyclotron as a source of primary neutrons, provides a strong flux of nearly pure thermal neutrons, and at the same time can be used with a resonance detector (such as indium) to measure the total fast-neutron output of any source which is placed in it. The fast-neutron measurement depends on the fact that an indium foil, covered with cadmium to eliminate thermal activity, when placed in the block will acquire an activity proportional to the flux of neutrons of \( 1.4 \) ev energy present, and therefore proportional to the slowing down density at \( 1.4 \) ev, the energy of indium resonance neutrons.

The slowing-down density at any given energy, \( q(E) \), is the number of neutrons passing from above to below that energy per cubic centimeter per second; it is therefore clear that if we surround a source by a slowing-down medium, the integral over all space of \( q(E) \) is equal to the number of neutrons of energy greater than \( E \) given out by the source per second, if there is no absorption in the medium. Since practically no neutrons of extremely low energy come from a fission source, one can measure a quantity proportional to the number of fission neutrons given off by making such an integral in a graphite block with cadmium-covered indium foils. The proportionality constant can then be determined by making a similar integral but replacing the fission source with a natural source of known strength.

The accuracy of the neutron counting, then, depends upon the standardization of a Ra-Be source. Two programs to make such a measurement have been launched, one in this laboratory and one at Chicago, and it was felt that rather
accurate results could eventually be expected from both of them.

The fission rate in the sample was measured by counting the fissions from a thin foil placed on the surface of the sample and containing a very small, known fraction of the total fissionable material in the sample. \( \psi \) is then given by the number of neutrons divided by the number of fissions;

\[
\psi = \frac{Q}{F} \frac{\sum A_f dV}{\sum A_{rb} dV} \cdot \frac{m_s}{m_f}
\]

where \( A_f \) and \( A_{rb} \) are the saturation activities of indium foils due to the fission source and the Ra-Be source, respectively; \( Q \) is the number of neutrons per second from the Ra-Be source; \( F \) is the number of fissions per second from the thin foil; and \( m_f \) and \( m_s \) are the masses of the thin foil and the sample, respectively. (Some small corrections have been omitted).

**General Arrangement**

Fig. 1 shows the arrangement of the indium foils and ion chamber in the graphite block. Our block was 7" wide, 6'8" high and 11' long. The fast neutrons from the cyclotron come in at one end and are slowed to a nearly pure thermal neutron flux in the first five feet. This leaves an approximate cube seven feet on a side at the end of the block away from the cyclotron in which to make the fast-neutron measurements. The foils were placed along the axis of the block, on the side of the chamber away from the cyclotron (to minimize the background of residual fast neutrons always present in the block). Their distances were approximately 10, 25, 40, 55, 70, and 85 cm from the source. The volume integral is made by taking the values of \( \psi \) along one radius and assum-
ing that the distribution of fission neutrons about the point source is spherically symmetric. The chamber lead runs up to the preamp on the top of the block.

An accurate measurement of the number of neutrons emitted by a source using the method of indium foils in a graphite block requires that the following conditions be fulfilled: 1) The leakage of fast neutrons out of the block before they are slowed to the indium resonance energy must be negligibly small. 2) The absorption of neutrons in the block during the slowing-down process must also be negligibly small. 3) The graphite block must be free from gaps and holes and of as uniform density as possible. Failure to meet these requirements introduces errors in the volume integral of the indium foil β-activity for which corrections may be calculated if they are sufficiently small.

The requirements for our problem are less rigorous because we wish to compare two neutron sources, Ra-Be and fission neutrons, which have substantially the same slowing-down ranges in graphite (although their neutron spectra are considerably different). This means that one expects the fractional neutron losses from absorption and leakage, and the magnitude of any gap corrections to be similar for the two. Nevertheless considerable care was taken to minimize leakage, absorption and gaps.

The indium foil counting followed the highly standardized Chicago technique, using 2.4 gm foils, .127 cm thick Cd shields, and thin aluminum-walled β-counters. The counting was reproducible to within the statistical accuracy expected from the number of counts.

Fission Counting

The ion chamber for counting fissions and its lead to the top of the block introduced into the block the only sources of absorption other than the
foils and the graphite itself. They introduced also the largest air gaps. It was therefore important that the volume of both chamber and lead be as small as possible and that they present as little neutron absorption as possible. The volume occupied by the chamber was reduced to $103 \text{ cm}^3$. Actually in the course of our measurements two chambers slightly different in design were used. The first contained about 100 gm each of paraffin and aluminum. Most of this weight was in the lead, hence only a third of these materials was within a foot of the neutron source within the chamber. The second chamber contained no paraffin but weighed 240 grams. Again much of the weight was in the lead to the top of the block. The use of such small amounts of materials and such small space for the chamber and leads was made possible by using air as a chamber gas and by operating with the collecting electrode at high potential, the case serving as both the fixed potential electrode and electrical shield.

