Computer Simulation
of the Sequential Probability Ratio Test
for Nuclear Safeguards
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Kenneth L. Coop
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COMPUTER SIMULATION OF THE SEQUENTIAL PROBABILITY RATIO TEST FOR NUCLEAR SAFEGUARDS

by

Kenneth L. Coop

ABSTRACT

A Fortran IV computer program called SPRTEST is used to simulate the Sequential Probability Ratio Test (SPRT). The program provides considerably more information than one can obtain from the approximate SPRT theory of Wald. For nuclear safeguards applications SPRTEST permits the equipment designer to optimize the input test parameters and, indeed, to determine whether the SPRT is the statistical test of choice. Using Monte Carlo techniques, SPRTEST simulates the use of the SPRT in a radiation monitor. The accumulation of monitoring data from a normal distribution is simulated by repeated sampling of a random number generator. In this way, SPRTEST determines the expected false-positive (α) and false-negative (β) detection probabilities and the average step number (ASN) for a particular SPRT. The report describes SPRTEST, provides a Fortran listing, and demonstrates SPRTEST applications. The report also compares results with those expected from the single-interval test (SIT) on which the SPRT is based; generally, the SPRT provides better detection probabilities for a wide range of source strengths and, at background levels, it takes less time, on average, to make decisions. To obtain optimal results with the SPRT, it must have the capability to detain the counting subject for longer than the SIT time. The SPRTEST program should be useful in choosing the best statistical test for a wide variety of applications, including safeguards, health physics monitoring, and general nuclear detection.

I. INTRODUCTION

The Sequential Probability Ratio Test (SPRT) of Wald is a statistical analysis method in use at Los Alamos for nuclear safeguards applications. The test, as used for portal safeguards monitors, consists of examining nuclear counting data sequentially in time and making one of three decisions after each step or increment of data is obtained.
1. Accept the hypothesis $H_0$ (background only).

2. Accept the hypothesis $H_1$ (count is above background).

3. Accept neither hypothesis; continue counting by obtaining another increment of data.

When either of the first two decisions is made, the counting sequence usually terminates and the result is indicated visually or audibly. Wald shows that eventually acceptance of either $H_0$ or $H_1$ will occur if the sequence continues long enough.

The average time required to make a decision for a properly designed SPRT may be considerably less than the time required for a single-interval test (SIT) of similar statistical strength for differentiating between background-only and above-background radiation levels.¹ That is the primary reason for using the sequential test. The primary disadvantages of the SPRT are that it is more complex to set up, that the time required for a particular trial or test may be longer than that required for the equivalent single-interval test, and that the analytic equations provided by Wald generally provide only approximate values for the statistical parameters of interest. These parameters are $\alpha$, $\beta$, and the average step number (ASN).

1. $\alpha$: error of the first kind, or the false-positive detection probability.

2. $\beta$: error of the second kind evaluated for a particular or nominal source strength; this is also referred to as the false-negative detection probability.

3. ASN: the average number of increments or steps required to reach a decision to accept $H_0$ or $H_1$.

The $\alpha$ and $\beta$ actually obtained using Wald's equations are generally somewhat different from the nominal (input) values (designated with a zero subscript), but the input values provide reasonably good approximations for many problems. However, those approximations may become considerably poorer if the testing sequence is forced to terminate after a set maximum number of steps. In practice, it is often desirable to force a termination to ensure that a counting period does not exceed some predetermined time. Doing so, however, also decreases the ASN, and Wald does not provide a method of estimating the magnitude of that effect.

Furthermore, the input value for $\beta$ ($\beta_0$) only approximates the true value for a particular or nominal source strength. (As described in Sec. II-B, that nominal source strength is determined by the input parameters for $\alpha$ and $\beta$, referred to as $\alpha_0$ and $\beta_0$, and, of course, the background count rates and counting times.) In safeguards applications, as well as many others, sources (i.e., above-background signals) of different strengths may be present, and it is desirable to know the false-negative detection probabilities for them even though the SPRT is set up to optimally detect the nominal source strength.

To determine the parameters estimated by Wald more accurately, a computer program, called SPRTEST, was devised to simulate the SPRT using
Monte Carlo techniques. While developed independently, presumably SPRTEST is similar in concept to other programs that have been written previously. Alternative methods for improving on Wald's theory were not pursued in this study.

Data similar to those obtained with SPRTEST could, in theory, be obtained experimentally, but results can be generated much more quickly by computer, without the potential uncertainties associated with experimental data. Of course, the fluctuations associated with sampling from statistical populations (i.e., sources of nuclear radiation) are preserved using the Monte Carlo technique. Thus, the results obtained with the computer simulation will, if properly performed, represent the best statistical test performance that can be expected experimentally.

Two versions of the Fortran IV code, SPRTEST and SPRTREP, used for simulating the SPRT on the Los Alamos computer system appear, respectively, in Appendixes A and B. These two programs run on a CDC Cyber-176 computer. Los Alamos users can obtain the programs from the MASS storage system under the directory root KLCQ2.

II. COMPUTER SIMULATION OF THE SPRT

This section describes the method used in the SPRTEST program, setting up a problem, and interpreting the program output.

A. Description of the Method

The basic computer program, SPRTEST, is designed to simulate actual experiments by using Monte Carlo sampling techniques described as follows.

The decision levels for accepting hypothesis $H_0$ and $H_1$ are set by the user's selection of nominal (input) parameters $\alpha_0$ and $\beta_0$, following Wald's approximations

\[
B = \ln \left( \frac{\beta_0}{1 - \alpha_0} \right)
\]

\[
A = \ln \left( \frac{1 - \beta_0}{\alpha_0} \right)
\]

At the start of any step in the sequential analysis, SPRTEST calls a random number generator RANF(1)\(^*\) twice to obtain two numbers uniformly distributed between 0 and 1. It uses these numbers to calculate $Y$, which corresponds to a point on the abscissa of a normal distribution with a mean of zero and a standard deviation of 1. This value is always positive; the probability of

\(^*\)RANF(1) is a standard random number generator widely used at Los Alamos, written by M. Steuerwalt. The generator uses the algorithm $S' = S \times F \mod 2^{48}$, and delivers $2^{-48} \times S'$ as a normalized fraction. It uses $F = 553645_{10}$ and starts with $S = 1274321477413155_8$. The value 1 in parentheses following RANF is a dummy argument of no significance.
obtaining a value from any region of the positive abscissa is proportional to the corresponding ordinate of the normal distribution. A third call to the random number generator is then made to determine whether to assign a positive or negative value to the abscissa, depending on whether the third random number is larger or smaller than 0.5.

This value, in nuclear counting applications, then corresponds to the detection of a number of photons or nuclear particles. Thus, it is assumed that in each step of the actual test being simulated, enough events are detected to approximate the population sampled by a normal distribution; fifty or more events detected per step would be adequate for most experimental applications. The SPRTEST never actually refers to a specific number of counts, but as will be described in Sec. II-B, the results can be related to a particular mean number of counts per step.

SPRTEST is set up such that the normal distribution just described, which has a mean of zero, corresponds to the background-only distribution. To simulate counts obtained from populations with means greater than zero (i.e., background plus a radiation source), a value, UADD, is added to the Y obtained previously to obtain the sum U. (The units of UADD are standard deviations of the normal distribution.) Thus, it is assumed that the standard deviation of all the populations sampled—background only and above background—are the same, which is a good approximation for many safeguards applications. For example, if one wishes to detect a source giving an average count per step of 100 plus a background mean of 1000, the approximate standard deviations are \((1000)^{1/2} = 31.6\) for the background and \((1100)^{1/2} = 33.2\) for the background plus source. Differences of this magnitude will generally not appreciably affect comparisons of experimental results derived from these calculations.

Next, the program computes \(Z = \log \left( \frac{f(U, \Theta_1)}{f(U, \Theta_0)} \right)\), which is the logarithm of the quotient of the two normal distributions' ordinates evaluated at the abscissa value, U, obtained previously. In the case of the normal distribution, \(Z\) takes the simple form \(Z = \Theta_1 \times U - 0.5 \Theta_2^2\), where \(\Theta_1\) is the abscissa of the distribution mean of a nominal (user-selected) source and \(U\) is the abscissa value obtained using the random number generator, as described previously.

Then \(Z\) is added to the \(Z\) value obtained in the previous step of the sequence and the sum is compared to A and B. If the sum is less than or equal to B, the hypothesis \(H_0\) (background only) is accepted; if the sum of \(Z\) is greater than or equal to A, the hypothesis \(H_1\) (above background) is accepted. In either case, the result is recorded by incrementing by +1 the value of the decision matrix \(I_HO(i)\) or \(I_HI(i)\), respectively, where \(i\) corresponds to the step number where the decision is made. Then another independent trial is begun.

*SPRTEST* program could be changed, rather easily, so that the effective width of the normal distribution would become a function of the mean count. This could be done by recasting the program to make counts the unit for the abscissa, instead of fractions of the standard deviation, as it now is. For very low count rates, it would be more appropriate to sample from a Poisson distribution, instead of the normal distribution.
If neither decision to accept $H_0$ or $H_1$ occurs, then another step is made by sampling again from the normal distribution. Another $Z$ is computed and added to the previous value. Then that sum is compared to $A$ and $B$ to determine whether to accept hypothesis $H_0$ or $H_1$, or to continue the trial. This process can be repeated for up to 98 steps (as now programmed), if necessary, to reach a decision to accept $H_0$ or $H_1$.

SPRT ES1 also provides for forcing a decision after NSTEP steps; the forced result is stored in IHO(100) or IH1(100), respectively, depending on whether $H_0$ or $H_1$ was accepted. The criterion used for this forced decision is to determine whether the sum of $Z$ is equal to or less than 0.0 (accept $H_0$) or greater than zero (accept $H_1$), as suggested by Wald. Other criteria can readily be substituted by editing SPRT ES1, and might be more appropriate in particular cases; see Ref. 8 for examples of such criteria. Whereas a decision can be forced at any step number and the result recorded as indicated, the trial also continues until a decision is made using the original, nonforcing decision points ($A$ and $B$) or until step 98 is completed. In the sample tests described in Sec. III, step 98 seldom is reached. However, if it is, a decision is forced (using the same criterion as at NSTEP) with the result recorded in IHO(99) or IH1(99), respectively, depending on whether $H_0$ or $H_1$ is accepted.

After completion of a trial, another independent trial begins and the process repeats until a total of 100,000 trials have been made. This typically takes less than 30 s of computer time, including compilation.

The value of 100,000 can, of course, be readily changed by editing SPRT ES1. Increasing the number of trials may be necessary to obtain sufficient statistical precision in some cases, such as, for example, when $\alpha_0$ is less than $10^{-3}$.

B. Setting Up Problems

The usual method for setting up an SPRT is to base it on a single-interval test with false-detection probabilities of $\alpha_0$ and $\beta_0$, as the SIT is relatively easy to visualize and set up. The intent, then, is that the SPRT will have a better $\alpha$ or $\beta$ or will require less time to run, on average, even though the nominal $\alpha_0$ and $\beta_0$ are the same as for the SIT.

