Simple EOS for new Polytetradentateroethylene

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A Simple EOS
for Linear Polytetradeuteroethylene

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A SIMPLE EOS FOR LINEAR POLYTETRADEUTEROETHYLENE

by

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ABSTRACT

A simple equation of state (EOS) for linear polytetraduteuroethylene (PTDE), with initial state density $\rho_0 = 1.093 \text{g/cm}^3$, was generated and added to the T-4 Sesame EOS Library as material number 7230. This EOS reproduces the experimental shock Hugoniot data for PTDE and for isotopically scaled linear polyethylene.

INTRODUCTION

In this report, the generation of a simple equation of state (EOS) for linear polytetraduteuroethylene (PTDE) is presented. The EOS presented here reproduces the experimental shock Hugoniot data for PTDE and for isotopically scaled linear polyethylene, thus fulfilling the primary purpose of this EOS to describe the compression region.

METHOD

A simple EOS for PTDE was generated from various theoretical models using the GRIZZLY\textsuperscript{1} computer code. (See the Appendix for a pedigree of GRIZZLY.)

For reasons of tractability, the models used to generate the simple EOS described here for PTDE do not explicitly treat PTDE as a polymer. However, experimental data for PTDE are used in these models, and thus the polymeric nature of PTDE is implicitly included in at least parts of the EOS.
The repeat unit in the PTDE polymer chain is CD$_2$. Therefore, PTDE was modeled as an average atom with an average atomic number of 8/3 and an average atomic weight $\bar{M}$ of 5.3469. An initial density $\rho_o = 1.093 \text{ g/cm}^3$ at $P = 0$ (P ~ 1 bar) and $T = 298.15$ K was used.

The total EOS is a sum of cold curve ($T = 0$ K isotherm), thermal electronic, and nuclear (ion) contributions.

Cold Curve:

The cold curve in the compression regime from $\eta = 1$ to $\eta = 2.042$ (where $\eta = \rho/\rho_o$, where $\rho$ is the density) was calculated from experimental shock Hugoniot data assuming a Mie-Grüneisen$^2$ EOS. Experimental shock Hugoniot data for PTDE$^3,4$ and for ISM [linear polyethylene$^4$ (Marlex) that has been isotopically scaled] were used. The $\eta$ of 2.042 is the largest $\eta$ for which there are experimental shock Hugoniot data.$^3,4$ As seen in Fig. 1, the $U_s$ vs $U_p$ data for PTDE and ISM fall on the same curve.

The following fits to this shock Hugoniot data were used:

\begin{equation}
U_s = 2.441 + 1.990 U_p - 0.1560 U_p^2, \quad U_p < 1.828
\end{equation}

and

\begin{equation}
U_s = 2.719 + 1.606 U_p - 0.02365 U_p^2, \quad U_p > 2.342
\end{equation}

where $U_s$ is the shock velocity and $U_p$ is the particle velocity; $U_s$ and $U_p$ are given in km/s. As with other polymers,$^5$ two $U_s$ vs $U_p$ fits for PTDE were made, one below and one above the slight break in the $U_s$ vs $U_p$ curve at $U_p \sim 2$ km/s. (See Fig. 1.)
For $\eta > 2.042$, the cold curve was calculated using an analytic form for the Thomas-Fermi-Dirac (TFD) electronic model. At $\eta = 2.042$, the energy, the pressure, and the first derivative of the pressure were required to be continuous.

For $\eta < 1$, the cold curve was calculated with an analytic Lennard-Jones (LJ) form with the term $F_{ACLJ} = 2$ (corresponding to an $r^{-6}$ attractive term, where $r$ is the separation distance involved in the LJ pair potential). At $\eta = 1$, the energy, the pressure, and the first derivative of the pressure were required to be continuous.

The cohesive energy $E_{coh}$ (the energy of vaporization of the solid at 0 K) used in this EOS for PTDE was calculated to be 0.302 MJ/kg, applying the estimation method of Bunn to the energy of vaporization of 680 cal/mole (at the normal boiling point) for the repeat unit, CH$_2$, of polyethylene.

At high temperatures, one expects the polymer to dissociate chemically. Thus, a dissociation energy (rather than a vaporization energy) might be more appropriate at higher temperatures. A dissociation $E_{coh}'$ for a CH$_2$ repeat unit was calculated using bond strengths for a C-C single bond and for two C-H bonds. This $E_{coh}'$ gave virtually the same numerical results for the total EOS in the compression region as did the $E_{coh}$ of 0.302 MJ/kg, and gave similar qualitative results—-but somewhat different numerical results—in the expansion region, especially for intermediate temperatures.