Whereas 25 foils do not give off a bothersome number of $\alpha$-particles, and slow amplifiers suffice, this is not true for 49. The counting of 25 fissions was done with a slow amplifier in the first measurements and a fast one in the last. The 49 fissions were counted using a fast amplifier throughout. The profitable use of fast amplifiers was possible because we found that collection of electrons in air without appreciable capture was possible at 2500 volts/cm when the electron path length was $\sim 1 \text{ cm}$. The slow amplifier and preamp were of the stable gain, inverse feedback design, while the fast amplifier and preamp were of the Crouch type, wherein the gain is kept constant only by a regulated plate voltage supply and constant A.C. line voltage. Both schemes gave good plateaus. However, the relatively higher noise in the fast amplifier made an extrapolation to zero bias of the pulse discriminator somewhat more difficult, but still good to less than one percent.
Samples and Foils

To find the number of fissions in the total amount of material present we must know the ratio of weight of active material in the thin foil to weight of active material in the sample. For $^{49}$ the thin foil was made by transferring quantitatively an aliquot of the total sample on to a thin platinum disk. The value thus obtained was checked by comparing the fission counts from this foil with the fission counts from a very small $^{49}$ foil which had been $\alpha$-counted accurately; and $\alpha$-counting a very small aliquot from the total sample (note that the ratio is independent of the specific activity of $^{49}$)\(^2\). The 25 foils were prepared by electrolysis from material of the same isotopic constitution as the 25 sample (E-10). Direct weighing of these foils proved unreliable, apparently because their large area encourages the deposition of impurities. The 25 foil for the first ion chamber, WL-I, was determined by $\alpha$-counting\(^2\) and checked by fission-counting\(^3\); thus the relative weights of the foil depend on the weights of small, accurately known E-10 foils. The second 25 foil, E-10 H-13, was determined by comparison of fission counts against well known E-10 foils\(^4\). Since all these measurements go back to a weight of E-10 oxide, the ratio of the weights is independent of the isotopic constitution of E-10.

The sample of 25 consisted of some 20 gm of E-10 oxide, spread out over 30 cm$^2$. Its aluminum container also served as the electrode of the ion chamber. The thin foil was fastened to one face of this container, 0.4 mm of aluminum

\(^2\) We are indebted to R. W. Dodson and members of his group for these determinations.

\(^3\) By O. Chamberlain.

\(^4\) These measurements were made by Wilson, DeWire, and Woodward.
being between the foil and the upper surface of the sample. The first ion chamber was square, and the corresponding 25 container had a square cross section. The second chamber was round; the 25 was transferred to a round container. The sample had 562 mg Pu in the form NaPu₂(Ac)₃ • xH₂O, and its container was exactly similar to the round 25 container. In all cases the thin foils were made the same area and shape as the sample.

Details of the Measurement

The measurements necessary to obtain the value of \( \gamma/Q \) were all repeated many times. The course of a typical experiment was as follows: the "block background" arising from the residual neutrons of greater than thermal energy which are always present in the graphite block, was measured by placing the Cd-covered indium foils in their usual positions, but without having the sample in the ion chamber. Small monitor foils of indium were placed in the block in such a position that they would not be affected by the presence or absence of the fissionable sample, and the cyclotron was then operated at maximum intensity for a time of the order of an hour.

The thin fissionable foil and sample in the ion chamber were then placed in the block and a number-bias curve taken. If the plateau was satisfactory, indium foils and monitors were then placed in the block, the counter was turned on, and the cyclotron operated as steadily as possible for a period of twenty to ninety minutes, the time being carefully noted. Counting the foils then took from two to three hours. This comparison of foil activity with fission counts was repeated a number of times.

Finally the sample was removed from the ion chamber and source 843, a 1 gm, pressed Ra-Be source in the standard cylindrical container, was placed in the ion chamber in the position that had been occupied by the sample. The cham-
ber was then put back in the block, and indium foils in the standard positions were bombarded with the neutrons from this source, again for a time of the order of an hour. Several such Ra-Be runs were made in each experiment.

Evaluation of Data

The complete determination of $\gamma/\theta$ as outlined above was performed three times for 25 and twice for 49. It is convenient to express these results in terms of the integrals

$$I_1 = \int_0^\infty \frac{A_F(r)}{F} r^2 \, dr$$

and

$$I_2 = \int_0^\infty A_{r_B}(r) r^2 \, dr$$

which have already appeared on page 6. The integrals were evaluated by plotting the average value (for a given determination) of $A/F$ at each of the six points, drawing a smooth curve through the points and integrating numerically. The fraction of the total area which was beyond 85 cm from the sample, the farthest point measured, was 3 percent for the fission curves and 4 percent for the Ra-Be curves, according to the extrapolation we made. Table I gives the number of runs and the value of the integrals for the five determinations. The probable errors listed for $I_1$ were determined from the dispersion of the data, and for $I_2$ from the counting statistics. The errors from counting statistics for $I_1$ were around 0.4 percent.

A comparison of the slowing-down density curves for 25 and 49 revealed no difference to within experimental error. This is not a very sensitive test, but it indicates that the fission spectra of the two substances do not differ

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widely. A similar result was obtained by Fermi\textsuperscript{5).}

Checks and Corrections

Since this experiment was intended to give rather high accuracy for the value of $\sqrt{Q}$, various checks were made to try to uncover possible sources of systematic error. The reproducibility of the fission counting and neutron counting is shown in Table I, where the error calculated from the deviations from the mean agrees very well with the error expected from counting statistics.

To make sure that the true center of the Ra-Be source had been located, runs were made with various orientations of the source. These measurements indicated that the neutron center of source #43 coincides with its geometrical center.