The following example will illustrate the general approach to setting up the SPRT based on a single-interval test. Assume that a safeguards radiation monitor has a mean background of 500 counts/s; you want to set up a 30-s single-interval test with an $\alpha = 0.01$ and $\beta = 0.05$. Thus, in 30 s the mean background will be $30 \times 500 = 15,000$ and the standard deviation will be $\sigma = (15000)^{1/2} = 122.5$. From a table of areas under the normal curve you

*Comparison of the sum of $Z$ with 0.0 corresponds, in nuclear counting applications, to making a decision at a count level halfway between the background mean and the nominal source mean.
find that the abscissa for $\alpha = 0.01$ is 2.326 and for $\beta = 0.05$ is 1.645 standard deviations. Therefore, the mean of the source that can be detected in 30 s with these errors must be $(1.645 + 2.326)\sigma = 486$ counts/30 s above background. These relationships are illustrated in Fig. 1. A source whose count rate is greater than 486 counts/30 s will give a smaller $\beta$, and vice versa. The decision level, of course, always remains at a count rate of $15000/30 \text{ s} + 2.326\sigma/30 \text{ s} = 15285/30 \text{ s}$. Every count will be 30 s in length, regardless of a source's presence or size.

To set up the SPRT, use the same $\alpha$ and $\beta$ (referred to here as $\alpha_o$ and $\beta_o$, the input values) and divide the 30-s interval, somewhat arbitrarily, into a number of steps. If the number of steps is too small, say 3 or less, the average length or time to make the test may be unnecessarily long. On the other hand, if there are too many steps, say more than 30 or 40, you may need to modify SPRTEST1 to keep the number of forced decisions after step 98 to a small fraction of the total. There is usually little, if anything, to gain by increasing the number of intervals beyond 30 or so. For purposes of illustration, let us choose to divide the 30-s interval into 10 steps and choose the step number, NSTEP = 15, to force a decision if neither hypothesis $H_0$ or $H_1$ is accepted based on the A or B decision criterion at the completion of the step. The forced result, as stated previously, is stored in $lH0(100)$ or $lH1(100)$, and the trial continues.

Another input parameter required is the location on the abscissa, in units of $\sigma$, of the mean of the source distribution of interest. If you wish to determine the actual $\alpha$ and ASN for background only, the abscissa location is 0.0.

![Sketch of normal distributions with means of background-only and above-background, as appropriate for a single-interval test with $\alpha_o = 0.01$ and $\beta_o = 0.05$.](Fig. 1.)
To test for the ASN and $\beta$ for the nominal source strength giving 486 counts/30 s above background, use an abscissa value of $1.645 + 2.326 = 3.971$. Of course, you can select other values in between or even greater than 3.971 to determine the ASN and $\beta$ for other source-strength values; you should do this for a complete comparison with other statistical tests. SPRTREP does this automatically for background and 10 other incremented values of the source strength (see Appendix B for a listing).

The last parameter to select is the starting argument for the random number generator. Normally, this is input as 0 (zero), which causes the generator to start at its default value. At the end of each run, a number related to the current argument of the random number generator is printed out. If this number is reinserted at the start of a subsequent run, the random number sequence will start at that point. This would be useful, for example, if you wish to compare two different runs using the same parameters, but using a different subset of random numbers. If you use 0 in both runs, the results will be identical, because the random numbers used are the same.

The preceding paragraphs give the complete set of parameters required to run a simulated SPRT. They are shown in Table I.

<table>
<thead>
<tr>
<th>Fortran Name</th>
<th>Value for Example</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA</td>
<td>0.01</td>
<td>Nominal $\alpha_0$ (false-positive detection probability)</td>
</tr>
<tr>
<td>BETA</td>
<td>0.05</td>
<td>Nominal $\beta_0$ (false-negative detection probability for $UADD = 3.971$)</td>
</tr>
<tr>
<td>Y1</td>
<td>2.326</td>
<td>Abscissa value corresponding to $\alpha_0$, in standard deviations</td>
</tr>
<tr>
<td>Y2</td>
<td>1.645</td>
<td>Abscissa value corresponding to $\beta_0$, in standard deviations</td>
</tr>
<tr>
<td>UADD</td>
<td>0.0 or 3.971</td>
<td>Abscissa value of the mean of the source to be sampled</td>
</tr>
<tr>
<td>N0</td>
<td>10</td>
<td>Number of steps corresponding to the nominal single-interval test length</td>
</tr>
<tr>
<td>NSTEP</td>
<td>15</td>
<td>Step after which a decision is forced</td>
</tr>
<tr>
<td>NSEED</td>
<td>0</td>
<td>Number that provides the starting argument for the random number generator</td>
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To run SPRTEST at the Los Alamos Central Computing Facility on the Livermore Time Sharing System (LTSS), store SPRTEST as a local file and issue the command

\[ \text{FTN (I=SPRTEST,GO) / t p} \]

The letters t and p stand for the maximum time in minutes allowed for the run and the priority assigned; normally, values of 1 (the default value) for both parameters will suffice.

After compilation, SPRTEST prompts the user for the parameter values, in the order listed in the table, with the Fortran name of the parameter. During and after completion of the run, the results are printed at the user's terminal, as explained in Sec. II-C.

C. Interpreting the Computer Output

The first 10 lines of output data constitute the IHO matrix, which is a record of decisions for accepting the \( H_0 \) hypothesis; i.e., decisions that the population sampled was background only. A sample printout appears in Fig. 2. The first element of the first row is the number of times, out of the 100,000 trials, that \( H_0 \) was accepted after step 1. The second element is the number of times \( H_0 \) was accepted after step 2, etc. Row 2 contains the number of decisions for \( H_0 \) after steps 11 through 20; row 3, steps 21 through 30; etc., for rows 4 through 9. In row 10, the ninth element corresponds to forced decisions for \( H_0 \) after completion of 98 steps in which no decision for either \( H_0 \) or \( H_1 \) was reached using the normal (A and B) decision criteria. Hence, IHO(99) is the number of decisions made to accept the hypothesis \( H_0 \) (background only) based on the sum of \( Z < 0.0 \) after step 98. Finally, IHO(100) represents the number of decisions for \( H_0 \) after step NSTEP, where a decision was forced (using the sum of \( Z < 0.0 \)).

The next 10 rows of data represent the decisions for \( H_1 \) (above background), arranged in the same manner as for \( H_0 \). Elements 99 and 100 represent forced decisions after steps 98 and NSTEP, based on the sum of \( Z > 0.0 \). Examination of the elements of these matrices can be very instructive regarding when decisions (correct or incorrect) are made in the sequential analysis.

The next row contains values labeled ASN and ASN(FORCED). The first is the average step number, when the only forced decisions, if any, occur after step 98. ASN(FORCED) is the average step number resulting from termination of the sequence after step NSTEP, made by forcing a decision after that step if a decision to accept \( H_0 \) or \( H_1 \) is not made sooner. Both are obtained by appropriate calculations using the IHO and IH1 matrix elements. These values, divided by NO, give the fraction of the single-interval test length that the average SPR1 takes to make a decision, shown in the next row. It is, of course, best that these fractions be less than 1 over the range of JUADD values of most interest to the user.
The next row contains \( N_{H0} \) and \( N_{H1} \), which are simply the total number of decisions in the matrices \( I_{H0} \) and \( I_{H1} \), respectively, excluding elements 100 in both cases. Then \( N_{H1}/(N_{H0} + N_{H1}) \) is the fraction of decisions accepting the hypothesis \( H_1 \). This represents \( \alpha \) (the false-positive probability) when the population being tested in the SPRT simulation is the background; i.e., for runs with \( U_{ADD} = 0.0 \). For runs with \( U_{ADD} > 0.0 \), \( N_{H0}/(N_{H0} + N_{H1}) \) is equal to \( \beta \), the false-negative probability. The computed \( \text{ALPHA} \) or \( \text{BEI}A \) is shown in the next row. The \( \beta \) obtained for \( U_{ADD} = Y_1 + Y_2 \), and the \( \alpha \) can be compared with the input, nominal \( \beta_0 \) and \( \alpha_0 \), respectively, to determine how the statistical performance of the SPRT compares with the single-interval test. These calculated \( \alpha \) and \( \beta \) values, of course, are based on no forced decisions (except possibly after step 98).
III. RESULTS FOR SAMPLE PROBLEMS

This section contains results for three sample problems, and a brief discussion of the results. The problems explore how different combinations of initial input parameters affect the SPR1 results.

Sample Problem 1: $\alpha_0 = 0.01$, $\beta_0 = 0.05$,
Sample Problem 2: $\alpha_0 = 0.01$, $\beta_0 = 0.01$,
Sample Problem 3: $\alpha_0 = 3.16 \times 10^{-5}$, $\beta_0 = 0.5$.

A. Problem 1

Problem 1 ($\alpha_0 = 0.01$, $\beta_0 = 0.05$) uses the values from Table I as input parameters to SPRTEST. (The problem is discussed in Sec. II.) Two runs were made: the first with UADD = 0.0, corresponding to background only, and the second with UADD = 3.971, which corresponds to a source giving a mean count of 486/30 s above the background mean. The computed results for UADD = 0.0 and 3.971 are shown in Figs. 2 and 3, respectively. Figure 4 shows selected portions of the printout obtained at the data input stage when the program was compiled and run for UADD = 0.0, showing the input of the parameters from Table I.

For the first run (UADD = 0.0), it can be seen (Fig. 2) that the ASN is just less than 5, regardless of whether a decision is forced after NSTEP = 15. Because the SPRT is based on a single-interval test of 10-step length, this means that for background only the SPR1 requires, on average, just one-half the length of the single-interval test, as shown by ASN/NO.

The false-positive probability, $\alpha$, is $\text{ALPHA} = 0.00503$ for the unforced case and $\text{ALPHA(FORCED)} = 0.00928$ for the test when the sequence is terminated no later than step 15. These values can be compared with the nominal $\alpha_0$ of 0.01 for the single-interval test. Thus, both versions of the SPRT give a lower (better) value for $\alpha$, with the nonforced value considerably better than that obtained when the decision is forced after step 15.
### MATRIX IHO (BACKGROUND-ONLY):

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### MATRIX IH1 (ABOVE-BACKGROUND):

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<td>3336</td>
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<td>1892</td>
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<td>1156</td>
<td>792</td>
<td>575</td>
<td>450</td>
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<td>177</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
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<td>0</td>
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<td></td>
</tr>
</tbody>
</table>

| ASN= 6.660 | ASN(FORCED)= 6.542 |
| ASN/NO= .6660 | ASN(FORCED)/NO= .6542 |
| NHO= 2405 | NH1= 97595 |
| BETA= .024050 |
| FNHO= 2637 | FNH1= 97363 |
| BETA(FORCED)= .026370 |

LAST RANDOM NO. STARTING SEED= 274530846076037529

---

**Fig. 3.**

Computer printout of calculated results for UADD = 3.971 for problem 1.

For UADD = 3.971, the ASN from Fig. 3 is about 6.6 for both the forced and unforced cases, whereas $\beta$ is about 0.025. So again, the average trial time is less than the SIT time and the $\beta$ is about half the nominal $\beta_0$.

Examination of the matrices shows that because element 99 is always zero, the nonforced decisions were all made before the completion of step 98. Element 100 contains the number of decisions forced at the completion of step 15 (NSTEP). For example, of the forced decisions in Fig. 3, 2814 were made to accept $H_1$ and 270 were made to accept $H_0$.

In summary, these results show that the SPRT for this case gives a better $\alpha$ and $\beta$, and requires less time, on average, for both the nonforced and forced...
Fig. 4.
Computer printout at the data input stage for problem 1. The question marks are computer prompts, requiring the user to type in the particular input parameter values.