**Thermal Electronic:**

The thermal electronic contributions to the total EOS were calculated using a Thomas-Fermi-Dirac theory with an exchange constant of 2/3. This particular TFD theory is the same as that described in Ref. 9, except that the TFD theory used here for PTDE uses a local density approximation to the exchange.
Nuclear (ion):

The nuclear (ion) contributions to the total EOS were calculated by a solid-gas interpolation scheme, which reduces to a Debye model at low temperatures and high densities and which reduces to an ideal gas at high temperatures or low densities.

The Grüneisen function $\gamma$ and the Debye temperature $\theta$ at a given $\rho$ were calculated from $\gamma_0$ and $\theta_0$ using the Thompson formulas:

$$\gamma = \gamma_0 \xi + [2(1 - \xi)^2/3]$$

and

$$\theta = \theta_0 \eta^{2/3} \exp\{\gamma_0(1 - \xi) - [(3 - 4\xi + \xi^2)/3]\} ,$$

where $\xi = \rho_0/\rho = 1/\eta$. An effective $\gamma_0$ of 0.4047 for PTDE was calculated using the Thompson formula and values of $\gamma$ at various $\rho$ calculated from a combination of off-Hugoniot data (sound speed and multishock measurements) and shock Hugoniot data for PTDE and ISM.

The $\theta_0$ was calculated in the following manner: The lattice (intermolecular) contribution $\theta_{0l}$ to $\theta_0$ was calculated from

$$\theta_{0l} = 444.12\left[\frac{(1 - 2\sigma)/(1 + \sigma)}{\eta}\right]^{1/2} c_0 \left[\frac{\rho_0}{(\gamma)}\right]^{1/3},$$

with

$$\lambda = 2 + [(0.5 - \sigma)/(1 - \sigma)]^{3/2}$$
using \( \rho = 1.093 \text{ g/cm}^3 \), the intercept \( c_o \) (in km/s) of the given quadratic fit for \( U_s \) vs \( U_p \) shock Hugoniot data for PTDE, and a Poisson’s ratio \( \sigma \) of 1/3; this equation\(^6\) can be derived from the usual relation between \( \theta \) and the longitudinal and transverse sound speeds. For the quadratic fit for \( U_p < 1.828 \text{ km/s} \), \( \theta_{o1} = 248.4 \text{ K} \); for the quadratic fit for \( U_p > 2.342 \text{ km/s} \), \( \theta_{o1} = 276.6 \text{ K} \).

The intrachain (intramolecular) contribution \( \theta_{o2} \) to \( \theta_o \) was calculated in the following manner: For \( \eta < 1.493 \) (corresponding to \( U_p < 1.828 \text{ km/s} \)),

\[
\theta_{o2} = [(\theta_1 \theta_2 \theta_3 \theta_4 \theta_5 \theta_6 \theta_7 \theta_8 \theta_9)^{2/9}(\theta_{10} \theta_{11} \theta_{12})^{1/3}]^{1/3},
\]

where these \( \theta_i \)’s correspond to the following vibrational frequencies\(^{12,13} \) \( v_i \)’s (in cm\(^{-1}\)):

\[
\begin{align*}
\theta_1 & : 2197 \quad \text{CD}_2 \text{ asymmetric} \\
\theta_2 & : 2192 \quad \text{stretch} \\
\theta_3 & : 2102 \quad \text{CD}_2 \text{ symmetric} \\
\theta_4 & : 2088 \quad \text{stretch} \\
\theta_5 & : 1146 \quad \text{CD}_2 \text{ bend} \\
\theta_6 & : 1093 \quad \text{CD}_2 \text{ rock} \\
\theta_7 & : 991 \quad \text{CD}_2 \text{ twist} \\
\theta_8 & : 522 \quad \text{CD}_2 \text{ twist}
\end{align*}
\]
\[ \nu_{10} : 1249 \quad \text{C-C stretch} \]

\[ \nu_{11} : 1146 \quad \text{C-C stretch and C-C-C bend} \]

\[ \nu_{12} : 522 \quad \text{C-C deformation.} \]

The \( \theta_{02} \) was calculated as a geometric mean \(^14\) of the \( \theta_1 \)'s, with each \( \theta_1 \) weighted according to the number of chemical bonds that involve the atoms in that vibrational mode \( \nu_1 \) in the repeat unit of PTDE.

In a similar manner for \( \eta > 1.572 \) (corresponding to \( U_p > 2.342 \text{ km/s} \)),

\[ \theta_{02} = \left( (\theta_{10} \theta_{11} \theta_{12})^{2/3} \theta_{14} \right)^{1/3}, \]

where \( \theta_{14} \) corresponds to the vibrational frequency \( \nu_{14} = 3053 \text{ cm}^{-1} \) for D-D, which was calculated from the vibrational frequency \(^{15}\) of \( \nu = 4315 \text{ cm}^{-1} \) for H-H using \(^{16}\) the ratio of the square roots of the masses of deuterium and hydrogen.

