AEC Research and Development Report

Los Alamos Scientific Laboratory
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Critical Assembly of Uranium Metal
At an Average U^{235} Concentration of 16-1/4%
CRITICAL ASSEMBLY OF URANIUM METAL
AT AN AVERAGE U\textsuperscript{233} CONCENTRATION OF 16-1/4%
ABSTRACT

A uranium metal critical assembly consisted of a 15 inch diameter core with an average $^{235}\text{U}$ content of 16-1/4%, surrounded by a 3 inch thick natural uranium reflector. The critical mass was 692 kg of core material.
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Before this investigation, critical assemblies of undiluted uranium metal had average $U^{235}$ concentrations ranging from 29 w/o to 94 w/o.\(^{(1)}\) With uranium plates that had been obtained for exponential columns, there was the opportunity to extend this concentration range to an appreciably lower value. One problem of interest in a lower-concentration assembly is deviation from $m_c = \text{const} \cdot c^{-1.7}$, the constant-power relation between critical mass of uranium and $U^{235}$ concentration that holds above 29% $U^{235}$. Also useful is the degree to which the neutron spectrum is degraded with increased $U^{238}$ content.

**Description**

The metal critical assembly with a core of 16-1/4% average $U^{235}$ concentration and 3 inch normal uranium reflector is a cylinder (Fig. 1, schematic) made up of discs of natural uranium 1.5 cm thick by 15 inches in diameter, discs of oralloy (93.4% $U^{235}$) 0.3 cm thick by 15 inches in diameter, and natural uranium reflector rings around the cylindrical surface of the core 1.8 cm thick, 15 inches I.D., and 21 inches O.D. The top reflector section is 2.95 inches
FIG. 1 Critical assembly with core composition averaging 16-1/4% U235.
thick, 21 inches in diameter, and includes a top safety block 2.95 inches thick by 15 inches in diameter. The bottom reflector section is 3 inches thick, 21 inches in diameter, and includes a control disc 3 inches thick by 12 inches in diameter. The control disc is mounted in a threaded sleeve in the bottom reflector, and is closed against the bottom of the core for control.

The assembly consists of two major parts with the top portion mounted on a fixed platform, and the bottom section mounted on a hydraulic cylinder. Safety is assured by interlocks and scram systems which prevent the assembly of a supercritical configuration, and automatically disassemble the machine by lifting the top safety block and dropping the bottom section of the assembly. An axial "glory-hole" 1.5 inches in diameter extends through the top portion of the assembly, and a radial "glory-hole" 1/2 x 1/2 inch in cross section extends from the vertical axis to the surface. Both glory holes have fillers available which allow the insertion of 1/2 inch cylindrical fission chambers or 1/2 inch diameter foil detectors.

**Critical Mass**

Critical mass for the reflected 16-1/4% assembly was estimated from multiplication measurements and delayed critical operation with "clean" configuration as $m_c = 692 \pm 4$ kg
for $\rho = 18.75$ gm/cm$^3$. Before estimates of $m_c$ for a bare, spherical 16-1/4% geometry could be made, it was necessary to determine values for the effective extrapolation length of the reflected system (beyond the core surface) and for the extrapolation length of a bare 16-1/4% sphere.

Experimental fission rate distributions for U$^{235}$, U$^{238}$, and Np$^{237}$ detectors indicated an effective extrapolation length of 2.70 inches $\pm$ 5% for the reflected assembly. A calculation by Hansen, made by determining the effective buckling value ($B^2$) for a 3 inch natural uranium reflector on a 16-1/4% core, subtracting the axial component, and obtaining the effective radial extrapolation length, gives a value of 2.7 inches, in agreement with the experimental results. Using this value, the measured critical height, and critical radius, a $B^2$ of 0.0135 cm$^{-2}$ was obtained. From the relation $B^2_{\text{sphere}} = \frac{\pi}{R_c^2}$, a value of $R_c$ ($r_c$ + extrapolation length) equal to 27.01 cm was obtained.