FTN (I=SPRTEST,GO) / 1 1

TYPE IN ALPHA (F10.8) ? .01
TYPE IN BETA (F10.8) ? .05
TYPE IN Y1 (F7.5) ? 2.326
TYPE IN Y2 (F7.5) ? 1.645
TYPE IN UADD (F7.5) ? 0.0
TYPE IN NO (12) ? 10
TYPE IN NSTEP (12) ? 15
TYPE IN NSEED (118) ? 0
RANDOM NO. STARTING SEED= 0

(NSTEP = 15) decision cases than the nominal single-interval test on which it was based, for the two distributions tested. For other values of source strength, the SPRT may or may not be a better test than the single-interval test; problem 2 illustrates this point.

B. Problem 2

Problem 2 (α₀ = 0.01, β₀ = 0.01) uses the input parameters shown in Table II. Thus, this SPRT is based on a single-interval test with α₀ = β₀ = 0.01, having a nominal length of 12 steps. Decisions will be forced after step 12; i.e., for the forced-decision situation, no trial will be longer than the single-interval test. To solve the problem took a total of 11 runs, starting with UADD = 0.0 and incrementing by Y1 + Y2 = 4.652/5 = 0.9304 for succeeding runs. These incremental runs will provide a range of source strengths ranging from zero to 9.3 times the standard deviation of the single-interval background. The run for UADD = 4.652 corresponds to the source strength on which the single-interval test was based; i.e., for that source strength the single-interval test is expected to result in Β = 0.01. By varying the source strengths in the above manner, we can determine the variation in actual ASN and the actual α and β; they can then be compared with the single-interval test values.

This result could be accomplished by running SPRTEST eleven times with the appropriate value of UADD input for each run. However, this type of problem can more readily be handled by the program SPR1REP, which is simply SPRTEST1 with a DO-LOOP added to automatically increment UADD and repeat the test for a total of 11 runs. Each run starts with the next random number, so that a different set of random numbers are sampled for each run. The input UADD is 0.9304, the increment value we want.
TABLE 11
INPUT VALUES FOR SAMPLE PROBLEM 2

<table>
<thead>
<tr>
<th>Fortran Name</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALPHA</td>
<td>0.01</td>
</tr>
<tr>
<td>BETA</td>
<td>0.01</td>
</tr>
<tr>
<td>Y1</td>
<td>2.326</td>
</tr>
<tr>
<td>Y2</td>
<td>2.326</td>
</tr>
<tr>
<td>UADD</td>
<td>((Y1 + Y2) \ast J)/5., J = 0, 10^b</td>
</tr>
<tr>
<td>N0</td>
<td>12</td>
</tr>
<tr>
<td>NSTEP</td>
<td>12</td>
</tr>
<tr>
<td>NSEED</td>
<td>0</td>
</tr>
</tbody>
</table>

^aSee Table I for the definition of the parameters.

^bThe actual input value is 0.9304, as discussed in the text.

Selected results are shown in Table III. The single-interval data were calculated by hand using standardized tables of the cumulative area under a normal curve.

The value of \( \alpha \) can be derived from the first row (UADD = 0.0000) of Table III as described previously. For the unforced case, \( \alpha = 0.0045 \); for the forced, it's 0.0118; and for the single-interval test, \( \alpha = 0.0100 \). Thus \( \alpha \) for the unforced problem is considerably better than that for the single-interval test and slightly worse for the forced SPR1 case.

By examining the second, fourth, and last columns of the other rows in Table III, whose values are all equal to \( 1 \times 10^5 \), one can compare the false-negative detection probabilities for the three different tests. For UADD less than about 2, the forced and single-interval tests give similar values for \( \beta \), whereas the unforced test gives poorer values. In the range of UADD from about 2 to 6, the unforced SPR1 gives better results for \( \beta \), whereas for larger UADD, the single-interval test appears to give a smaller \( \beta \). (Because the statistics in the table are poor for small \( \beta \), runs using SPRTEST were made with \( 10^6 \) trials at UADD = 6.5128 and 7.4432 to confirm the latter conclusion.)

Figures 5–7 show the computer output for runs with UADD = 0.0, 2.7912, and 9.3040, respectively. Comparison of the matrices in Figs. 5 and 6 shows that decisions are generally made more quickly in the case of background only (UADD = 0.0), as can also be seen from the ASN values. From Fig. 5, in fact, it is evident that all decisions are made before step 50, whereas in Fig. 6, that is not the case. Based on this observation, it is apparent that the unforced
TABLE III

RESULTS FOR SAMPLE PROBLEM 2

<table>
<thead>
<tr>
<th>UADD</th>
<th>NH0</th>
<th>ASN</th>
<th>SPRT Unforced&lt;sup&gt;a&lt;/sup&gt;</th>
<th>FNHO</th>
<th>ASN Forced&lt;sup&gt;b&lt;/sup&gt;</th>
<th>SPRT Forced&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Single-Interval Test&lt;sup&gt;c&lt;/sup&gt;</th>
<th>β x 10&lt;sup&gt;5&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>99554</td>
<td>6.22</td>
<td>98815</td>
<td>5.99</td>
<td>99000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.9304</td>
<td>96045</td>
<td>9.49</td>
<td>91243</td>
<td>7.87</td>
<td>91900</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.8608</td>
<td>74346</td>
<td>14.87</td>
<td>67561</td>
<td>9.35</td>
<td>67900</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>2.7912</td>
<td>25610</td>
<td>14.87</td>
<td>32520</td>
<td>9.34</td>
<td>32100</td>
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<tr>
<td>3.7216</td>
<td>3877</td>
<td>9.43</td>
<td>8543</td>
<td>7.85</td>
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<td>4.6520</td>
<td>426</td>
<td>6.17</td>
<td>1166</td>
<td>5.95</td>
<td>1000</td>
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<td></td>
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<tr>
<td>5.5824</td>
<td>45</td>
<td>4.54</td>
<td>96</td>
<td>4.52</td>
<td>56</td>
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<td></td>
<td></td>
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<tr>
<td>6.5128</td>
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<td>3.62</td>
<td>2</td>
<td>3.62</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.4432</td>
<td>1</td>
<td>3.02</td>
<td>1</td>
<td>3.02</td>
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<tr>
<td>9.3040</td>
<td>0</td>
<td>2.33</td>
<td>0</td>
<td>2.33</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> Decisions were actually forced after step 98 if the trial continued that long; this occurred only 33 times out of 100,000 trials, in the worst case.

<sup>b</sup> Decisions were forced after step 12, if the trial continued that long.

<sup>c</sup> Based on a single-interval test corresponding in length to 12 steps.

SPRT could be improved somewhat, by forcing a decision at, say, step 50 to accept H₁; i.e., if the sequence does not terminate before reaching step 50, force termination with the decision that the trial is sampling background plus a source (above background). Not only would that result in a somewhat decreased β for UADD between 2 and 3, but the ASN in that region would also decrease slightly. Moreover, the maximum possible length of a trial would be reduced by a factor of 2. So, there would appear to be several advantages to making such a forced termination of the sequence, and no apparent disadvantages.

Figure 6 shows that a few trials did not result in a decision after completion of 98 steps. Thus, a decision was forced and the result recorded in element 99. In this case, the SPR1 made 11 decisions to accept H₀ (background only) and 16 to accept H₁ (above background). Generally, the SPR1 has
the most difficulty making a decision—and thus, the largest ASN—for UADD values about midway between 0 and (Y1 + Y2). When the corresponding mean count rates are lower or much higher, the SPRT can make decisions more quickly, which, at higher count rates, are more frequently correct. It can be seen, for example, in Fig. 7, where UADD = 9.304, that all decisions are made before step 9, with the majority made at the end of step 2, and all decisions were made correctly to accept H1.

Fig. 5.
Computer output for problem 2, with UADD = 0.0.
**Fig. 6.**

Computer output for problem 2, with UADD = 2.7912.

<table>
<thead>
<tr>
<th>MATRIX IHO (BACKGROUND-ONLY):</th>
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</thead>
<tbody>
<tr>
<td>16</td>
</tr>
<tr>
<td>1118</td>
</tr>
<tr>
<td>439</td>
</tr>
<tr>
<td>207</td>
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<td>76</td>
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<tr>
<td>6</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>3</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>MATRIX IH1 (ABOVE-BACKGROUND):</th>
</tr>
</thead>
<tbody>
<tr>
<td>56</td>
</tr>
<tr>
<td>3292</td>
</tr>
<tr>
<td>1426</td>
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<td>576</td>
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<td>23</td>
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<tr>
<td>10</td>
</tr>
<tr>
<td>3</td>
</tr>
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</table>

ASN = 14.869  \quad \text{ASN (FORCED)} = 9.344

ASN/NO = 1.2391  \quad \text{ASN (FORCED) / NO} = .7787

NHO = 25611  \quad \text{NH1} = 74389

BETA = .256110

FNHO = 32524  \quad \text{FNH1} = 67476

BETA (FORCED) = .325240

UADD = 2.79120

LAST RANDOM NO. STARTING SEED = 274706265348229153
Fig. 7.
Computer output for problem 2, with UADD = 9.3040.
C. Problem 3

Problem 3 ($\alpha = 3.16 \times 10^{-5}$, $\beta = 0.5$) involves computer simulations of a vehicle portal monitor used in a nuclear safeguards application, as the monitor was initially set up. The monitor's decision logic requires some changes in SPRTEST. Only part of the results are described in this report; a listing of the modified program is not included because of the program's specialized nature.

The actual monitor consists of four detector modules, each performing the SPRT using identical parameters. The simulated SPRT for a single module is described first, then the simulation for the four modules combined.

For the single module, $N_0 = 12$ and $N_{STEP} = 15$. But, SPRTEST was modified so that $A$ is equal to 8.0, and after step 15 the forced decision always accepts hypothesis $H_0$ (background only). The results for $\beta$ and the ASN as a function of $UADD$ are plotted in Fig. 8.

The ASN for background only ($UADD = 0.0$) is 2.4, meaning an average time savings of a factor of 5 over the nominal (12-step) single-interval test for a monitoring situation where no source is present. The ASN increases to almost 9 for $UADD = 2.0$, then declines for higher values of $UADD$. Because the actual monitoring that is being simulated is almost always of vehicles without sources, the value of the ASN for $UADD = 0.0$ is, by far, the most important one.

The actual $\alpha$ determined by the simulation is $(1.07 \pm 0.10) \times 10^{-4}$, which is considerably larger than the nominal $\alpha$. This larger $\alpha$ is due primarily to the use of the modified value of 8.0 for $A$ (instead of the value 9.67, which would have been calculated by the normal equation used in SPRTEST and SPRTREP).

To compare the power of the SPRT with the (12-step) single-interval test, the latter was calculated using the same $\alpha$ as determined above; i.e., $\alpha = 1.07 \times 10^{-4}$. The results for $\beta$ are also plotted in Fig. 8, where it can be seen that they are very close to the SPRT values for $UADD$ less than 4.0. At higher values of the abscissa, the single-interval values of $\beta$ are superior (i.e., lower).

