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TITLE  The Empirical Connection Between $(p,n)$ Cross Sections and Beta Decay Transition Strengths

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THE EMPIRICAL CONNECTION BETWEEN (p,n) CROSS SECTIONS AND BETA DECAY TRANSITION STRENGTHS

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ABSTRACT

A proportionality is assumed to exist between 0° (p,n) cross sections and the corresponding beta decay transition strengths. The validity of this assumption is tested by comparison of measured (p,n) cross sections and analogous beta decay strengths. Distorted waves impulse approximation calculations also provide useful estimates of the accuracy of the proportionality relationship.

INTRODUCTION

Standard reaction theory and experimental observations support the idea that there should be a measure of proportionality between (p,n) cross sections and beta decay transition strengths. This correspondance derives from the similarity of the operators involved in each type of reaction. The central isovector terms in the effective nucleon nucleon interaction that mediate low momentum transfer spin flip (S - 1) transitions,

\[ V_{\sigma t (1ip)}^{\dagger} \sigma_{1} \cdot \sigma_{p}, \]

and non spin flip (S = 0) transitions,

\[ V_{1 (1ip)}^{\dagger} t_{1} \cdot t_{p}, \]
are similar to the corresponding operators

\[ G_A \sum_{I} q_1 i_1^+ \]

and

\[ G_V \sum_{I} q_1 i_1^+ \]

for Gamow-Teller (GT) and Fermi (F) beta decay, respectively. There are numerous beta transitions with known decay rates for which the analogous (p,n) cross section can also be measured. The existence or absence of a useful proportionality can therefore be tested empirically. The initial results of such an investigation have been reported by Goodman et al.\(^2\) for \(E_p = 120\) MeV and were highlighted at the very first Telluride conference. Much additional data has become available since then, and the experimental situation up to about 1986 has been reviewed in another paper.\(^4\) In this article I will summarize previous results and present some new data.

From general considerations, the (p,n) cross section should depend on the bombarding energy \(E_p\), the number and type of nucleons in the target nucleus \(A(N,Z)\), the asymptotic three-momentum transfer \(q\), the energy loss \(\omega\), where

\[ \omega = E_x + (N'_A - N_A + N_n + N_p)c^2, \]

and the specific nuclear structure relationship between initial and final nuclear states. The dependence on these quantities can be expressed as a product of three factors:

\[ \sigma = \hat{\sigma}_\alpha(E_p,A)F_\alpha(q,\omega)B(\alpha), \tag{1} \]

where \(\alpha = F\) or GT. The proportionality factor \(\hat{\sigma}\), which I shall call the "unit cross section," can be bombarding energy and target dependent and is the factor of primary interest. The factor \(F(q,\omega)\) describes the shape of
the cross section distribution. At fixed $\omega$ it is approximately an exponential function of $q^2$ (for $q < 0.5 \, fm^{-1}$),

$$ F(q, \omega) = C(\omega) \exp(-\frac{1}{3} \langle r^2 \rangle q^2) $$

and goes to unity in the limit of zero momentum transfer and energy loss:

$$ F(q, \omega) \rightarrow 1 \quad \text{as} \quad (q, \omega) \rightarrow 0. $$

The beta decay transition strengths $B(\alpha)$ are obtained from beta decay lifetimes according to

$$ (G_V)^2 B(F) + (G_A)^2 B(GT) = \frac{K}{\tau} $$

where

$$ \frac{K}{(G_V)^2} = 6166 \pm 2 \, \text{sec} $$

and

$$ \left( \frac{G_A}{G_V} \right)^2 = (1.260 \pm 0.008)^2. $$

The coupling constant values used here are those recommended by Wilkinson.\(^5\)

It is useful to distinguish between comparisons of $(p,n)$ and beta decay for different transitions starting from the same parent state, which I shall call specific proportionality, and comparisons of cross sections for transitions originating from different target nuclides, which I shall call general proportionality. In the former case a knowledge of the $A$ dependence of $\delta$ is not required. Application of the more general proportionality relationship will require the $A$ dependence of $\delta$ to be smooth or at least calculable.
REACTION MODEL CALCULATIONS

The distorted waves impulse approximation (DWIA) calculations described here include "exact" knock-on exchange amplitudes and were performed with the code DV81.6 The calculations employed relativistic kinematics but are otherwise consistent with the standard non-relativistic Schrödinger equation. The effective interaction used was the nucleon-nucleon t-matrix parametrization of Franey and Love (FL).7 The 175-MeV version of this interaction was used for the reaction calculations at 160 MeV. Single-particle wave functions were calculated in a harmonic oscillator basis and are labeled by the notation \( j_{\pi} = \Omega + 1/2 \) and \( j_{\zeta} = \Omega - 1/2 \), where \( \Omega \) is the orbital angular momentum. The optical potential parameters used in the DWIA calculations were those of Meyer et al.8 for \( A = 6-18 \), Olmer et al.9 for \( A = 28 \), and Schwandt et al.10 for \( A = 40-208 \). The Schwandt parameters were extended to the lower mass range for comparison purposes.