Extrapolation lengths for 0.7%, 4.2%, 9.2%, and 94.5% concentrations (unreflected) were calculated from equilibrium spectra furnished by LASL Group T-4, cross sections from W-2-577, (2) and the relation:

$$\text{Extrapolation length} = \frac{0.71\lambda_{tr}}{1 + \frac{(\nu-1)\sigma_f - \sigma_c}{\sigma_{tr}}}$$
where $\lambda_{tr}$ is the transport mean free path for core material, 
$\nu$ is the number of neutrons emitted per fission, and $\sigma_f$, $\sigma_c$, 
and $\sigma_{tr}$ are the fission, capture, and transport cross sections. The term $\lambda_{tr}$ was obtained from:

$$\lambda_{tr} = \frac{\sum_i \phi_i \lambda_{tr_i}}{\sum_i \phi_i},$$

where $\phi_i$ and $\lambda_{tr_i}$ are six-group fluxes and transport mean free paths. Necessary cross-section values for 16-1/4% $^{235}\text{U}$ concentration were interpolated from the available information. The value of the extrapolation length for an un-reflected 16-1/4% sphere was computed to be 2.16 cm, giving

$$r_c = 24.85 \text{ cm}, \text{ and } m_c (\text{bare sphere}) = 1205 \pm 60 \text{ kg for }$$

$$\rho = 18.75 \text{ gm/cm}^3.$$ This mass compares with the value of 1000 kg from power-law extrapolation of critical masses at $^{235}\text{U}$ concentrations of 29% and greater.\(^{(1)}\)

**Calibration**

Calibration of the control disc in terms of "cents" units* was accomplished by positive period measurements. Since all periods were relatively long (~15 sec), the excess reactivity was obtained from the inhour equation in

*100 cents is the reactivity increment between delayed and prompt critical.
the form

$$\Delta k = 100 \sum_{i} \frac{a_i}{1 + (T/\tau_i)}$$

where $a_i$ and $\tau_i$ represent the relative abundance and the mean life of the $i^{th}$ group of delayed neutrons, and $T$ is the pile period. The $a_i$'s and $\tau_i$'s were weighted for a 16-1/4% $^{235}U$ concentration, using delayed neutron abundances and mean lives for $^{235}U$ and $^{238}U$ as reported by Keepin and Wimett. A plot of "cents" vs control disc inches is shown in Fig. 2.

**Rossi Alpha**

A measurement of Rossi alpha at delayed critical was made by the statistical method described in LA-744. A value of $\alpha_{dc} = 1.75 \times 10^5 \pm 5\% \text{ sec}^{-1}$ was obtained.

The relation for fraction of delayed neutrons

$$(f \sum_f)_{16-1/4\%} = (f \sum_f)_{^{235}U} + (f \sum_f)_{^{238}U}$$

reduces to

$$f(16-1/4\%) = f(^{235}U) + \left[ f(^{238}U) - f(^{235}U) \right] \frac{\sigma_f(^{238}U)/\sigma_f(^{235}U)}{1 + [\sigma_f(^{238}U)/\sigma_f(^{235}U)] [N(^{238}U)/N(^{235}U)]}$$

where $\Sigma$ and $N$ denote, respectively, macroscopic cross section
FIG. 2 Control disc calibration (from positive pile periods).
and atomic proportion in the 16-1/4% material. Using Keepin-Wimett values for \( f(U^{235}) \) and \( f(U^{238}) \) (3) and the observed fission cross section ratio, one obtains the value
\[
f(16-1/4\%) = 0.0091.
\]
A two-group estimate in which the low-energy group characterizes delayed neutrons gives for \( \gamma \), the relative effective-ness of delayed neutrons, about 0.89 or
\[
\gamma f(16-1/4\%) \sim 0.0081.
\]
Taking \( \Delta a/\Delta k = -a_{dc}/\gamma f \), this gives
\[
\Delta a/\Delta k = 2.16 \times 10^7 \text{ sec}^{-1}.
\]
According to an extrapolated end-point calculation by Hansen, the value for a bare system would be 13\% greater or \( (\Delta a/\Delta k)_{\text{bare}} = 2.44 \times 10^7 \text{ sec}^{-1} \).