To model the simultaneous use of the four detector modules, further modifications of SPRTEST were made to simulate the logic of the system controller. That logic is basically as follows. A background indication is given only when all four modules accept hypothesis $H_0$. An alarm results as soon as any of the modules makes a decision to accept $H_1$. Thus, for the $H_0$ hypothesis, the length of time required to complete the trial is governed by the module that takes the longest time to make a decision. For the $H_1$ hypothesis, the module making the decision in the shortest time controls the overall time for the trial.
The results of this simulation are shown in Fig. 9. The problem assumed that all modules had the same background intensity and were exposed to the same source strength; the plot is in terms of the UADD for a single detector module. A comparison of Fig. 9 to Fig. 8 shows that the ASN goes up considerably for small values of UADD, and is smaller for large values, as would be expected based on the controller logic. The ASN for UADD = 0.0 is 4.8, which is twice the single-module value. Still, it is only 40% of the nominal single-interval time. The calculated $\alpha$ for the four-module SPRT is $(4.3 \pm 0.2) \times 10^{-4}$, which, as would be expected, is four times the single-module value.

The single-interval test results for $\beta$ are also plotted in Fig. 9 for comparison with the SPRT values. Again, for UADD less than about 4 they are quite similar to the SPRT values, but diverge at larger values with the single-interval $\beta$ being lower. The single-interval values shown here for $\beta$ were simply calculated from the single-interval values in Fig. 8 by taking those values to the fourth power. The 4-module SPRT values for $\beta$ were obtained from the computer simulation, but similar values could also have been obtained from the one-module SPRT values by the same method used to calculate the single-interval results.
IV. PARAMETER COMPARISONS

This section describes selected results of a series of runs made with SPRTEST to provide a systematic comparison of the parameters $\alpha$, $\beta$, and the ASN. Runs were made for $\alpha_0 = 0.1, 0.05, 0.01, 0.001, \text{ and } 0.0001$, while for each $\alpha_0$, $\beta_0$ took on the values of 0.5, 0.1, 0.05, and 0.01. For each of these combinations, a run was made with $UADD = 0.0$, corresponding to background, and $UADD = Y_1 + Y_2$, corresponding to background plus a source that would give $\beta = \beta_0$ for the nominal single-interval test.

One-hundred thousand trials were made for each run, except for those with $\alpha_0 = 0.001$ and 0.0001 with $UADD = 0.0$, where the number of trials was set at $4 \times 10^5$ and $2 \times 10^6$, respectively. Changes were made in the Fortran code to obtain reasonable statistical precision for the low-probability tallies in II-1 for those values of $\alpha_0$ and $UADD$. NO and NS1EP were set at 10 and 15 respectively, for all the runs.
The values of $\alpha_o$ and $\beta_o$ chosen cover a range of practical use in most safeguards applications. The NO and NSTEP were selected somewhat arbitrarily, but again they are typical of what might be used in actual applications. Although the results in the following paragraphs strictly apply only for these parameter values, similar results and conclusions would be expected for other parameter choices similar to these.

A. False-Positive Probability

Table IV shows the values obtained for $\alpha$ for various values $\alpha_o$ and $\beta_o$ from the various computer runs when no forced decisions were made (except in a few rare and insignificant number of trials where a decision was forced after step 98).

In all cases $\alpha$ is less than $\alpha_o$, ranging in value from about 30 to 98% of $\alpha_o$. The ratio of $\alpha/\alpha_o$ is largest for large $\beta_o$ and decreases as $\beta_o$ decreases. Although not shown in the table, runs were made for the extreme cases of $\beta_o = 0.5$ and $\alpha_o = 0.25$ and 0.40; even in those cases $\alpha$ was not greater than $\alpha_o$, within the statistical uncertainties of the 100,000-trial runs.

Table V shows the results for $\alpha$ when a decision is forced after step 15. In many cases $\alpha$ is greater than $\alpha_o$; indeed, in some cases it is greater by more than an order of magnitude. On the other hand, for some sets of $\alpha_o$ and $\beta_o$, $\alpha$ is less than the nominal $\alpha_o$ by almost 50%. This wide difference in the $\alpha/\alpha_o$ ratio for forced decisions clearly illustrates the need for caution when you force the sequential test to terminate prematurely.

<table>
<thead>
<tr>
<th>$\alpha_o$</th>
<th>$\beta_o$ (0.5)</th>
<th>$\beta_o$ (0.1)</th>
<th>$\beta_o$ (0.05)</th>
<th>$\beta_o$ (0.01)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.098</td>
<td>0.064</td>
<td>0.062</td>
<td>0.051</td>
</tr>
<tr>
<td>0.05</td>
<td>0.048</td>
<td>0.031</td>
<td>0.028</td>
<td>0.024</td>
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<tr>
<td>0.01</td>
<td>0.0091</td>
<td>0.0056</td>
<td>0.0046</td>
<td>0.0042</td>
</tr>
<tr>
<td>0.001</td>
<td>0.00084</td>
<td>0.00052</td>
<td>0.00042</td>
<td>0.00038</td>
</tr>
<tr>
<td>0.0001</td>
<td>0.00009</td>
<td>0.00005</td>
<td>0.00004</td>
<td>0.00003</td>
</tr>
</tbody>
</table>
B. False-Negative Probability

Table VI shows the calculated values of $\beta$ for various values of $\alpha_0$ and $\beta_0$ for unforced decisions. These are the calculated $\beta$ values for a source strength corresponding to $Y_1 + Y_2$; i.e., a source that would give the nominal $\beta_0$ in the single-interval test used to set up the particular SPRT.

**TABLE V**
CALCULATED VALUES FOR $\alpha$ FOR FORCED DECISIONS
AT NSTEP = 15

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.152</td>
<td>0.081</td>
<td>0.072</td>
<td>0.054</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.096</td>
<td>0.045</td>
<td>0.037</td>
<td>0.026</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.038</td>
<td>0.013</td>
<td>0.0085</td>
<td>0.0057</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.011</td>
<td>0.0026</td>
<td>0.0016</td>
<td>0.00069</td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td>0.0078</td>
<td>0.0020</td>
<td>0.00036</td>
<td>0.00012</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE VI**
CALCULATED VALUES FOR $\beta$ FOR UNFORCED DECISIONS$^a$

<table>
<thead>
<tr>
<th>$\alpha_0$</th>
<th>$\beta_0$</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.392</td>
<td>0.064</td>
<td>0.030</td>
<td>0.0056</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.367</td>
<td>0.059</td>
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<td>0.0053</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.322</td>
<td>0.053</td>
<td>0.024</td>
<td>0.0046</td>
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</tr>
<tr>
<td>0.001</td>
<td>0.273</td>
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<td>0.021</td>
<td>0.0038</td>
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<tr>
<td>0.0001</td>
<td>0.239</td>
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<td>0.0033</td>
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</tbody>
</table>

$^a$Evaluated at a source strength corresponding to $Y_1 + Y_2$ for each $\beta_0$. 

22
The values for $\beta$ are all less than the $\beta_o$ values, ranging from about 33 to 78% of $\beta_o$. In Sec. IV-A for the unforced case, $\alpha$ was always less than $\alpha_o$ for the range of $\alpha_o$ and $\beta_o$ covered, therefore it follows that $\alpha + \beta \leq \alpha_o + \beta_o$, which is the relationship derived by Wald for the general case. The trend observable in the table is for $\beta/\beta_o$ to decrease as $\alpha_o$ decreases.

Table VII shows the calculated values of $\beta$ when a decision is forced after NSTEP = 15. The trend here is the same as in the preceding table, namely, $\beta/\beta_o$ decreases as $\alpha_o$ decreases. However, for $\beta_o \leq 0.1$, the values of $\beta$ here are somewhat greater than those in the preceding table, and in the case of $\alpha_o = 0.1$ and $\beta = 0.01$, $\beta/\beta_o$ is greater than 1. For $\beta_o = 0.5$, the values of $\beta$ are less than those in Table VI. So, the actual $\beta$ for forced decisions can be smaller or larger than the unforced $\beta$ values, depending on $\beta_o$.

A different decision criterion for forced decision could markedly change the results shown in Tables V and VII for $\alpha$ and $\beta$, respectively. For example, if hypothesis $H_0$ is always accepted after NSTEP (= 15 or otherwise), then the forced-decision values for $\alpha$ will be lower than those shown in Table V, while the values for $\beta$ will be higher than in Table VII; in fact, the forced-decision $\alpha$ values will be equal to or lower than the unforced values.

TABLE VII
CALCULATED VALUES FOR $\beta$
FOR DECISIONS FORCED AT NSTEP = 15$^a$

<table>
<thead>
<tr>
<th>$\alpha_o$</th>
<th>$\beta_o$</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0.380</td>
<td>0.081</td>
<td>0.043</td>
<td>0.0126</td>
<td></td>
</tr>
<tr>
<td>0.05</td>
<td>0.356</td>
<td>0.069</td>
<td>0.036</td>
<td>0.0095</td>
<td></td>
</tr>
<tr>
<td>0.01</td>
<td>0.316</td>
<td>0.056</td>
<td>0.027</td>
<td>0.0058</td>
<td></td>
</tr>
<tr>
<td>0.001</td>
<td>0.272</td>
<td>0.047</td>
<td>0.021</td>
<td>0.0041</td>
<td></td>
</tr>
<tr>
<td>0.0001</td>
<td>0.238</td>
<td>0.041</td>
<td>0.019</td>
<td>0.0034</td>
<td></td>
</tr>
</tbody>
</table>

$^a$Evaluated at a source strength corresponding to $Y1 + Y2$ for each $\beta_o$.

C. Average Step Number

Table VIII shows the ASN values versus $\alpha_o$ and $\beta_o$ for unforced decisions with UADD = 0.0 (background). These values range from 24 to 75% of NO, the
nominal length of the single-interval test on which the SPRT is based. The obvious trends are that the ASN decreases as \( \alpha_o \) decreases and as \( \beta_o \) increases. The lowest ASN is for \( \alpha_o = 0.0001 \) and \( \beta_o = 0.5 \).

For \( \text{UADD} = Y1 + Y2 \), the results are shown in Table IX. These values are higher, on average, than for \( \text{UADD} = 0.0 \), but they are always less than \( N_0 \) (= 10). However, for some values of UADD between 0.0 and \( Y1 + Y2 \), the ASN might be greater than \( N_0 \), as is apparent from some of the sample problems discussed in Sec. III.

As expected, for those entries corresponding to \( \alpha_o = \beta_o \), the ASN values in Tables VIII and IX are equal, because the analysis of \( \text{UADD} = 0.0 \) and \( \text{UADD} = Y1 + Y2 \) is symmetrical in that situation. Similarly, the values for \( \alpha \) in Tables IV and V are equal (within statistical variations) to the values of \( \beta \) in Tables VI and VII, respectively, for \( \alpha_o = \beta_o \).