\textbf{TABLE I}

<table>
<thead>
<tr>
<th>Determination</th>
<th>Material</th>
<th>Foil #</th>
<th># Fission runs</th>
<th># Ra-Be runs</th>
<th>$I_1 \pm \xi_1$</th>
<th>$I_2 \pm \xi_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>WL-I</td>
<td>5</td>
<td>5</td>
<td>$13,500 \pm 100$</td>
<td>$(4.578 \pm 0.003) \times 10^8$</td>
</tr>
<tr>
<td>2</td>
<td>25</td>
<td>WL-I</td>
<td>6*</td>
<td>7</td>
<td>$13,530 \pm 70$</td>
<td>$(4.545 \pm 0.003) \times 10^8$</td>
</tr>
<tr>
<td>3</td>
<td>25</td>
<td>E-10 H-13</td>
<td>6</td>
<td></td>
<td>$20,890 \pm 170$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>49</td>
<td>C-7 B-A</td>
<td>8</td>
<td></td>
<td>$40,640 \pm 270$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>49</td>
<td>C-7 B-A</td>
<td>6</td>
<td>1</td>
<td>$41,960 \pm 170$</td>
<td>$(4.580 \pm 0.009) \times 10^8$</td>
</tr>
</tbody>
</table>

* fissions counted on only three of these

\textsuperscript{5) Fermi, CP-1592, April, 1944.}
The source was then moved laterally a distance equal to the radius of the sample of fissionable material, to see what effect the finite extension of the sample would have on the neutron counting. A slight decrease (about 1 percent) in foil activity was observed, which was the order of magnitude expected. Since the correction is small and easily calculable, it was decided that the calculation would be more reliable than the experiment. This calculation will be found in Appendix I; it makes about 0.3 percent correction.

The 25 sample used in this experiment contained about 15 gm of 28. To check on the estimate that the capture by this amount of 28 would be negligible, we placed 15 gm of normal alloy in the chamber with the Ra-Be source. No effect on the Ra-Be curve was observed. Similarly, to check on the effect of the paraffin in the electrical lead of the first ion chamber, 8 gm of paraffin was placed in with the Ra-Be source, and again no effect was observed.

Between measurement 1 and measurement 2 the graphite block was partially taken apart and restacked with slightly better stacking density. Results of these two measurements agree satisfactorily.

The hole in which the chamber was placed extended 1.2 cm from the neutron source toward the indium foils. The question of extrapolating the slowing-down density to zero is complicated by this fact, but it can be shown that to a first approximation one should simply draw the curve in parabolically as if there were no hole. Moreover, the area under the curve from \( r = 0 \) to \( r = 1.2 \text{ cm} \) is \( \sim 0.02 \) percent of the total.

The use of indium foils (which have a 54 minute half-life) to compare an "artificial" source with a natural one requires either that the time of bombardment be quite short compared with 54 minutes, or that something be done about possible variations in intensity of the artificial source. For intensity reasons
our bombardments were usually long, so that we had to monitor the cyclotron beam
and make a correction for any fluctuations that occurred. The details of this
correction will be found in Appendix II. The largest correction made was 1.9 per-
cent, and the average of all corrections was 0.2 percent.

Because of these checks, and also because the effect of any fast-neutron
absorbers would only be noticeable insofar as it was not the same for fission neu-
trons as for Ra-Be neutrons, it is felt that the comparison of fast-neutron outputs
is quite reliable.

The principal check on the count of the number of fissions was the flat-
ness and reproducibility of plateaus on the number-bias curve. The integrals in
Table I have been corrected for the extrapolation of plateaus to zero bias. As
an additional precaution against possible failure of the amplifier, etc., the
same monitors which were used to make the number-bias counts were compared with
the ion chamber during the actual course of the runs in measurements 3, 4, and 5.
An 0.8 percent thickness correction was applied to the fission counts on the 25
foils, which were 50 $\mu$g/cm$^2$.

The ratios of weight of sample to weight of foil were:

Square 25: $m_s/m_f = (1.070 \pm .005) \times 10^4$

Round 25: $m_s/m_f = (1.742 \pm .017) \times 10^4$

$49$: $m_s/m_f = (2.840 \pm .042) \times 10^4$

We feel that the estimated probable errors used here are fairly liberal.

To establish the relation between the thin-foil fissions and the fissions in the sample, runs were made with the thin foil on the side of the sample
toward the cyclotron and on the side of the sample away from the cyclotron. The counting rate with foil away from cyclotron was found to be 3.34 percent less by measurements done on the square 25 sample, and 3.2 percent less by measurements on the 49 and round 25 samples. Part of this rather large effect probably arose from the presence of a slow-neutron sink caused by the cadmium foil holders on the "weak" side. The average counting rate has consistently been used in these calculations; the integrals in Table I have been corrected to average counting rate.

The self-absorption correction was calculated on the basis of this average counting rate plus the assumption of an isotropic neutron flux around the sample. The straight-through self-absorption in the sample was about 11 percent in the worst case (round 25 sample); it becomes larger when averaged over all angles, but is partially monitored by the detector foil, one side of which is shaded by the sample. The net correction was 5 percent for the round 25 sample. Details of this calculation are given in Appendix III.