The results of the DWIA calculations for \( E_p = 160 \) MeV are shown in Fig. 1. The variations in \( \delta \) for different particle-hole configurations are not large for \( 1^+ \) transitions with the full single-particle GT strength. The dashed line in this figure represents the average mass dependence of \( \delta_{\text{GT}}(A) \) and can be used to assess the implicit accuracy of the proportionality of Eq. (1), as predicted in the DWIA. The standard deviation of the DWIA values of \( \delta_{\text{GT}} \) with respect to this average mass dependence is \( \Delta \delta / \delta = 7\% \). To the extent that the DWIA variations model the expected variations in nature, this value of \( \Delta \delta / \delta \) thus represents the smallest level of uncertainty that can be achieved in the experimental determination of GT transition strengths through the use of a proportionality relation such as Eq. (1).

In contrast to the GT unit cross sections, large variations are observed in the calculated Fermi unit cross sections. Central interaction exchange amplitudes alone cause \( j_{\pi} j_{\zeta}^{1-0^+} \) transitions to have unit cross sections larger than those for \( j_{\zeta} j_{\zeta}^{1-1^+} \) transitions. This difference increases with target mass from about 5\% for \( A = 17 \) to about 14\% for \( A = 90 \) and vanishes when the abnormal parity \( j_{\pi} j_{\zeta} = 0 \) \( |1, 1^+ \) amplitude is set to zero. Love, Nakayama, and Franey11 pointed out that an interference between the microscopic \( V_{LS\pi} \) and \( V_{\pi} \) interactions in the
Fig. 1 Distorted-waves impulse approximation unit cross sections (squares). Multiple boxes for a given value of \(A\) correspond to different particle-hole configurations; the smallest Fermi cross sections for a given value of \(A\) correspond to \(|\ell|\frac{1}{2}-\frac{1}{2}\) transitions. The dashed line represents the average \(A\) dependence of the GT unit cross sections. (See Ref. 4).

The calculations of Fig. 1 employ full single particle strengths. However, an important concern in the discussion of proportionality is the range of transition strengths over which the relationship is valid. The
proportionality must obviously fail when the L=0 central interaction amplitude becomes so weak that competing amplitudes are comparable in magnitude. This issue is best addressed by comparing individual beta decay strengths to relevant single-particle strengths, given by:

\[
B(\text{GT})_{sp} = \begin{cases} 
\frac{(j_+ + 1)}{j_+} & j_+ j_+^{-1} \\
\frac{(2j_+ + 1)}{j_+} & j_+ j_-^{-1} \\
\frac{(2j_+ - 1)}{j_+} & j_- j_+^{-1} \\
\frac{j_-}{(j_- + 1)} & j_- j_-^{-1} 
\end{cases}
\]  \hspace{1cm} (3)

Figure 2 shows calculations of the unit GT cross section for two different mass values at 160 MeV. In these calculations the GT amplitude was decreased while holding other amplitudes constant. The dotted horizontal lines represent a ±10% variation from the average value of \(\theta_{\text{GT}}\) for full single-particle strength. With the exception of \(j_- j_+^{-1}\) transitions, which are strongly affected by tensor exchange amplitudes, the calculated unit cross sections remain within the 10% limit to quite small values of the GT strength relative to the full single-particle strength.

The \(^{27}\text{Al}(p,n)\) reaction provides a good empirical example of weak transitions for which the proportionality appears to be valid. Very good correspondence is observed between beta transition strengths and \((p,n)\) cross sections for the transitions to the 7.16 MeV and 7.65 MeV levels in \(^{27}\text{Si}\) (Fig. 3). These transitions carry only 5.1% and 7.6% of the \(d_{3/2} d_{5/2}^{-1}\) and \(d_{5/2} d_{5/2}^{-1}\) single particle strength, respectively.
Fig. 7 Values of the GT unit cross section for A=12 and A=29. Starting with the three pure single particle transitions indicated \((B_{GT}/B_0(GT) - 1)\), the GT amplitude was decreased while holding the other amplitudes fixed. \(B_0(GT)\) represents the full single particle strength. The dotted horizontal lines indicate a 10% variation from the average value at \(B_{GT}/B_0(GT) = 1\). The F1 1/2 MeV interaction containing central, spin orbit, and tensor terms was used in the calculations.
Fig. 3 Cross section spectrum for $^{27}\text{Al}(p,n)$ at $0^\circ$ and 120 MeV. The vertical bars represent the GT transition strengths for analogous beta decays. The Fermi strength is indicated by the dashed vertical line.