### TABLE VIII
**THE AVERAGE STEP NUMBER FOR UADD = 0.0 (BACKGROUND)**

<table>
<thead>
<tr>
<th>( \alpha_o )</th>
<th>( \beta_o )</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
<td>7.1</td>
<td>7.3</td>
<td>7.4</td>
<td>7.5</td>
</tr>
<tr>
<td>0.05</td>
<td></td>
<td>6.1</td>
<td>6.3</td>
<td>6.5</td>
<td>6.7</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td>4.3</td>
<td>4.7</td>
<td>4.9</td>
<td>5.3</td>
</tr>
<tr>
<td>0.001</td>
<td></td>
<td>3.0</td>
<td>3.5</td>
<td>3.7</td>
<td>4.1</td>
</tr>
<tr>
<td>0.0001</td>
<td></td>
<td>2.4</td>
<td>2.8</td>
<td>3.0</td>
<td>3.4</td>
</tr>
</tbody>
</table>

### TABLE IX
**THE AVERAGE STEP NUMBER FOR UADD = Y1 + Y2**

<table>
<thead>
<tr>
<th>( \alpha_o )</th>
<th>( \beta_o )</th>
<th>0.5</th>
<th>0.1</th>
<th>0.05</th>
<th>0.01</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td></td>
<td>9.7</td>
<td>7.3</td>
<td>6.3</td>
<td>4.7</td>
</tr>
<tr>
<td>0.05</td>
<td></td>
<td>9.7</td>
<td>7.4</td>
<td>6.5</td>
<td>4.9</td>
</tr>
<tr>
<td>0.01</td>
<td></td>
<td>9.7</td>
<td>7.5</td>
<td>6.7</td>
<td>5.3</td>
</tr>
<tr>
<td>0.001</td>
<td></td>
<td>9.8</td>
<td>7.7</td>
<td>6.9</td>
<td>5.6</td>
</tr>
<tr>
<td>0.0001</td>
<td></td>
<td>9.9</td>
<td>7.9</td>
<td>7.2</td>
<td>5.9</td>
</tr>
</tbody>
</table>
In fact, for $\alpha_o$ and $\beta_o$ in Tables IV, V, and VIII equal to $\beta_o$ and $\alpha_o$ in Tables VI, VII, and IX, respectively, the entries should be equal, within statistical variation. For example, the entry in Table VIII for $\alpha_o = 0.01$, $\beta_o = 0.1$ is equal to the Table IX entry for $\alpha_o = 0.1$, $\beta_o = 0.01$. As another example, the entry in Table IV for $\alpha_o = 0.01$, $\beta_o = 0.05$ is 0.0046, whereas the equivalent value in Table VI for $\alpha_o = 0.05$, $\beta_o = 0.01$ is 0.0053. Because these values are each based on $10^5$ trials, they represent approximately 460 and 530 decisions, respectively. Thus, their standard deviations are approximately $(460)^{1/2} \approx 21$ and $(530)^{1/2} \approx 23$. To determine if these entries are within reasonable agreement, the normal distribution test may be applied to yield $t = \frac{530 - 460}{(530 + 460)^{1/2}} = 2.22$. This means that a difference at least this large would be expected with a frequency of 2.6%. Considering the number of entries being compared in the tables, these two entries seem to be in reasonable agreement. Most of the other entries appropriate for comparison are in closer agreement.

V. EFFECT OF VARYING THE NOMINAL STEP NUMBER

To gain some insight into the effect of varying $N_0$, the number of steps corresponding to the nominal single-interval test length, a series of runs was made with $N_0 = 1, 2, 4, 8, 16,$ and $32$. For all runs the value $\alpha_o = \beta_o = 0.01$ was used, while $UADD$ took on values from 0.0 to 6.0 in increments of 1.0. Each run was 100,000 trials in length.

The results for $\alpha$ and $\beta$ are shown in Table X for the unforced decision case. (Although a decision was actually forced after step 98 for some trials, this did not have a significant effect on the results shown except for $N_0 = 32$, where the values for $UADD = 2.0$ and 3.0 would have been, respectively, somewhat larger and smaller.) It can be seen that smaller $N_0$ values resulted in smaller values for $\alpha$. However, for small values of $UADD$, $\beta$ is poorer (larger) for smaller $N_0$ values; this is, of course, always the case for very small values of $UADD$, because in the limit as $UADD$ goes to zero, $\beta = 1 - \alpha$.

Because $\alpha_o = \beta_o = 0.01$, it follows that for $UADD = Y_1 + Y_2 = 2.326 + 2.326 = 4.652$, $\beta = \alpha$; and for $UADD = 2.326$, $\beta = 0.5$ for all values of $N_0$. Also, for any $N_0$, the $\beta$ for any $UADD' = 4.652 - UADD$ is equal to $1 - \beta$ for $UADD$. For example, the $\beta$ for $UADD' = 4.652 - 2.0$ is equal to $1 - 0.685 = 0.315$ for $N_0 = 8$. Thus additional values for $\beta$ may be derived from the table for $UADD'$ = 0.652, 1.652, 2.652, 3.652, and 4.652.

Based on these characteristics, it follows that for values of $UADD$ between 2.326 and 4.652, the smaller $N_0$ is, the smaller (relatively) is $\beta$. This is clear from the table for $UADD = 3.0$ and 4.0, and, indeed, the table indicates that this might be the trend for considerably larger values of $UADD$.

The statistical cost of the lower $\alpha$ as a function of lower $N_0$ is demonstrated in Table XI, where the ratio of the ASN to $N_0$ is shown for the unforced decision case. (Again, a decision was actually forced after step 98, if
no decision had been reached by then. This only had a noticeable effect on the runs with NO = 32 and with UADD = 2.0 and 3.0, where otherwise the values for ASN/NO would have been somewhat larger.)

The average time for a test (relative to the nominal single-interval test) increases with decreasing NO. For example, if these tests were based on a single-interval test that took 10 s, the average length of the SPRT test for UADD = 0.0 would be 10.9 s for NO = 1, but only 4.7 s for NO = 32. Actually, every trial for the SPRT test for NO = 1 takes as long or longer than the single-interval test because no decision can be made until the end of step 1, which is exactly the length of the single-interval test.

**TABLE X**

CALCULATED RESULTS FOR $\alpha$ AND $\beta$ FOR UNFORCED DECISIONS

<table>
<thead>
<tr>
<th>NO</th>
<th>UADD 0.0</th>
<th>UADD 1.0</th>
<th>UADD 2.0</th>
<th>UADD 3.0</th>
<th>UADD 4.0</th>
<th>UADD 5.0</th>
<th>UADD 6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0004</td>
<td>0.986</td>
<td>0.736</td>
<td>0.106</td>
<td>0.0048</td>
<td>0.0002</td>
<td>$&lt;10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>0.0016</td>
<td>0.975</td>
<td>0.713</td>
<td>0.134</td>
<td>0.0098</td>
<td>0.0006</td>
<td>$&lt;10^{-4}$</td>
</tr>
<tr>
<td>4</td>
<td>0.0027</td>
<td>0.967</td>
<td>0.699</td>
<td>0.153</td>
<td>0.0149</td>
<td>0.0012</td>
<td>0.0001</td>
</tr>
<tr>
<td>8</td>
<td>0.0038</td>
<td>0.959</td>
<td>0.685</td>
<td>0.165</td>
<td>0.0185</td>
<td>0.0016</td>
<td>0.0002</td>
</tr>
<tr>
<td>16</td>
<td>0.0048</td>
<td>0.952</td>
<td>0.675</td>
<td>0.179</td>
<td>0.0213</td>
<td>0.0021</td>
<td>0.0003</td>
</tr>
<tr>
<td>32</td>
<td>0.0061</td>
<td>0.946</td>
<td>0.664</td>
<td>0.191</td>
<td>0.0255</td>
<td>0.0025</td>
<td>0.0003</td>
</tr>
</tbody>
</table>

*aEntries under the column with UADD = 0.0 are the calculated values for $\alpha$; all other columns contain the calculated $\beta$ values.

**TABLE XI**

ASN/NO VALUES FOR UNFORCED DECISIONS

<table>
<thead>
<tr>
<th>NO</th>
<th>UADD 0.0</th>
<th>UADD 1.0</th>
<th>UADD 2.0</th>
<th>UADD 3.0</th>
<th>UADD 4.0</th>
<th>UADD 5.0</th>
<th>UADD 6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.09</td>
<td>1.48</td>
<td>2.56</td>
<td>2.12</td>
<td>1.28</td>
<td>1.05</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.75</td>
<td>1.14</td>
<td>1.96</td>
<td>1.66</td>
<td>0.96</td>
<td>0.68</td>
<td>0.56</td>
</tr>
<tr>
<td>4</td>
<td>0.62</td>
<td>0.97</td>
<td>1.60</td>
<td>1.39</td>
<td>0.81</td>
<td>0.55</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>0.55</td>
<td>0.87</td>
<td>1.38</td>
<td>1.21</td>
<td>0.72</td>
<td>0.48</td>
<td>0.36</td>
</tr>
<tr>
<td>16</td>
<td>0.50</td>
<td>0.79</td>
<td>1.24</td>
<td>1.10</td>
<td>0.66</td>
<td>0.44</td>
<td>0.33</td>
</tr>
<tr>
<td>32</td>
<td>0.47</td>
<td>0.74</td>
<td>1.11</td>
<td>1.00</td>
<td>0.63</td>
<td>0.41</td>
<td>0.31</td>
</tr>
</tbody>
</table>
So, although $\alpha$ is better for small $N0$ than large, the length of time required to make a decision is larger. It is, thus, not apparent from these two tables that there is a universally best $N0$ for the SPRT with $\alpha_0 = \beta_0 = 0.01$. This general problem of a best $N0$ requires further study.

For the same runs discussed previously, but for forced decisions at $N0 = NSTEP$, the results are shown in Tables XII and XIII. Setting $NSTEP = N0$ ensures that the SPRT never takes longer than the single-interval test on which it is based. In fact, because of the forced-decision criteria used in the program, for $\alpha_0 = \beta_0$, the run with $N0 = NSTEP = 1$ is exactly equivalent to the single-interval test. In Table XII, the theoretical results of the single-interval test, as determined from cumulative probability tables for the normal distribution, are shown in the first row, while the values obtained from the computer program are shown in the second row ($N0 = 1$). The agreement between the two rows is excellent. The trends noticeable in Table XII are that $\alpha$ increases slightly with increasing $N0$, and the $\beta$ values for particular source strengths are very similar for a large range of $UADD$ values, increasing somewhat with $N0$ as $UADD$ increases above 2.326.

Table XIII shows that for $N0 = 1$, $ASN/N0 = 1$; in fact, one and only one step is always required. For the other values of $N0$, the ASN is always less than 1. Of particular interest is the ASN/N0 ratio for $UADD = 0.0$. This is, for example, equal to 0.48 for $N0 = 16$; i.e., the SPRT with a decision forced after step 16 takes only half as long on average, as the single-interval test. It never takes longer than the single-interval test for any value of $UADD$, and

### TABLE XII

CALCULATED RESULTS FOR $\alpha$ AND $\beta$ FOR FORCED DECISIONS AT $NSTEP + N0$

<table>
<thead>
<tr>
<th>$N0$</th>
<th>0.0$^a$</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)$^b$</td>
<td>(0.0100)</td>
<td>(0.908)</td>
<td>(0.628)</td>
<td>(0.250)</td>
<td>(0.0470)</td>
<td>(0.00375)</td>
<td>(0.00012)</td>
</tr>
<tr>
<td>1</td>
<td>0.0104</td>
<td>0.908</td>
<td>0.627</td>
<td>0.250</td>
<td>0.0454</td>
<td>0.0038</td>
<td>0.0001</td>
</tr>
<tr>
<td>2</td>
<td>0.0108</td>
<td>0.907</td>
<td>0.629</td>
<td>0.255</td>
<td>0.0492</td>
<td>0.0042</td>
<td>0.0002</td>
</tr>
<tr>
<td>4</td>
<td>0.0112</td>
<td>0.905</td>
<td>0.629</td>
<td>0.255</td>
<td>0.0496</td>
<td>0.0045</td>
<td>0.0002</td>
</tr>
<tr>
<td>8</td>
<td>0.0115</td>
<td>0.903</td>
<td>0.627</td>
<td>0.252</td>
<td>0.0504</td>
<td>0.0049</td>
<td>0.0004</td>
</tr>
<tr>
<td>16</td>
<td>0.0122</td>
<td>0.900</td>
<td>0.622</td>
<td>0.256</td>
<td>0.0502</td>
<td>0.0049</td>
<td>0.0004</td>
</tr>
<tr>
<td>32</td>
<td>0.0133</td>
<td>0.899</td>
<td>0.624</td>
<td>0.254</td>
<td>0.0533</td>
<td>0.0052</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

$^a$Entries under the column with $UADD = 0.0$ are the calculated values for $\alpha$; all other columns contain the calculated $\beta$ values.