**Reactivity Contributions of Various Materials**

The reactivity change when a central void was filled with a sample has been determined for a limited number of elements. Results are listed in Table I, as \( \Delta k_{\text{obs}} (\varphi/\text{gm atom}) \) observed, \( \Delta k_{\text{O}} (\varphi/\text{gm atom}) \) corrected for sample size and density, and as apparent absorption cross sections, \( \sigma_a \). All values of \( \sigma_a \) are relative to \( \sigma_a (Oy) \) taken as -1.83 barns, the value of \( [(\nu - 1) \sigma_f - \sigma_c] \) \( Oy \). The necessary cross sections were obtained as described in the section dealing with critical mass. The sample size correction used was
\[
\Delta k_{\text{O}} = \frac{\Delta k_{\text{obs}}}{1 - (3/4)R \sigma_a},
\]
where \( R \) is the radius of an equivalent sphere and \( \sigma_a \) is in units of \( \text{cm}^{-1} \). (5)
### TABLE I.

**REACTIVITY CONTRIBUTIONS OF VARIOUS MATERIALS IN CENTRAL CAVITY**

<table>
<thead>
<tr>
<th>Element</th>
<th>$\Delta k_{\text{obs}}$ ($\varphi$/gm atom)</th>
<th>$\Delta k_0$ ($\varphi$/gm atom)</th>
<th>$\sigma_a$ (barns)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oy (93.7%)</td>
<td>$+34.9 \pm 0.9$</td>
<td>$+32.8$</td>
<td>$-1.83$</td>
</tr>
<tr>
<td>Tu</td>
<td>$-1.3 \pm 0.2$</td>
<td>$-1.3$</td>
<td>$+0.070$</td>
</tr>
<tr>
<td>$\text{Li}_2\text{SO}_4$</td>
<td>$-26.8 \pm 1.5$ ($\varphi$/mole)</td>
<td>$-27.0$</td>
<td>$+1.51$</td>
</tr>
<tr>
<td>$B^{10}$ (~85%)</td>
<td>$-14.7 \pm 0.3$</td>
<td>$-15.6$</td>
<td>$+0.87$</td>
</tr>
<tr>
<td>Au</td>
<td>$-4.1 \pm 0.4$</td>
<td>$-4.1$</td>
<td>$+0.23$</td>
</tr>
<tr>
<td>W</td>
<td>$-2.5 \pm 0.3$</td>
<td>$-2.5$</td>
<td>$+0.14$</td>
</tr>
</tbody>
</table>
Fission Rate Distributions

Both axial and radial distributions for fission rates of U\textsuperscript{235}, U\textsuperscript{238}, and Np\textsuperscript{237} were obtained using multiple foil, high resolution fission counters. A high resolution spiral fission counter containing U\textsuperscript{235} was used as an external monitor. Data were taken by the methods described in LA-1669.\(^{(6)}\)

The relative fission rate distributions are shown in Figs. 3 and 4, and are plotted with the Bessel function \(J_0(0.236 \text{ in.}^{-1} r)\) and with \(\cos (0.163 \text{ in.}^{-1} h)\). It should be noted that while the value of the argument of the Bessel function plotted leads to an effective extrapolation length in agreement with the calculated value, the argument of the cosine does not (2.7 inches calculated vs 3.3 inches experimental). This disagreement is attributed to the fact that the axial glory hole was empty above the chamber except for one Tu-Oy unit, and to the asymmetry of the core structure. The value of the extrapolation length obtained from the radial distributions is considered most reliable, since the assembly had full glory holes, and the asymmetry of the core would have little effect.

Structural Effects

Structural effects were measured by the "U\textsuperscript{238} foil catcher X-ray film" technique described in LA-1487.\(^{(7)}\)
FIG. 3 Radial fission-rate distributions on median plane.
FIG. 4 Axial fission-rate distributions (detectors imbedded in Tu).
Results of the measurements were used to obtain the structural effect corrections for spectral indices described in the next section. A curve of the relative $^{238}\text{U}$ activation obtained is shown in Fig. 5. As the $^{235}\text{U}$ foil activation curve does not reflect core inhomogeneity, the influence of structure on critical mass should be small. According to an independent estimate by Hansen, the critical mass of a homogeneous system would be about 1% greater than that observed.