One check on the results of the self-absorption correction was obtained as follows; a small "spot" of enriched material was placed at the center of the face of the square 25 sample, in the ion chamber, and its counting rate noted. It was then placed successively at each of the four corners of the sample. The counting rate was the same, within statistical accuracy, at all the corners, and 5 percent lower than this in the center. This is approximately the result expected from the leakage of thermal neutrons past the sample on to the side of the "spot" toward the sample, which is of course greater when the spot is out near the edge of the sample.

In measurement 2, a background of fissionable material in the ion chamber, resulting from a small leak in the aluminum container of the sample which
allowed some of the oxide to escape, was discovered when the measurement was partially completed. The chamber was then cleaned out, and only subsequent runs were used to determine the \( \text{(A/F)} \)'s; previous data were used to help establish the shape of the slowing-down density curve.

The strong \( \alpha \)-activity of \( ^{49} \) raised the possibility that the \( ^{49} \) sample might have a neutron background from \( \alpha-n \) reactions. The indium foils were, therefore, left in the block with the cyclotron off, exposed to the \( ^{49} \) sample for an hour. They had not acquired a measurable activity.

**Results**

The value \( \sqrt{q_{49}} \) is now given by

\[
\frac{\psi}{q_{49}} = \frac{I_1}{I_2 \times \left( m_s/m_f \right) \times \beta \times \gamma \times \delta}
\]

where \( \beta = \) finite-size-of-source correction (Appendix I), \( \gamma = \) thickness correction (assumed unity for the \( ^{49} \) foil, which was very thin), and \( \delta = \) self-absorption correction (Appendix III). We have attempted to assign reasonable probable errors to these correction factors, and have considered these with the errors shown in Table I and the errors in mass ratios to get the following values:

**Measurement**

\[
\frac{\sqrt{25}}{q_{49}} = \frac{13500}{4.578 \times 10^8 \times 1.070 \times 10^4 \times 0.9959 \times 1.008 \times 0.9521}
\]

\[
(2.88 \pm 0.05) \times 10^{-7} \text{ sec.}
\]
Measurement

\[ \frac{\nu_{25}}{Q_{43}} = \frac{13530}{4.545 \times 10^8 \times 1.070 \times 1.04 \times 0.9959 \times 1.008 \times 0.9521} = (2.80 \pm 0.05) \times 10^{-7} \text{ sec.} \]

\[ \frac{\nu_{25}}{Q_{43}} = \frac{20890}{4.550 \times 10^8 \times 1.742 \times 1.04 \times 0.9965 \times 1.008 \times 0.9502} = (2.78 \pm 0.05) \times 10^{-7} \text{ sec.} \]

\[ \frac{\nu_{19}}{Q_{43}} = \frac{4.0640}{4.545 \times 10^8 \times 2.84 \times 1.04 \times 0.9965 \times 1.000 \times 0.974} = (3.25 \pm 0.07) \times 10^{-7} \text{ sec.} \]

\[ \frac{\nu_{19}}{Q_{43}} = \frac{4.1960}{4.550 \times 10^8 \times 2.84 \times 1.04 \times 0.9965 \times 1.000 \times 0.974} = (3.35 \pm 0.07) \times 10^{-7} \text{ sec.} \]

These lead to the average values

\[ \frac{\nu_{25}}{Q_{43}} = (2.82 \pm 0.03) \times 10^{-7} \text{ sec.} \]

\[ \frac{\nu_{19}}{Q_{43}} = (3.30 \pm 0.05) \times 10^{-7} \text{ sec.} \]

The ratio of the number of neutrons per fission for the two substances
is therefore

\[ \frac{\gamma_{49}}{\gamma_{25}} = 1.17 \pm 0.02 \]

The best value for a Ra-Be source calibration available at this date (June 26) is the recent result of R. Walker and O. Frisch, giving \( \epsilon_{44} = 571 \times 10^7 \) neutrons per second. One correction, presumably quite small, has not yet been applied to this. \( \frac{\epsilon_{49}}{\epsilon_{44}} \) is accurately known to be 1.519, so

\[ \epsilon_{49} = 867 \times 10^7 n/sec, \]

which gives

\[ \gamma_{25} = 2.44, \quad \gamma_{49} = 2.86. \]

**Additional Experiments**

Using the neutron-counting technique already described, it would be possible to make a measurement of \( \bar{\nu} = \sqrt{1 + \chi_l} \) if one knew the absorption cross section \( (\sigma_a' = \sigma_f' + \sigma_r') \) of the material and the neutron density at the point at which the material is placed:

\[ \bar{\nu} = \text{number of neutrons per neutron absorbed} \]

\[ \bar{\nu} = \frac{Q_{49} \int A_f r^2 dr}{\int A_r b r^2 dr \cdot N \nu \sigma_a'} \]

Here \( N \) is the number of 25 or 49 atoms in the sample, and \( \nu \sigma_a' \) should properly
be integrated over velocity, but since 25 and 49 are nearly $1/v$ in the thermal region, we may consider $(v v' a)$ as a constant quantity, and inquire as to the value of $n = \int_0^\infty n(v) dv$. In principle this can be found with the aid of a material, such as indium, whose activity is $1/v$ in the thermal region, and which has been calibrated in some way. Actually, the type of calibration we have used is subject to serious objection on theoretical grounds, and experimental results obtained with it are therefore considerably less accurate than those obtained by the straightforward methods described above.