$^b$Values in parentheses are for the nominal single interval test; $\beta$ values were obtained from standard statistical tables.
TABLE XIII

ASN/NO VALUES FOR FORCED DECISIONS AT NSTEP = NO

<table>
<thead>
<tr>
<th>NO</th>
<th>0.0</th>
<th>1.0</th>
<th>2.0</th>
<th>3.0</th>
<th>4.0</th>
<th>5.0</th>
<th>6.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>2</td>
<td>0.70</td>
<td>0.83</td>
<td>0.92</td>
<td>0.89</td>
<td>0.79</td>
<td>0.66</td>
<td>0.62</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
<td>0.75</td>
<td>0.85</td>
<td>0.83</td>
<td>0.69</td>
<td>0.54</td>
<td>0.42</td>
</tr>
<tr>
<td>8</td>
<td>0.52</td>
<td>0.69</td>
<td>0.81</td>
<td>0.78</td>
<td>0.63</td>
<td>0.47</td>
<td>0.36</td>
</tr>
<tr>
<td>16</td>
<td>0.48</td>
<td>0.65</td>
<td>0.78</td>
<td>0.75</td>
<td>0.59</td>
<td>0.43</td>
<td>0.33</td>
</tr>
<tr>
<td>32</td>
<td>0.46</td>
<td>0.62</td>
<td>0.75</td>
<td>0.72</td>
<td>0.56</td>
<td>0.41</td>
<td>0.31</td>
</tr>
</tbody>
</table>

has similar \( \beta \) values (Table XII) for a range of UADD of interest to many safeguards problems. The \( \alpha \) is, however, somewhat larger, and \( \beta \) for large values of UADD is also larger than that for the single-interval test. Tests such as this may well be useful in particular applications, because they allow considerably faster tests on average, are never longer, and have only a slight decrease of statistical power, compared to the single-interval test.

VI. SELECTION OF THE INPUT FALSE-NEGATIVE PROBABILITY VALUE

The input parameter \( \alpha_o \) is selected to provide the (approximate) desired false-positive detection probability; to maximize detection sensitivity, it is generally chosen to be as large as tolerable for field conditions. However, selecting the input false-negative probability value \( \beta_o \) may be less straightforward, especially if you expect to encounter a range of source strengths. This difficulty arises because the choice for \( \beta_o \) affects the value of \( \beta \) for all source strengths (in contrast to the single-interval test, where the choice of \( \alpha_o \) fixes \( \beta \) for all source strengths).

To gain some understanding of this effect, a series of runs was made using SPRTREP for \( \alpha_o = 0.0228 \), and with \( \beta_o = 0.5, 0.1587, 0.0228, 0.00135, \) and \( 3.167 \times 10^{-5} \), corresponding to \( Y2 = 0.0, 1.0, 2.0, 3.0, \) and \( 4.0 \), respectively. For each of the five runs, NO equaled 10 while UADD varied from 0.0 to 6.0 in increments of 0.5.

The results for \( \alpha \) and \( \beta \) are shown in Table XIV for all five runs and are plotted in Fig. 10 for three runs. Examination of these data shows that, in general, each column has one region with a \( \beta \) lower than in any other column; this is near the region of UADD corresponding to the mean of the distribution appropriate for \( \beta_o \). Thus, for example, in Fig. 10 the curve for \( \beta_o = 0.0228 \) is best in the vicinity of UADD = \( Y1 + Y2 = 2.0 + 2.0 = 4.0 \). The other obvious generality is that the larger \( \beta_o \) is, the better (lower) \( \beta \) is at lower source strengths and the poorer it is at high source strengths. The converse is also
TABLE XIV
VALUES FOR \( \alpha \) AND \( \beta \) VERSUS \( \beta_0 \)

<table>
<thead>
<tr>
<th>UADD</th>
<th>( \beta_0 )</th>
<th>( \beta_0 )</th>
<th>( \beta_0 )</th>
<th>( \beta_0 )</th>
<th>( \beta_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5</td>
<td>0.1587</td>
<td>0.0228</td>
<td>0.00135</td>
<td>3.167 \times 10^{-5}</td>
</tr>
<tr>
<td>0.0(^a)</td>
<td>0.0213</td>
<td>0.01447</td>
<td>0.01125</td>
<td>0.00914</td>
<td>0.00760</td>
</tr>
<tr>
<td>0.5</td>
<td>0.9165</td>
<td>0.9486</td>
<td>0.9660</td>
<td>0.9761</td>
<td>0.9825</td>
</tr>
<tr>
<td>1.0</td>
<td>0.7686</td>
<td>0.8462</td>
<td>0.9043</td>
<td>0.9402</td>
<td>0.9612</td>
</tr>
<tr>
<td>1.5</td>
<td>0.5447</td>
<td>0.6351</td>
<td>0.7530</td>
<td>0.8517</td>
<td>0.9113</td>
</tr>
<tr>
<td>2.0</td>
<td>0.3457</td>
<td>0.3839</td>
<td>0.5001</td>
<td>0.6697</td>
<td>0.8070</td>
</tr>
<tr>
<td>2.5</td>
<td>0.2056</td>
<td>0.1960</td>
<td>0.2456</td>
<td>0.3842</td>
<td>0.5962</td>
</tr>
<tr>
<td>3.0</td>
<td>0.1243</td>
<td>0.0917</td>
<td>0.1116</td>
<td>0.1501</td>
<td>0.3011</td>
</tr>
<tr>
<td>3.5</td>
<td>0.07265</td>
<td>0.0426</td>
<td>0.0340</td>
<td>0.0430</td>
<td>0.0902</td>
</tr>
<tr>
<td>4.0</td>
<td>0.04379</td>
<td>0.0191</td>
<td>0.0112</td>
<td>0.0105</td>
<td>0.0177</td>
</tr>
<tr>
<td>4.5</td>
<td>0.02681</td>
<td>0.0088</td>
<td>0.0038</td>
<td>0.0023</td>
<td>0.00320</td>
</tr>
<tr>
<td>5.0</td>
<td>0.01581</td>
<td>0.0043</td>
<td>0.0011</td>
<td>0.00061</td>
<td>0.00043</td>
</tr>
<tr>
<td>5.5</td>
<td>0.00942</td>
<td>0.0020</td>
<td>0.0040</td>
<td>0.00012</td>
<td>0.00008</td>
</tr>
<tr>
<td>6.0</td>
<td>0.00582</td>
<td>0.00094</td>
<td>0.00015</td>
<td>0.00003</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

\(^a\)Values in columns 2–6 of this row correspond to \( \alpha \); all other rows are \( \beta \) values.

true; i.e., small \( \beta_0 \) results in relatively high values of \( \beta \) for small UADD and low \( \beta \) values for large UADD. The choice of \( \beta_0 \) also affects \( \alpha \), as described in Sec. IV. The values for \( \alpha \) are shown in the first row of Table XIV, for UADD = 0.0.

Table XV shows the ASN/NO values obtained for all five runs and Fig. 11 shows plots for three of them. It appears that for each run there is a region of UADD where the ASN/NO value is less than for any other run. This is near, but not identical to the region corresponding to \( \beta_0 \) for that run.

From this limited amount of data, it is obvious that the choice of \( \beta_0 \) can significantly influence the statistical parameters \( \alpha \), \( \beta \), and ASN. To determine the exact effect to expect for a particular \( \alpha_0 \), you might think it necessary to perform a series of Monte Carlo runs as I did. However, to the extent that these data can be generalized, it appears that a particular choice for \( \beta_0 \) gives the best test for source strengths corresponding to that value, as expected from the theory. If your concern is primarily with detecting sources of that
Fig. 10.
Plot of $\beta$ versus UADD for selected SPRT runs with $\alpha_0 = 0.0228$ and $N_0 = 10$.

Intensity, the choice of $\beta_0$ then is obvious. Because the actual problem is not always (or even usually) that simple, a more detailed examination of the expected results, using the technique demonstrated here may be appropriate.

For example, examination of the curves in Fig. 10 shows that the one for $\beta_0 = 3.167 \times 10^{-5}$ has the poorest detectability at low values of source strength. In most safeguards applications, this would be undesirable and, therefore, a larger $\beta_0$ would be chosen. However, this feature may be useful in some radiation monitoring applications, when, as here, it is coupled with very good capabilities at larger source strengths. Such features might be useful, for example, in a contamination monitor where only significant levels of contamination are of interest, and you don't want an alarm for levels just above background.
TABLE XV
CALCULATED VALUES FOR ASN/NO VERSUS $\beta_o$

<table>
<thead>
<tr>
<th>UADD</th>
<th>$\beta_o$</th>
<th>$0.5$</th>
<th>$0.1587$</th>
<th>$0.0228$</th>
<th>$0.00135$</th>
<th>$3.167 \times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.506</td>
<td>0.536</td>
<td>0.581</td>
<td>0.623</td>
<td>0.660</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0.706</td>
<td>0.712</td>
<td>0.730</td>
<td>0.752</td>
<td>0.775</td>
<td></td>
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<tr>
<td>1.0</td>
<td>0.930</td>
<td>0.934</td>
<td>0.936</td>
<td>0.931</td>
<td>0.929</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>1.022</td>
<td>1.121</td>
<td>1.176</td>
<td>1.172</td>
<td>1.140</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>0.974</td>
<td>1.123</td>
<td>1.287</td>
<td>1.411</td>
<td>1.408</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>0.866</td>
<td>0.985</td>
<td>1.168</td>
<td>1.443</td>
<td>1.646</td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td>0.751</td>
<td>0.814</td>
<td>0.932</td>
<td>1.177</td>
<td>1.577</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>0.655</td>
<td>0.676</td>
<td>0.725</td>
<td>0.863</td>
<td>1.168</td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td>0.572</td>
<td>0.567</td>
<td>0.577</td>
<td>0.639</td>
<td>0.794</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>0.506</td>
<td>0.486</td>
<td>0.477</td>
<td>0.496</td>
<td>0.564</td>
<td></td>
</tr>
<tr>
<td>5.0</td>
<td>0.453</td>
<td>0.424</td>
<td>0.403</td>
<td>0.405</td>
<td>0.433</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>0.410</td>
<td>0.377</td>
<td>0.351</td>
<td>0.343</td>
<td>0.353</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>0.375</td>
<td>0.338</td>
<td>0.312</td>
<td>0.298</td>
<td>0.300</td>
<td></td>
</tr>
</tbody>
</table>

VII. SUMMARY AND CONCLUSIONS

SPRTTEST simulates the SPRT for populations described by the normal distribution. SPRTTEST and its variation SPRTREP are listed in the appendices; Los Alamos users can obtain them directly from the MASS storage system using the command GET/KLCQ2/name.