EMPIRICAL EVIDENCE FOR SPECIFIC PROPORTIONALITY

Specific proportionality implies that all GT (p,n) transitions originating from a given target nuclide will have the same beta-decay proportionality factor (unit cross section). Equivalently, there will be a fixed proportionality between these Gf (p,n) cross sections and the Fermi component of the cross section for the isobaric analog state transition. In contrast to the large configuration dependent variations in the ratio $\frac{\sigma_{GT}}{\sigma_{F}}$ as displayed in Fig. 1, experimental studies of GT and F transitions have shown a well defined ratio between GT and F cross sections in the energy range 50 - 200 MeV. This ratio can be conveniently parametrized as
\[ \frac{\hat{\sigma}_{GT}}{\hat{\sigma}_F} = \left( \frac{E_p}{\epsilon_0} \right)^2 \]  

(4)

where \( \epsilon_0 = 55.0 \pm 0.4 \) MeV. A summary of the data available up to 1986 is shown in Fig. 4. The standard deviation of the data points plotted in this figure is \( \Delta \epsilon_0 = 1.7 \) MeV. Note that this implies an uncertainty in the ratio of unit cross sections of about 6%.

In addition to the \(^{27}\)Al(p,n) transitions illustrated in Fig. 3, some more examples that appear to demonstrate the validity of specific proportionality are \(^{13}\)C(p,n), \(^{14}\)O(p,n), \(^{26}\)Mg(p,n), and \(^{34}\)S(p,n). Spectra for (p,n) reactions on these target nuclides are shown in Figs. 5-8 for a bombarding energy of 120 MeV. The ratio of unit cross sections defined by Eq. (4) is assumed in plotting the relative sizes of the GT and F transition strength bars in these figures. That is, to within an overall

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**Fig. 4** The parameter \( \epsilon_0 = E_p / (\frac{\hat{\sigma}_{GT}}{\hat{\sigma}_F})^{1/2} \). Solid circles are alternated with open squares to indicate data points for different nuclides. For a given nuclide, the bombarding energy increases from left to right. From the left, the data correspond to \(^7\)Li, \(^{14}\)C, \(^{18}\)O, \(^{26}\)Mg, \(^{27}\)Al, and \(^{47}\)Ca. The solid horizontal line represents the weighted average \( \epsilon_0 = 55.0 \pm 0.4 \).
Fig. 5 Cross section spectrum for $^{13}$C(p,n) at 0° and 120 MeV. The vertical bars represent the GT transition strengths for analogous beta decays. The Fermi strength is indicated by the dashed vertical line.

Scale factor the quantities plotted are $F(q,\omega)B(GT)$ (solid bars) and $F(q,\omega)B(F)(E_0/E_p)^2$ (dashed bars).

**Empirical Evidence for General Proportionality**

If the nuclide specific proportionality factors between (p,n) cross sections and beta decay transition strengths were to follow a smooth or at least predictable trend, then a more generally useful proportionality would exist. In such a case a limited number of (p,n) transitions could be calibrated against analogous beta decay transitions to determine the proportionality constant $\hat{\alpha}_{GT}$, and this empirically established
Fig. 6 Cross section spectrum for $^{18}$O(p,n) at 0° and 120 MeV. The solid vertical bars represent the GT transition strengths for analogous beta decays and the dashed bar represents the Fermi strength. The overall normalization for this spectrum is poorly determined.

...proportionality factor could then be used to measure GT strengths for transitions in other nuclides.

The empirical results for $\delta_{\text{GT}}(A)$ and $\delta_{\text{F}}(A)$ are plotted for $E_p$ - 160 MeV in Fig. 9. Also plotted is the average value of $\delta_{\text{GT}}(A)$ determined from the DWIA calculations of Fig. 1. While the experimental points seem to follow the general trend of the DWIA mass dependence, it is very obvious that the scatter in the experimental values is much larger than the scatter in the calculated values (Fig. 1). The relative uncertainty weighted standard deviation of the experimental GT points with respect to the normalized average DWIA mass dependence is $\Delta \delta/\delta$ - 77% for $E_p$ - 160 MeV. This spread, if treated as statistical, is too
Fig. 7 Cross section spectrum for $^{26}$Mg(p,n)$^{26}$Al at 0° and 120 MeV. The vertical bars represent the GT transition strengths for analogous beta decays. The Fermi strength is indicated by the dashed vertical line.

large to be accounted for by the estimated experimental relative normalization uncertainty of about 8%.