We used very thin indium alloy foils (1/9 indium, 8/9 tin) which had been calibrated in the graphite block in G-Building. This block contains a Ra-Be source, and the thermal flux arising from this source has been calculated in the usual way. Essentially one finds from such a calculation, assuming only one velocity at thermal energies, $\nu v = Q_G f (L^2 a r_o^2 E(v))$, where $f$ is a complicated function of quantities related to the block, and $Q_G$ is the strength of the Ra-Be source. Integrating over a Maxwellian distribution and ignoring the small variation in $f$ over this region, one finds $n = Q_f (L^2 a r_o^2 E_{th})/\tilde{v}$, i.e., the total neutron density is expressed in terms of the average Maxwellian velocity, $\tilde{v} = \sqrt{\hbar/\pi} v_m$. One finds in the G calibration the thermal activity of the thin indium foil corresponding to the calculated neutron density, $(A_{th}/G) = K n_G$

Then, by putting the thin indium on the face of the sample of fissionable material in our block in Building X one measures $(A_{th}/X)$ and finds the neutron density there.

---

so

$$n_X = \left[ \frac{\langle A_{th} \rangle_X}{\langle A_{th} \rangle_g} \right] n_g$$

so

$$\gamma = \frac{Q_{49} \int A_r r^2 dr}{\int A_r b r^2 dr \cdot (v_{\sigma_a}) \cdot N \left( \frac{\langle A_{th} \rangle_X}{\langle A_{th} \rangle_g} \right) \frac{Q_g f(L^2, \text{etc.})}{\nu}}$$

So $\gamma$ does not depend on an absolute value of $Q_g$ but only on a ratio which can be measured accurately.

If $(v_{\sigma_a})$ is not constant over the energy range of the slow neutrons present, the average value of $(v_{\sigma_a})$ over the Maxwell distribution must be employed. Using the transmission data on $^{49}$Ti by the time of flight method 7) (see Fig. 2) we found $(v_{\sigma_a})_{^{49}Ti} = 2.46$ barns/meters/$\mu$sec for a Maxwellian distribution at 20.4° C, if $\sigma_a(0.025 v) = 1057$ barns.

Three measurements of $\gamma_{^{49}Ti}$ gave good agreement, their average value being $\gamma_{^{49}Ti} = 1.97$. If \( \omega \) varies with energy over the region in question, this represents $\sqrt{1 + \omega}$, where

$$\sqrt{1 + \omega} = \sqrt{\int (1 + \omega)(v\sigma_f) n(v) dv / \int (v\sigma_f) n(v) dv}$$

and $n(v)$ is a Maxwell distribution. The relation arises from our method of measuring

7) MoDaniel et al, LA-82.
\[ \gamma = \int \left[ \sigma_a n(v) \, dv \right] / \int \left[ \sigma_f n(v) \, dv \right]. \]

This is not too helpful for \( h_9 \), since we do not know \( \sigma_f(v) \) except that it apparently exhibits at least part of the strong resonance \(^8\) shown by \( \sigma_a \).

The variation of \( (\nu \sigma)_9 \) in the thermal region is certainly much less severe than that of \( h_9 \). However, time of flight measurements \(^7\) indicate a difference between the energy dependence of \( (\nu \sigma_a)_9 \) and that of \( (\nu \sigma_f)_9 \) which is difficult to explain; \( (\nu \sigma_a)_9 \) was constant in the thermal region, while \( (\nu \sigma_f)_9 \) appeared to drop down to a minimum at 0.15 ev. Accepting \( (\nu \sigma_a)_9 \) as constant and using Fermi's value \(^9\) of 1.408 barn-meters/\mu sec (1.42 barn-m/\mu sec was found by McDaniel) we find from a single measurement \( \gamma_{AV} = 2.26 \), the average being of the type described above.

From these values of \( \gamma \) and the previously quoted values of \( \nu \) we find

\[ (1 + \alpha)_9 = \gamma_9 \gamma_9 = 1.45, \quad (1 + \alpha)_9 = 1.08. \]

We complete the circle of measurement by placing the thin indium foil on the back of a foil of fissionable material, in the ion chamber, and immersing the chamber in the thermal flux of the block. Thus we count the fissions and measure the flux which produces them, obtaining a value of \( (1 + \alpha) \) more directly. This

\(^8\) Anderson, Lavatelli, McDaniel and Sutton, LA-91; and Anderson, CK-1761.

\(^9\) Fermi, CP-1389.
procedure, which is clearly equivalent except in experimental detail to the one used above to find \((1 + \alpha)\), gave the values \((\frac{1 + \alpha}{1 + \alpha})_{49} = 1.41, 1.42, 1.43, (\frac{1 + \alpha}{1 + \alpha})_{25} = 1.13, 1.15\). The best averages of these values seem to us to be \((\frac{1 + \alpha}{1 + \alpha})_{49} = 1.44, (\frac{1 + \alpha}{1 + \alpha})_{25} = 1.13\). This assumes that \(\alpha_{43} = 0.67 \times 10^{-7} \text{n/sec}\).

To obtain the best precision on \(\gamma\), we divide our values of \(\gamma\) by these average values of \((1 + \alpha)\). This gives \(\gamma_{49} = 1.99, \gamma_{25} = 2.16\).