The SPRTTEST program should prove useful in deciding whether to use the SPRT or another statistical test in various applications, in selecting parameters for the test, and in determining what experimental results would be expected ideally using a particular SPRT. Its current use is primarily for nuclear safeguards testing, but it should also be useful in other fields involving random sampling from populations approximated by the normal distribution. The various tables and figures in this report provide some insights into the usefulness and limitations of the SPRT for such applications.

For the domain of $\alpha$ and $\beta$ of most interest in safeguards applications, it was shown that for $N0 = 10$, $\alpha$ is always equal to or less than the nominal $\alpha_o$ for unforced decisions, and $\beta < \beta_o$ for $UADD = Y1 + Y2$. For other values of $UADD$, $\beta$ may be greater or lesser than the single-interval test $\beta$, but a number of trends were noted.
The average length of time required to complete an SPRT is usually less than that for the single-interval test on which it is based for background (UADD = 0.0) sampling and for UADD \( \geq Y_1 + Y_2 \). In between, however, it is often longer.

The effect of dividing the nominal single-interval period into different numbers of steps, NO, was investigated and trends were noted. For NSTEP = NO = 1, the SPRT was shown to be equivalent to the nominal single-interval test on which it is based, for the forced decision criteria used in the program.

A maximum time may be imposed on the SPRT by forcing a decision after NSTEP steps of the sequence. This never improves \( \alpha \) and \( \beta \) simultaneously and may increase both, while the ASN decreases (or in extreme cases, remains the same). In general, NSTEP should be as large as tolerable to maximize the power of the SPRT. However, even when NSTEP = NO, the SPRT may be preferred to the single-interval test for particular applications; this choice for NSTEP ensures that the SPRT is never longer than the single-interval test on which it is based.
The effect of varying $\beta_0$ was investigated over a limited range. In general, if it is most important to detect the source strength corresponding to a particular $\beta_0$, then input of that value provides the best SPRT. However, if a broad range of source strengths is of more or less equal importance, then it may be desirable to investigate the effect of varying $\beta_0$, using the Monte Carlo technique, before deciding on which $\beta_0$ to use in the particular safeguards monitor. That type of investigation was demonstrated in this report.

While not described in this report, SPRTEST can be easily modified to examine more complex safeguards problems. For example, the source strength can be varied during a test sequence to simulate passage of a source through a radiation monitor. The frequency of detection with the SPRT can then be compared with that for the single-interval test, or other commonly used tests such as the sliding-interval procedure. SPRTEST may also be readily modified to use a Poisson distribution instead of the normal distribution used in this report.

ACKNOWLEDGMENTS

I am grateful to Paul E. Fehlau of Los Alamos who introduced me to the subject of the SPRT. The Monte Carlo Theory and Application Course, taught by Tom Booth also of Los Alamos, provided me with the background necessary to conceive this study and the basic technique to carry it out.

REFERENCES


APPENDIX A

SPRTEST FORTRAN LISTING
Los Alamos Identification No. LP-1732

1 $ FTN (I=SPRTEST,GO,SET,SYM=)
2 PROGRAM SPRTEST(TTY,INPUT=TTY,OUTPUT=TTY)
3 KEN COOP'S PROGRAM TO TEST WALD'S SEQUENTIAL PROB. RATIO TEST
4 GROUP Q-2, LOS ALAMOS NATIONAL LABORATORY, MAIL STOP U-562
5 WRITTEN IN FORTRAN IV FOR THE LOS ALAMOS LTSS COMPUTER SYSTEM
6 JANUARY 3, 1985 VERSION
7
8 INTEGER FNHO,FNH1
9 DIMENSION IHO(IOO),IHO1(IOO)
10 C
11 C INITIALIZE SOME PARAMETERS
12 DO 10 J=1,100
13 IHO(J)=0
14 10 IH1(J)=0
15 NH1=0
16 NHO=0
17 ASN=0.0
18 LOOP=1
19 C
20 C READ IN PARAMETERS FROM KEYBOARD
21 C
22 C READ IN THE NOMINAL ALPHA
23 PRINT 12
24 READ 14,ALPHA
25 C READ IN THE NOMINAL BETA
26 PRINT 16
27 READ 18,BETA
28 C READ IN Y1, THE ABSCISSA VALUE CORRESPONDING TO ALPHA(NOMINAL)
29 PRINT 20
30 READ 22,Y1
31 C READ IN Y2, THE ABSCISSA VALUE CORRESPONDING TO BETA(NOMINAL)
32 PRINT 24
33 READ 22,Y2
34 C READ FROM KEYBOARD VALUE TO ADD TO U TO GET MEAN OF DISTRIBUTION
35 C THAT IS BEING TESTED OR SIMULATED
36 C PROPERLY LOCATED FOR HYPOTHESIS H0, THE VALUE IS 0.0
37 PRINT 30
38 READ 60,UA00
39 C READ IN NO, NO. OF STEPS CORRESPONDING TO NOMINAL SINGLE-INTERVAL TEST
40 PRINT 26
41 READ 28,NO
42 C READ IN STEP NO. AFTER WHICH A DECISION IS FORCED
43 PRINT 40
44 READ 70,NSTEP
45 C READ IN SEED FOR RANDOM NO. GENERATOR;
46 C USUALLY THIS WILL BE 0 (ZERO)
47 PRINT 50
48 READ 80,NSEED
49 PRINT 90,NSEED
50 12 FORMAT(/,30H TYPE IN ALPHA (F10.8) )
51 14 FORMAT(F10.8)
52 16 FORMAT(/,30H TYPE IN BETA (F10.8) )
53 18 FORMAT(F10.8)
54 20 FORMAT(/,30H TYPE IN Y1 (F7.5) )
55 22 FORMAT(F7.5)
56 24 FORMAT(/,30H TYPE IN Y2 (F7.5) )
57 26 FORMAT(/,30H TYPE IN NO (I2) )
58 28 FORMAT(I2)
59 30 FORMAT(/,30H TYPE IN UA00 (F7.5) )
60 40 FORMAT(/,30H TYPE IN NSEED (I18) )
61 50 FORMAT(/,30H TYPE IN NSEED (I18) )
62 60 FORMAT(F7.5)
63 70 FORMAT(I2)
64 80 FORMAT(I18)
65 90 FORMAT(5X,25HRANDOM NO. STARTING SEED=,I20)
66 C ALPHA IS THE FALSE POSITIVE PROBABILITY (ERROR OF FIRST KIND)
67 C BETA IS FALSE NEGATIVE PROB. (ERROR OF SECOND KIND)
68 C Y1 IS THE ABSCISSA OF THE NORMAL DIST. CORRESPONDING TO ALPHA
69 C Y2 IS THE ABSCISSA (ABSOLUTE VALUE) FOR BETA
70 C NO IS THE NOMINAL NUMBER OF STEPS CORRESPONDING TO THE SO-CALLED
71 C (BY WALD) "CURRENT BEST SINGLE TEST PROCEDURE"
72 C I REFER TO IT AS THE "SINGLE-INTERVAL" TEST OR "SIT"
73 C CALCULATE SOME VALUES USED FOR ALL TRIALS BELOW
74 C
75 C  
76 A=ALOG((1.0-BETA)/ALPHA) 
77 B=ALOG(BETA/(1.0-ALPHA)) 
78 UADD=UADD/NO**.50 
79 THETA=(Y1+Y2)/NO**0.50 
80 C INITIALIZE RANDOM NUMBER GENERATOR, USING RANSET( ), IF CALLED 
81 IF(NSEED.EQ.0) GO TO 100 
82 CALL RANSET(NSEED) 
83 C 
84 C MAIN LOOP STARTS 
85 C 
86 100 LOOP=LOOP+1 
87 X=0.0 
88 IF(LOOP.GE.100000) GO TO 300 
89 DD 200 K=1.98 
90 C FIND EFFECT OF STOPPING AFTER NSTEP STEPS 
91 IF(K.NE.NSTEP+1) GO TO 120 
92 IF(Z.LE.0.0) IHO(100)=IHO(100)+1 
93 IF(Z.GT.0.0) IHI(100)=IHI(100)+1 
94 120 CONTINUE 
95 C OBTAIN ABSISSA VALUES FROM NORMAL DISTRIBUTION SAMPLING 
96 R=(-ALOG(RANF(1)) )**0.5 
97 TNU=1.5707963*RANF(1) 
98 Y=1.4142136*R*COS(TNU) 
99 IF(RANF(1).GT.0.5000) GO TO 150 
100 Y=-Y 
101 150 CONTINUE 
102 C 
103 C CALCULATE Z, THE LOGARITHM OF THE PROBABILITY RATIO 
104 M=K 
105 U=Y+UADD 
106 X=X+THETA*U 
107 Z=X - M*THETA*THETA**.50 
108 C COMPARE Z WITH LIMITS, REPEAT TEST OR STORE RESULT 
109 C 
110 IF(Z.LE.B) GO TO 280 
111 IF(Z.GE.A) GO TO 290 
112 200 CONTINUE 
113 IF(Z.LE.0.0) IHO(99)=IHO(99)+1 
114 IF(Z.GT.0.0) IHI(99)=IHI(99)+1 
115 GO TO 100 
116 280 IHO(M)=IHO(M)+1 
117 GO TO 100 
118 290 IHI(M)=IHI(M)+1 
119 GO TO 100 
120 C PRINT OUT MATRICES 
121 C 
122 300 PRINT 380 
123 PRINT 400, (IHO(K),K=1,100) 
124 PRINT 390 
125 PRINT 400, (IHI(K),K=1,100) 
126 380 FORMAT((//,10X,"MATRIX IHO(BACKGROUND-ONLY): ")) 
127 390 FORMAT((//,10X,"MATRIX IHI(ABOVE-BACKGROUND): ")) 
128 400 FORMAT(5X,10I6)
129 C  CALCULATE AVERAGE NUMBER OF STEPS
130 C  ASN IS THE NUMBER WITH 98 STEPS PERMITTED
131 C  FASN IS THE NUMBER WITH A MAX. OF NSTEP STEPS PERMITTED
132 C  NHO IS TOTAL NUMBER OF RUNS ENDING WITH HO FOR 98 STEP MAX.
133 C  NH1 IS TOTAL ENDING IN DECISION H1 FOR 98 STEP MAX.
134 C
135 DO 500 J=1,99
136 IF(J.NE.NSTEP+1) GO TO 450
137 FASN=ASN
138 FNHO=NHO
139 FNHI=NH1
140 CONTINUE
141 500 CONTINUE
142 NHO=NHO+IHO(J)
143 NH1=NH1+IH1(J)
144 500 ASN=ASN+(IHO(J)+IH1(J))*J
145 ASN=ASN/LOOP
146 FASN=FASN+(IHO(100)+IH1(100))*NSTEP
147 FASN=FASN/LOOP
148 C  FNHO IS THE NUMBER OF TESTS ACCEPTING HO FOR A MAX. OF NSTEP STEPS
149 C  FNHI IS THE NO. OF TESTS REJECTING HO FOR A MAX. OF NSTEP STEPS
150 FNHO=FNHO+IHO(100)
151 FNHI=FNHI+IH1(100)
152 C
153 PRINT OUT CALCULATED RESULTS AND NEXT RANDOM GEN.-SEED USING RANGET( )
154 C
155 PRINT 550,ASN,FASN
156 550 FORMAT(/,10X,6H ASN= ,F10.3,10X,"ASN(FORCED)= ",F10.3)
157 PRINT 560,ASN/N0,FASN/N0
158 560 FORMAT(/,11X,"ASN/NO=",F7.4,11X,"ASN(FORCED)/NO=",F7.4)
159 PRINT 600,NHO,NH1
160 600 FORMAT(/,10X,6H NHO= ,17,5X,6H NH1= ,17)
161 ANHO=NHO+1.0
162 ANH1=NH1+1.0
163 AFNH1=FNH1+1.0
164 AFNHO=FNHO+1.0
165 IF(UAOD.GT.0.0) GO TO 635
166 620 PRINT 630,ANH1/(ANHO+ANH1)
167 630 FORMAT(/,11X,"ALPHA=",F9.6)
168 GO TO 645
169 635 PRINT 640, ANHO/(ANHO+ANH1)
170 640 FORMAT(/,10X,"BETA=" ,F9.6)
171 645 PRINT 650,AFNH1,AFNHO
172 650 FORMAT(/,10X,6HFNH1= ,17,5X,6HFNH1= ,17)
173 IF(UAOD.GT.0.0) GO TO 685
174 PRINT 680,AFNH1/(AFNH1+AFNHO)
175 680 FORMAT(/,10X,"ALPHA(FORCED)= ",F9.6)
176 GO TO 700
177 685 PRINT 690,AFNHO/(AFNHO+AFNH1)
178 690 FORMAT(/,10X,"BETA(FORCED)= ",F9.6)
179 700 RAN=RANF(1)
180 CALL RANGET(NUM)
181 PRINT 800,NUM
182 800 FORMAT(/,10X,30H LAST RANDOM NO. STARTING SEED=,I20,////)
183 END
APPENDIX B