For \( \eta > 1.572 \), the polymer is being treated as a slightly different polymer, i.e., a crosslinked polymer with a repeat unit of C:

\[ -\text{C} - \]

Here, deuterium atoms on neighboring polymer chains have come off as D\(_2\), and carbon-carbon single bonds have formed between neighboring chains, thus crosslinking the original polymer chains through side bonds. This very simplistic treatment is conservatively consistent with analyses \(^{17}\) of the products recovered from shock Hugoniot experiments on various polymers, which imply that the original polymer has undergone some chemical modification (which seems to involve some extent of irreversible dissociation) on the shock Hugoniot in the
regime above the break in the $U_s$ vs $U_p$ curve. That the crosslinked structure used here for $\eta > 1.572$ is only slightly different from the original PTDE structure used for $\eta < 1.493$ is highly consistent with the fact that, as with other saturated polymers, there is only a rather small change in the slope of the $U_s$ vs $U_p$ curve at the slight break at $U_p \sim 2$ km/s in the curve for PTDE. (See Fig. 1.) By the methods described above, $\theta_{o2}$ was calculated to be 1680 K for $\eta < 1.493$ and 1956 K for $\eta > 1.572$.

The $\theta_o$ was calculated as the geometric mean of $\theta_{o1}$ and $\theta_{o2}$:

$$\theta_o = (\theta_{o1} \theta_{o2})^{1/2}.$$  

Here, three normal modes have been assumed for the lattice vibrations and three normal modes have been assumed for the intrachain vibrations, thus leading to an equal weighting of $\theta_{o1}$ and $\theta_{o2}$ in $\theta_o$. In this manner, $\theta_o$ was calculated as 646.0 K for $\eta < 1.493$ and 735.6 K for $\eta > 1.572$.

**Final EOS:**

Tables of pressure, energy, and Helmholtz free energy for PTDE as a function of temperature and density were calculated in the manner described above, first using the input terms appropriate for $\eta < 1.493$ and second using the input terms appropriate for $\eta > 1.572$. GRIZZLY automatically sets the zero of energy at $P = 0$ and $T = 298.15$ K. The final tabular EOS is composed of the table values from the first EOS for $\eta < 1.493$ and of the table values from the second EOS for $\eta > 1.572$. This final EOS has been added to the T-4 Sesame EOS Library under material number 7230. The following kinds of pressure, energy, and Helmholtz free energy tables were added: 301 (total EOS), 304 (thermal electronic), 305 (ion, including zero point contributions), and 306 (cold curve, with no zero point contributions).
DISCUSSION

As seen in Fig. 1, the EOS described in this report for PTDE reproduces the experimental shock Hugoniot data\(^3\,^4\) for PTDE and ISM. Values of pressure, energy, and negative Helmholtz free energy for every other isotherm from the total EOS tables calculated for PTDE are plotted vs density in Figs. 2-4.

APPENDIX

PEDIGREE OF GRIZZLY

GRIZZLY\(^1\) is a computer code put together by J. Abdallah (Group T-4) with contributions from J. D. Johnson (T-4) and B. I. Bennett (T-4). GRIZZLY uses subroutines taken, with appropriate modifications, from the computer codes PANDA,\(^6\) EOSCRAY, CANDIDE, and CHART D.\(^10\)

PANDA is a computer code made by G. I. Kerley (then in T-4), using ideas and some coding from the computer code EOSLTS, developed by Bennett and modified by Bennett, Johnson, Kerley, and R. C. Albers (then in T-4). Some of the ideas and models in EOSLTS came from the following sources: the computer code SESAME, developed by J. F. Barnes and G. T. Rood (both then in T-4), and the CHART D radiation-hydrodynamic computer code\(^10\) of S. L. Thompson and H. S. Lauson (both of Sandia National Laboratories).

EOSCRAY is a version of EOSLTS. CANDIDE is a TFD computer code developed by D. A. Liberman (then in T-4) and modified by Bennett and Johnson.
REFERENCES


Fig. 1. Comparison of model and experimental shock velocity $U_s$ vs particle velocity $U_p$ on the Hugoniot. The solid curve is generated from the total EOS tables for PTDE with $\rho_o = 1.093 \text{ g/cm}^3$. The experimental shock Hugoniot data$^{3,4}$ are indicated by X's for PTDE and squares for isotopically scaled linear polyethylene.
Fig. 2. Some pressure vs density isotherms from the total EOS tables for PTDE. Pressure is in GPa; density is in Mg/m$^3$ = g/cm$^3$. 
Fig. 3. Some energy vs density isotherms from the total EOS tables for PTDE. Energy is in MJ/kg; density is in Mg/m$^3$. 
Fig. 4. Some negative Helmholtz free energy vs density isotherms from the total EOS tables for PTDE. Helmholtz free energy is in MJ/kg; density is in Mg/m$^3$. 
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