It is perhaps convenient at this point to summarize the principal quantities that affect the various measurements described above. These are; \(\sqrt{\alpha}\), fission count, indium foil count, weight ratio of sample to foil; \(\overline{\gamma}\), indium foil count, absolute weight of sample, absorption cross section, average velocity of neutrons in block, calculation of thermal flux in G block; \((1 + \alpha)\), fission count, standardization of Ra-Be source, absolute weight of sample, absorption cross section, average velocity of neutrons in block, calculation of thermal flux in G block.

Since the \(\gamma\) and \((1 + \alpha)\) measurements involve quantities whose accuracy is very difficult to determine, we have not tried to assign probable errors to them. In particular, the calculation of thermal flux in a graphite block involves the assumption of a single mean free path for thermal neutrons in graphite. Recent researches \(^{10}\) have indicated that the mean free path varies sharply with energy in this region. Although no quantitative investigation of this effect has been made, it seems possible that the calculation might be off by perhaps 10 percent.

\(^{10}\) Anderson, CP-1592.
Summary of Some Other Measurements Related to $\alpha$

The quantities related to $\alpha$ have been measured by different methods, and the results of some of these, as of June 22, will be summarized here for comparison with our results.

The effect on pile reactivity when (a) $^{25}$ and (b) boron, are placed in the pile, in amounts which absorb equal numbers of neutrons, will give a value for $\alpha$. This was done at Chicago $^{11}$ and the value $\alpha_{25} = 2.15$ obtained.

$(1 + \alpha)_{25}$ has been measured by comparing $\sigma_F(25)$ to the cross section of a substance in which it is believed every capture leads to a disintegration.

The substances used were lithium and boron, which have the additional required property of $1/\nu$ cross sections. Measurements at Chicago with lithium gave $(1 + \alpha)_{25} = 1.18$ $^{12}$. Measurements in this laboratory by Bailey and Williams $^{13}$, with both lithium and boron, give $(1 + \alpha)_{25} = 1.16$.

The ratio $\gamma_{49}/\gamma_{25}$ was measured by Wilson, DeWire, and Woodward $^{14}$ using a coincidence method, and found to be $1.18 \pm 0.01$.

Wilson, DeWire and Woodward $^{15}$ found, for thermal flux in the graphite block, $(1 + \alpha)_{49}/(1 + \alpha)_{25} = 1.24$. Our result of 1.27 is not entirely independent of this; however, an independent measurement at Chicago $^{10}$ also gave 1.24.

It will be noted that if one accepts a value for $\alpha_{\text{thermal}}$ of about

---


14) Wilson, DeWire and Woodward, LA-104.

.17, these results are brought into remarkably good agreement. Sharing the
remaining error between our \( \gamma \) and Fermi's \( \gamma \) puts \( \gamma_{25} \) between 2.45 and
2.50. Similarly, \( \gamma_{49} \) would lie between 2.87 and 2.95. From \( \frac{(1 + \alpha_{49})}{(1 + \alpha_{25})} = 1.24 \), we find \( \alpha_{\text{thermal}} \) \( \gamma_{49} \) = 1.45.
APPENDIX I - FINITE-SIZE-OF-SAMPLR CORRECTION

The distance-dependence of the slowing-down density for fission neutrons from a point-source can be represented, with sufficient accuracy for this correction, by \( q(r) = C e^{-r^2/r_0^2} \). If the source is a disk of radius \( R \), the contribution from a point on the disk \((\rho, \theta)\) to the \( q \) at a point on the axis of the disk, at a distance \( r \) from the disk is

\[
dq = \left( \frac{\rho \ d\rho \ d\theta}{\pi R^2} \right) e^{-(r^2 + \rho^2)/r_0^2}
\]

So

\[
q(r) = C \frac{r_0^2}{R^2} e^{-r^2/r_0^2} (1 - e^{-R^2/r_0^2})
\]

For our case \( R^2/r_0^2 < .01 \), so

\[
q(r) = C e^{-r^2/r_0^2} (1 - R^2/2r_0^2)
\]

For fission neutrons in graphite \( r_0^2 = 1225 \text{ cm}^2 \). For the square sample, 5.6 x 5.6 cm, we take \( \pi R^2 = 5.6^2 \text{ cm}^2 \), \( R^2 = 9.98 \text{ cm}^2 \). Correction is therefore \( 1 - R^2/2r_0^2 = .9959 \), and the volume integral which we find for the fission neutrons from this sample must be divided by this number in order to be compared with the integral from the Ra-Be point-source. For the round sample, \( R^2 = 8.6 \text{ cm}^2 \), \( 1 - (R^2/2r_0^2) = .9965 \).
APPENDIX II - BEAM UNSTEADINESS CORRECTION

The use of indium foils to count neutrons always involves adjustment to a certain (usually infinite) time of bombardment. This is easy for a natural source such as Ra-Be, which puts out a very steady flow of neutrons, but is more complicated when the cyclotron is used, since it cannot always be persuaded to give a very uniform output. If one knows a function \( f(t) \), proportional to the instantaneous output of the cyclotron at any time during the bombardment, then the activity the foil would have had, had it received the same number of neutrons in the same time with a steady beam, is

\[
\frac{(e^{\lambda t} - 1) \int_0^t f(t) \, dt}{\lambda t \int_0^t f(t) e^{\lambda t} \, dt}
\]

times the activity it actually has, where \( t \) is the time of bombardment and \( \lambda \) the decay constant of the foil.