SPRTREP FORTRAN LISTING

1 $ FTN (I=SPRTREP,GO,SET,SYM=^)
2 C PROGRAM SPRTREP(TTY,INPUT=TTY,OUTPUT=TTY)
3 C KEN COOP'S PROGRAM TO TEST WALD'S SEQUENTIAL PROB. RATIO TEST
4 C GROUP Q-2, LOS ALAMOS NATIONAL LABORATORY, MAIL STOP J-562
5 C WRITTEN IN FORTRAN IV FOR THE LOS ALAMOS LTSS COMPUTER SYSTEM
6 C JANUARY 3, 1985 VERSION
7 C THIS VERSION REPEATS SPRTEST 11 TIMES WITH INCREMENTED UADD VALUES
8 C
9 INTEGER FNHO,FNH1
10 DIMENSION IHO(100),IH1(100)
11 C READ IN PARAMETERS FROM KEYBOARD
12 C READ IN THE NOMINAL ALPHA
13 PRINT 12
14 READ 14,ALPHA
15 C READ THE NOMINAL BETA
16 PRINT 16
17 READ 18,BETA
18 C READ IN Y1, THE ABSCISSA VALUE CORRESPONDING TO ALPHA(NOMINAL)
19 PRINT 20
20 READ 22,Y1
21 C READ IN Y2, THE ABSCISSA VALUE CORRESPONDING TO BETA(NOMINAL)
22 PRINT 24
23 READ 22,Y2
24 C READ IN UAOD, WHICH IN THIS PROGRAM IS THE INCREMENT FOR THE ABSCISSA
25 PRINT 30
26 READ 60,UAOD
27 C READ IN NO. NO. OF STEPS CORRESPONDING TO NOMINAL SINGLE-INTERVAL TEST
28 PRINT 40
29 READ 28,N0
30 C READ IN STEP NO. AFTER WHICH A DECISION IS FORCED
31 PRINT 50
32 READ 70,NSTEP
33 C READ IN SEED FOR RANDOM NO. GENERATOR.
34 PRINT 60
35 PRINT 90,NSEE0
36 FORMAT(/,30H TypE IN ALPHA (F10.8) )
37 FORMAT(/,30H TypE IN BETA (F10.8) )
38 FORMAT(/,30H TypE IN YI (F7.5) )
39 FORMAT(/,30H TypE IN Y2 (F7.5) )
40 FORMAT(/,30H TypE IN NO (I2) )
41 PRINT 50
42 PRINT 90,NSEE0
43 12 FORMAT(/,30H TYPE IN ALPHA (F10.8) )
44 14 FORMAT(/,30H TYPE IN BETA (F10.8) )
45 18 FORMAT(/,30H TYPE IN UADD (F7.5) )
46 20 FORMAT(/,30H TYPE IN Y1 (F7.5) )
47 22 FORMAT(/,30H TYPE IN Y2 (F7.5) )
48 26 FORMAT(/,30H TYPE IN NO (I2) )
49 28 FORMAT(/,30H TYPE IN NSEE0 (I18) )
50 30 FORMAT(/,30H TYPE IN UAOD (F7.5) )
51 40 FORMAT(/,30H TYPE IN NSTEP (I2) )
52 50 FORMAT(/,30H TYPE IN NSEE0 (I18) )
53 60 FORMAT(/,30H TYPE IN NSEE0 (I18) )
54 70 FORMAT(/,30H TYPE IN NSEE0 (I18) )
55 80 FORMAT(/,30H TYPE IN NSEE0 (I18) )
56 90 FORMAT(/,30H TYPE IN NSEE0 (I18) )
57 C ALPHA IS THE FALSE POSITIVE PROBABILITY (ERROR OF FIRST KIND)
58 C BETA IS FALSE NEGATIVE PROB. (ERROR OF SECOND KIND)
59 C Y1 IS THE ABSCISSA OF THE NORMAL DIS. CORRESPONDING TO ALPHA
60 C Y2 IS THE ABSCISSA (ABSOLUTE VALUE) FOR BETA
61 C NO IS THE NOMINAL NUMBER OF STEPS CORRESPONDING TO THE SO CALLED
62 C (BY WALD) "CURRENT BEST SINGLE TEST PROCEDURE"
63 C I REFER TO IT AS THE "SINGLE-INTERVAL" TEST OR "SITU"
CALCULATE SOME VALUES USED FOR ALL TRIALS BELOW

A = ALOG((1.0 - BETA)/ALPHA)
B = ALOG(BETA/(1.0 - ALPHA))
UADDIJ = UADD/NO**.50
THETA = (Y1 + Y2)/NO**.50

INITIALIZE RANDOM NUMBER GENERATOR USING RANSET(I), IF CALLED
IF(NSEED.EQ.0) GO TO 97
CALL RANSET(NSEED)

THIS VERSION REPEATS SPRTEST 11 TIMES WITH INCREMENTED UADD VALUES
DD 1000 IJ = 1, 11
UADD = (IJ - 1) * UADDIJ

INITIALIZE SOME PARAMETERS
DD 98 J = 1, 100
IHO(J) = 0
IH1(J) = 0
NH1 = 0
NHO = 0
ASN = 0.0
LOOP = -1

MAIN LOOP STARTS
100 LOOP = LOOP + 1
X = 0.0
IF(LOOP.GE.100000) GO TO 300

FIND EFFECT OF STOPPING AFTER NSTEP STEPS
120 CONTINUE

OBTAIN ABSCISSA VALUES FROM NORMAL DISTRIBUTION SAMPLING
150 CONTINUE

CALCULATE Z, THE LOGARITHM OF THE PROBABILITY RATIO
M = K
U = Y + UADD
X = X + THETA + U
Z = X - M * THETA - THETA**.50

COMPARE Z WITH LIMITS, REPEAT TEST OR STORE RESULT
140 CONTINUE

PRINT OUT MATRICES
300 PRINT 380
301 PRINT 380, IHO(K), K = 1, 100
302 PRINT 390
303 PRINT 390, IH1(K), K = 1, 100
304 FORMAT(///, 10X, "MATRIX IHO(BACKGROUND-ONLY): ", /)
305 FORMAT(///, 10X, "MATRIX IH1(ABOVE-BACKGROUND): ", /)
306 FORMAT(5X, 10I8)
135 C CALCULATE AVERAGE NUMBER OF STEPS
136 C ASN IS THE NUMBER WITH 98 STEPS PERMITTED
137 C FASN IS THE NUMBER WITH A MAX. OF NSTEP STEPS PERMITTED
138 C NHO IS TOTAL NUMBER OF RUNS ENDING WITH HO FOR 98 STEP MAX.
139 C NH1 IS TOTAL ENDING IN DECISION H1 FOR 98 STEP MAX.
140 C
141 DO 500 J=1,99
142 IF(J.NE.NSTEP+1) GO TO 450
143 FASN=ASN
144 FNHO=NHO
145 FNH1=NH1
146 450 CONTINUE
147 NH1=NH1+IH1(J)
148 ASN=ASN+(IH0(J)+IH1(J))*J
149 ASN=ASN/LOOP
150 FASN=FASN+(IH0(100)+IH1(100))*NSTEP
151 FASN=FASN/LOOP
152 C FNHO IS THE NUMBER OF TESTS ACCEPTING HO FOR A MAX. OF NSTEP STEPS
153 C FNH1 IS THE NO. OF TESTS REJECTING HO FOR A MAX. OF NSTEP STEPS
154 FNHO=FNHO+IH0(100)
155 FNH1=FNH1+IH1(100)
156 C PRINT OUT CALCULATED RESULTS, UADO, AND NEXT RANDOM GEN. SEED
157 C
158 C
159 C
160 PRINT 550,ASN,FASN
161 550 FORMAT(///,10X,6H ASN=,FI0.3,10X,*ASN(FORCED)=",F10.3)
162 PRINT 560,ASN/NO,FASN/NO
163 560 FORMAT(/,11X,"ASN/NO="*F7.4,11X,"ASN(FORCED)/NO="*F7.4)
164 PRINT 600,NHO,NH1
165 600 FORMAT(/,10X,6H NHO=,17,5X,6H NH1=,17)
166 ANHO=NHO*1.0
167 ANH1=NH1*1.0
168 AFNH1=FNH1*1.0
169 AFNHO=FNHO*1.0
170 IF(UADO.GT.0.0) GO TO 635
171 620 PRINT 630,ANH1/ANHO
172 630 FORMAT(/,11X,"ALPHA="*F9.6)
173 GO TO 645
174 635 PRINT 640,ANHO/(ANHO+ANH1)
175 640 FORMAT(/,11X,"BETA="*F9.6)
176 645 PRINT 650,FNHO,FNH1
177 650 FORMAT(/,11X,6HFNHO=,17,5X,6HFNH1=,17)
178 IF(UADO.GT.0.0) GO TO 665
179 680,AFNH1/(AFNH1+AFNHO)
180 680 FORMAT(/,11X,"ALPHA(FORCED)=",F9.6)
181 GO TO 700
182 685 PRINT 690,AFNHO/(AFNHO+AFNH1)
183 690 FORMAT(/,11X,"BETA(FORCED)=",F9.6)
184 700 RAN=RANF(1)
185 CALL RANGET(NUM)
186 750 FORMAT(/,11X,7HUADD=",F9.5,//)
187 C THE VALUE PRINTED OUT FOR UADD HAS THE INTERPRETATION OF BEING
188 C THE ABCISSA VALUE OF THE MEAN OF THE DIST. BEING TESTED
189 C
190 PRINT 800,NUM
191 800 FORMAT(11X,3OhLAST RANDOM NO. STARTING SEED=,I20,////////)
192 1000 CONTINUE
193 END
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*Contact NTIS for a price quote.