Spectral Indices

Ratios of $\sigma_f(U^{235})/\sigma_f(U^{238})$ were obtained by fission counting, gamma counting of irradiated foils, and radiochemistry (LASL Group J-11). Ratios of $\sigma_f(Np^{237})/\sigma_f(U^{238})$ were obtained by fission counting. The ratios from fission counting were obtained simultaneously with the fission rate distributions, using the same multiple foil chamber. Relative ratios as functions of radius and height (not corrected for structure) are shown in Figs. 6 and 7.

The structural correction for spectral indices was obtained as follows: the ratio of $\sigma_f(U^{235})/\sigma_f(U^{238})$ measured at a known position in a unit, the measured axial distribution ($U^{235}$), and the $U^{238}$ relative fission rate over a unit as obtained from photographic measurements were used to obtain an average value for $\sigma_f(U^{235})/\sigma_f(U^{238})$ in a unit. Since
FIG. 5  Detailed axial $^{238}$U fission-rate distribution showing effect of structure.
FIG. 6 Spectral indices on median plane as functions of radius.
FIG. 7 Spectral indices as function of axial position (detectors imbedded in Tt).
the positions of the foils for $\gamma$-counting and radiochemistry were known in a unit, they could then be corrected. The correction for $\sigma_f(Np^{237})/\sigma_f(U^{238})$ is assumed to be half that for $\sigma_f(U^{235})/\sigma_f(U^{238})$, varying as the average cross sections for inelastic scattering through the $U^{238}$ and $Np^{237}$ thresholds. Corrected central values averaged over a structural unit are given in Table II. Adjustments to a bare assembly are expected to increase $\sigma_f(U^{235})/\sigma_f(U^{238})$ a few percent and to affect $\sigma_f(Np^{237})/\sigma_f(U^{238})$ by less than 1%.

**TABLE II.**

**CENTRAL SPECTRAL INDICES AVERAGED OVER A STRUCTURAL UNIT**

<table>
<thead>
<tr>
<th>Spectral Index</th>
<th>Method</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_f(U^{235})/\sigma_f(U^{238})$</td>
<td>$\gamma$-counting</td>
<td>$20.4 \pm 10%$</td>
</tr>
<tr>
<td></td>
<td>Radiochemistry</td>
<td>$17.0 \pm 5%$</td>
</tr>
<tr>
<td></td>
<td>Fission counting</td>
<td>$18.2 \pm 5%$</td>
</tr>
<tr>
<td>$\sigma_f(Np^{237})/\sigma_f(U^{238})$</td>
<td>Fission counting</td>
<td>$6.27 \pm 5%$</td>
</tr>
</tbody>
</table>

**Correlations with Data for Bare Assemblies at Higher $U^{235}$ Concentrations**

Values of critical mass, $\Delta \alpha/\Delta \kappa$, and spectral indices which apply to a bare, homogeneous sphere at 16-1/4% $U^{235}$ may be used to extend previously reported relations between
these parameters and $U^{235}$ concentration. (1) Figure 8, critical masses of bare uranium spheres vs $U^{235}$ concentration, indicates departure from a constant-power law in the neighborhood of 20% $U^{235}$. Spectral indices and $\Delta a/\Delta k$, the reciprocal mean lifetime of a neutron, are given as functions of concentration in Figs. 9 and 10. The expected softening of neutron spectrum with decreasing $U^{235}$ concentration is clearly indicated.
FIG. 8 Critical masses of bare uranium spheres at various U^{235} concentrations.
FIG. 9 Central spectral indices of bare uranium spheres as functions of U\textsuperscript{235} concentration.

\[ \sigma_f (U^{235}) / \sigma_f (U^{238}) \]

\[ \sigma_f (Np^{237}) / \sigma_f (U^{238}) \]

O - FISSION CHAMBER
\[ \triangle - RADIOCHEMISTRY \]
\[ \times - \gamma - COUNTING \]
FIG. 10 $\Delta a/\Delta k$ (reciprocal neutron mean life) for bare uranium spheres vs $U^{235}$ concentration.
REFERENCES