To find such a function we used one of Watts' "safety circuit" devices, consisting of a \( \text{BF}_3 \) chamber driving a recording milliammeter through a D.C. amplifier. This proved to be linear, to the degree of accuracy needed, and very stable in operation. The record was divided into six-minute intervals and integrated numerically.

The cyclotron was always operated as steadily as possible, and we found it possible to tell by examination of the records which runs would not require any correction. Accordingly only the apparently "bad" ones were calculated after this discovery was made, and even in some of them the fluctuations cancelled out.
Following is a summary of the runs calculated:

<table>
<thead>
<tr>
<th>Runs</th>
<th>Correction factor (reciprocal)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.0013</td>
</tr>
<tr>
<td>2</td>
<td>0.9993</td>
</tr>
<tr>
<td>3'</td>
<td>0.9937</td>
</tr>
<tr>
<td>3'</td>
<td>0.9958</td>
</tr>
<tr>
<td>4</td>
<td>1.011</td>
</tr>
<tr>
<td>5</td>
<td>1.0191</td>
</tr>
<tr>
<td>5</td>
<td>0.9909</td>
</tr>
<tr>
<td>5</td>
<td>0.9979</td>
</tr>
<tr>
<td>5</td>
<td>1.0161</td>
</tr>
<tr>
<td>5</td>
<td>1.0010</td>
</tr>
<tr>
<td>5</td>
<td>1.0076</td>
</tr>
<tr>
<td>5</td>
<td>0.9905</td>
</tr>
</tbody>
</table>
We desire the relation between the number of fissions in a disk-shaped sample "a" of active material, transmission about 80-90 percent, and the number of fissions in a thin detector, "D" of the same material and same radius, placed on one face of "a", when the whole is immersed in a uniform flux of thermal neutrons (Fig. 3).

Consider first the number of disintegrations occurring in the sample. If the sample is a disk of radius R and height h, then the thickness measured in mean free paths is $t = \frac{h}{\lambda} = \frac{N}{\sigma' \lambda R^2}$, where N is the number of nuclei in the sample and $\sigma'$ the capture cross section. The fractional depletion of a beam of neutrons incident on the sample at angle $\theta$ is $1 - e^{-t/\cos \theta}$. Since the 25 and 49 cross sections are approximately $1/\nu$ in this region, we assume the distribution of neutrons in the block Maxwellian and calculate a correction similar to the "Bethe correction"\(^\text{16}\) which tells us the "effective velocity" $v_e$ for

\[^{16}\text{Bethe, Rev. Mod. Phys. 9, 135 (1937).}\]
the particular thickness of absorber we have. This turns out to be \( v_e = 0.89 v_m \) for 25, and \( v_e = 0.87 v_m \) for 49 \( (v_m = 2200 \text{ m/sec at } 20.4^\circ \text{C}) \), calculated on the basis of the transmission averaged over all angles.

Now we may speak of one velocity, \( v \), and one thickness, \( t \). If the sample is an infinitesimally thick disk (i.e., if \( h/R \to 0 \)), the activity it acquires in an isotropic flux, \( \text{nv} \), is

\[
A_{2, o} = 2 \int_0^{\pi} \int_0^{\pi/2} \int_0^{t/2} f(t) \text{nv} \cos \theta \sin \theta \, d\theta \, d\phi / 4\pi
\]

\[
= \text{nv} R R^2 \int_0^1 (1 - e^{-t/x}) x \, dx,
\]

which can be evaluated using

\[
E_i (-x) = \int_0^{-x} (e^{-u/u}) du,
\]

the logarithmic integral, tabulated, for example, in Jahnke and Emde. Thus, when the result is put in a form which will prove convenient for later comparisons, we have

\[
A_{2, o} = \text{nv} \nu' (1/2t) \left[ 1 - (1 - t) e^{-t} + tE_i (-t) \right].
\]

Actually, for the worst case (the round 25 sample) \( h/R = 0.086 \), which is not close enough to the conditions assumed above. We must therefore, estimate the increase in activity arising from the finite edges of the sample. A neutron which enters the top of the sample at angle \( \theta \) and leaves through the edge will have its exact counterpart in a neutron which enters the edge of the sample at a
corresponding point on the opposite side, along a parallel path, and leaves through the bottom. The total path traversed by this pair (measured in m.f.p.'s) is the same as that traversed by a neutron which goes through from top to bottom, \( t/\cos \theta \), and it is convenient to consider this as one path, and the edge effect as arising from the extra activity such a path will cause. We ignore for the moment the neutrons which enter the edge and leave through the edge.

