EFFECT OF VARIABLE ENTROPY IN THE THEORY
OF SPHERICALLY CONVERGENT DETONATION WAVES

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ABSTRACT

The results of two numerical computations of the motion of a convergent detonation wave, in one of which the effect of entropy is neglected, and in the other the internal energy is taken to be $pv/(\gamma-1)$, are compared. Up to the time when the radius of the wave is one third its initial value, no significant difference is observed.
It is frequently assumed that for certain explosions the end products of an explosion have adiabatics which can be approximately represented by the law

\[ p = k v^\gamma, \quad \gamma \sim 3 \]  

(1)

Here, \( p \) is of course pressure, \( v \) specific volume. This law is not in itself enough to determine the motion following a detonation wave; in addition, one must specify the manner in which \( k \) depends on entropy \( S \), or alternatively, the caloric equation of state of the exploded gases. In this connection, two simple alternatives leading to computationally feasible problems immediately suggest themselves. One can, of course, neglected entirely the effect of entropy— that is, take \( k \) in (1) as a constant—which is equivalent to assuming that the internal energy \( E \) of the exploded gases is given by an equation

\[ E = \frac{kv^{(\gamma-1)}}{\gamma-1} + \Psi(S) \]  

(2)

On the other hand, one can assume that the explosion products behave thermodynamically like perfect gases (with the modification, \( \gamma \sim 3 \)) and thus take for the internal energy the expression

\[ E = pv/(\gamma-1) \]  

(3)

obtaining then in the usual way,

\[ \log k = \text{const} \cdot S \]  

(4)

In LA-143, the results of a detailed computation for the problem of a convergent detonation wave, under the first assumption, are set forth, while in Report No. 11,
Bureau of Ships (U.S.N.) Computation Project, Harvard University, a similar computation under the second assumption is described. In both cases \( \gamma = 3 \), and since the two assumptions in a sense represent two extremes -- the truth lying, in all probability, somewhere between -- it is possible on the basis of the two reports to assess the effect of temperature on the hydrodynamic problem.

The effect appears to be surprisingly small over a considerable portion of the motion. The second computation was carried to a stage where the radius of the wave had become about one third its initial value, the other somewhat further. Up to this point, no effective difference is apparent in the two solutions, except to some extent in the behavior of specific volume \( v \) along the wave-front; the figures which follow indicate this clearly.

Thus in Fig. 1, which shows the position of the front in the two cases as a function of time, the largest difference attained is of the order of two percent. In this figure the unit of distance is the initial radius of the detonation, the unit of velocity the initial (Chapman-Jouget) velocity. In Fig. 2, the detonation velocity in the two cases is shown as a function of time. Here, of course, the two curves look rather different, but one must bear in mind that what is graphed is in effect the percent difference from the Chapman-Jouget velocity in the two cases.

In Figs. 3, 4, 5, the specific volume, pressure, and material velocity at the detonation front in each case are shown as a function of position. Here the unit of pressure is the initial (Chapman-Jouget) pressure, and the unit of velocity is the initial detonation velocity. Evidently only the first function has appreciably different values in the two cases over any part of the interval shown.

Finally, the possibility of appreciable differences in the pressure distribution behind the front in the two cases was considered. In order to avoid an
extensive interpolation, since both calculations were not carried out with the same time interval, it was necessary for this purpose to find a cycle of each computation corresponding to approximately the same time, and yet late enough to be significant. With the same unit of velocity as above, the computation with variable $k$ was found to have a cycle at $t = .5808$, the other at $t = .58$. For these two cycles, the pressure is shown as a function of radius in Fig. 6. Again, there is no significant difference.

It may be remarked in conclusion that the two solutions will, of course, differ markedly from each other in the later stages of the motion. In particular, in the case of variable $k$, the specific volume tends asymptotically to the value 0.5, as the radius $R$ of the detonation front approaches zero, and the problem has the asymptotic solution discussed in LA-242. In the case of constant $k$, on the other hand $v$ will tend to zero with $R$, and the motion tends to behave in the limit like that behind a convergent free surface (cf. LA-210).
Fig. 1
Radius-time curve of detonation wave

Upper curve: \( pv^y = \text{const.} \)
Lower curve: \( E = pv/(y-1) \)
Fig. 2

Deposition velocity vs. time

Upper curve: \( h = \frac{\rho v}{(\gamma - 1)} \)

Lower curve: \( \rho v' = \text{const.} \)
Fig. 3

Specific volume at detonation front vs. position.

Upper curve: \( \Phi = \frac{\text{vol.}}{\text{vol.}} \)

Lower curve: \( \Phi = \text{const.} \)
Fig. 4

Detonation pressure vs. position

Upper curve: $P = \frac{1}{(r-1)}$

Lower curve: $P_0 = \text{ const.}$
Fig. 5

Material velocity at detonation front vs. position.

Upper curve: $\rho v^y = \text{const.}$

Lower curve: $E = \rho v^y/(\gamma-1)$
Fig. 5
Pressure distribution behind interaction front

Upper curve: $e = y_t(1 - t)$, the plane.

Lower curve: $y_t$ is constant, $t = .55$. 