The activity arising from such a composite path is proportional to

\[ 1 - e^{-x} + 1 - e^{-(t/\cos \theta - x)}, \]

where \( x \) is the length, in m.f.p.'s, of the first part of the path. For this calculation we can take all paths \"x\" as equally probable between \( x = 0 \) and \( x = t/\cos \theta \). This is not quite true for very large values of the azimuthal angle \( \phi \) (\( \phi \) is taken as zero when the projection of the path in the plane of the disk is radial) but is a good approximation for our geometry. Therefore, the average excess activity which such a composite path has over a normal one is proportional to

\[ 2 - \frac{2 \cos \theta}{t} \left( 1 - e^{-t/\cos \theta} \right) = \left( 1 - e^{-t/\cos \theta} \right). \]

We integrate over the area of the edge, weighting by the cosine of the angle the neutron makes with the edge surface, to get the excess activity arising from the finite edge:

\[ A_{s, e} = 2 \int_0^{2\pi} \frac{h^2}{2} \int_0^{\pi R} \left[ 2 - \frac{2 \cos \theta}{t} \left( 1 - e^{-t/\cos \theta} \right) = \left( 1 - e^{-t/\cos \theta} \right) \right] \]

\[ \times \left[ n \nu \sin \theta \cos \phi \, d\phi \, \frac{\sin \phi \, d\phi \, d\phi}{4\pi} \right] \]

\[ = \frac{2n \nu \cos \theta}{\pi R} \frac{1}{t} \int_0^1 \left[ 2 - \left( 1 - e^{-t/u} \right) \frac{2u}{t} + 1 \right] \sqrt{1 - u^2} \, du. \]
The integral has been evaluated numerically for the values of \( t \) we are interested in. For large values of \( \theta \) an appreciable number of neutrons enter the edge and leave through the edge, creating composite paths of more than two parts, and increasing the excess activity. To put a limit on the error thus made, the value of the integral as written, from \( \theta = \tan^{-1}(3/2)(R/h) \) to \( \theta = \pi/2 \), was compared with that of a similar integral assuming the sample were completely black to neutrons. This made about a percent difference in the final correction factor, so it was felt that our approximation, which is certainly closer than the "black" case, is adequate.

We now consider the effect which the absorption of the sample has on the activity of the thin detector "D" (cf. Fig. 3). We again assume first that for the sample \( h/R \to 0 \); later a correction will be made for the effect of the edge of the sample. Let \( d \) be the distance from the detector to the sample. If \( \rho \) is the radius-vector from the center of the detector to a point on the detector, the flux incident on the detector at that point (from the side the sample is on) is a function of \( \rho, \theta, \) and \( \phi \). We can integrate over \( \phi \) immediately by observing that for any \( \rho \) and \( \theta \), the fraction, \( \rho \), of flux coming in which does not have to pass through the sample is given by geometry:

\[
\rho = \frac{1}{\pi} \cos^{-1} \left( \frac{R^2 - \rho^2 - d^2}{2 \rho d \tan \theta} \right)
\]

So the activity of the detector is;
in the ring of radius \( \rho \), width \( \delta \rho \). This is easy for \( \rho \ll R \), and was evaluated numerically for \( \rho = R \) and \( \rho \approx 0.75 \, R \). These three points determine the curve of activity as a function of \( \rho \) remarkably well—it is almost flat. A slight complication is added by the fact that the detector foils were not radially uniform, being heavier near the edges. This non-uniformity was measured and appropriate weighting used when the curve was integrated over \( \rho \).

The activity thus obtained depends on \( "d" \), and we want to find the correct activity taking into account the finite height of the sample. This can be done approximately by assuming that the absorption of an element of the sample
at distance $d$ is independent of the absorption of another element at $d'$. That
this is an excellent approximation can be seen by observing that we are essentially
replacing the average of $e^{-x}$ from 0 to $t/\cos \theta$ by the average of absorp-
tion and no absorption, that is, $(1 + e^{-t/\cos \theta})/2$. For the worst possible
case, that of the most oblique neutron that can come through the thicker of the
25 samples and hit the detector, $t/\cos \theta = 2.4$ and the two quantities mention-
ed above are .38 and .54. For straight-through neutrons they are .942 and
.944. Since the total variation in activity of detector with distance is only
11 percent and is fairly linear, it is plainly impossible to make an appreciable
error by using the simpler type of averaging.

We therefore plot activity as a function of $d$, and find by numerical
integration the average activity between the limits $d_1$ and $d_2$. The activity
of the detector is the sum of $\frac{1}{2} N_{\text{nv}}$.

In addition to radial non-uniformity of detector foil, a correction
should be made to the integration over $\phi$ because the neutron flux is presumably
somewhat weaker near the center of the disk than toward the edges owing to the
neutron sink effect of the sample. However, radial non-uniformity of 10 percent
in the foil made a difference of 0.1 percent in the final answer; since the total
sink for the largest sample used was 8 to 10 percent, the variation of this sink
across the sample would make very little difference to the correction, and it has
therefore been ignored.

In all the above calculations the square 25 sample was assumed to be
a round sample of equal area.

The results of the calculations, with activities expressed as fractions
of $N_{\text{nv}}$, are:
<table>
<thead>
<tr>
<th>Sample</th>
<th>t</th>
<th>( A_{s, o} )</th>
<th>( A_{s, e} )</th>
<th>( A_D )</th>
<th>( A_{s, o} + A_{s, e} = \delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Square</td>
<td>.111</td>
<td>.8243</td>
<td>.026</td>
<td>.892</td>
<td>.954</td>
</tr>
<tr>
<td>Round</td>
<td>.128</td>
<td>.8031</td>
<td>.026</td>
<td>.879</td>
<td>.943</td>
</tr>
<tr>
<td>4.9</td>
<td>.063</td>
<td>.8841</td>
<td>.012</td>
<td>.9205</td>
<td>.980</td>
</tr>
</tbody>
</table>
Fig 1

**Experimental Arrangement In Graphite Block**