The scatter in the experimental points is central to the investigation of general proportionality; it is therefore important to establish the relative cross sections accurately. Present evidence strongly supports a nonstatistical origin for the observed scatter. A subset of the points displayed in Fig. 9 consists of several independent measurements (i.e., different targets and detector configurations) which yield consistent results. In particular, I shall focus the discussion on the results for $^{12}$C, $^{14}$C, and $^{16}$C, which exhibit variations in $\delta_{GT}$ of as much as 50% from isotope to isotope.
Fig. 8 Cross section spectrum for $^{34}\text{S}(p,n)^{34}\text{Cl}$ at 0° and 120 MeV. The vertical bars represent the GT transition strengths for analogous beta decays. The Fermi strength is indicated by the dashed vertical line.

The relative cross sections for $^{12}\text{C}(p,n)$ and $^{11}\text{C}(p,n)$ have been verified through measurements with natural carbon targets, which are 1.11% $^{11}\text{C}$. The large difference in reaction 0 values for these two isotopes allows a very clean observation of the low excitation $^{11}\text{C}$ peaks in the natural carbon spectrum. The relative cross sections determined in this way agree well with cross sections measured with isotopically enriched carbon targets.

The $^{14}\text{C}(p,n)$ cross sections were measured with a target constructed by mixing amorphous carbon (enriched to 98.9% $^{14}\text{C}$) with a polystyrene (C$_2$H$_5$) binder and pressing the resulting mixture into a solid disk. The target thus contained known quantities of both $^{12}\text{C}$ and $^{14}\text{C}$. The $^{12}\text{C}$ cross
Fig. 9 Experimental unit cross sections for Gamow-Teller and Fermi transitions at $E_p = 160$ MeV. The dashed line represents the mass dependence of the DWIA GT unit cross sections calculated with the FL 175 MeV interaction and has not been normalized to the data. The dotted line is the dashed line divided by $(160/55)^2$.

sections obtained from measurements with this target agree well with the cross sections obtained with isotopically pure $^{12}$C and natural carbon targets.

The evidence summarized in the preceding discussion lends strong
support to the relative experimental values of \( \delta \) determined for \(^{12}\text{C} \), \(^{13}\text{C} \), and \(^{14}\text{C} \). An explanation of the observed differences must therefore be sought in the reaction dynamics.

The surprisingly large value of \( \delta \) for \(^{13}\text{C} \) compared to that for \(^{12}\text{C} \) or \(^{14}\text{C} \) is not easily explained in the context of the standard DWIA. Reasonable variations of model parameters, e.g., optical potentials, harmonic oscillator parameter, etc., cannot reproduce the observed difference. Indeed, it appears that even unreasonable parameter variations cannot explain the difference! It would be easy to dismiss the effect as an overlooked subtlety of nuclear structure or reaction mechanism unique to the ground state of \(^{13}\text{C} \) were it not for the fact that the 15.1-MeV transition shows the same large value of \( \delta \). It is also significant that a similar enhancement is seen in other nuclei such as \(^{15}\text{N} \) and possibly \(^{19}\text{K} \). Additionally, cross sections for the analogous \(^{13}\text{C}(p,p')\) 15.1-MeV transition also appear to be enhanced relative to \(^{12}\text{C}(p,p')\) 15.1-MeV cross sections.\(^{12,13}\)

**SUMMARY AND CONCLUSIONS**

Two major problems are yet unresolved in the comparison of \((p,n)\) cross sections to analogous beta decay strengths. First, the proportionality constant that relates \((p,n)\) cross sections to beta decay transition strengths does not have a smooth target nuclide dependence, nor is the dependence presently calculable in some cases to better than about 50\% in the context of the standard DWIA reaction model. This observation has several important implications. Until the origin of the fluctuations is understood, extrapolation or interpolation of proportionality constants from one target nuclide to another must be regarded as uncertain at the 20\%–50\% level. Quantitative conclusions based upon comparisons of measured cross sections and DWIA calculations should be especially unreliable. This uncertainty must apply as well to related reactions such as \((p,p')\).

A second problem is the predicted sensitivity of 0\(^{+}\) transitions, particularly those of the \( |1\rangle \leftrightarrow |0\rangle \) type, to tensor exchange and spin orbit amplitudes. Relative cross section systematics for 0\(^{+}\) and 1\(^{+}\) transitions show no clear evidence for this effect. Two Fermi transitions which ought
to show the effect are those in $^{14}$C(p,n) and $^{34}$S(p,n). These should be predominantly $d_{1/2}p_{1/2}^{-1}$ and $d_{3/2}d_{3/2}^{-1}$, respectively, yet exhibit $\delta_{G}/\delta_{F}$ ratios consistent with other Fermi transitions of $j\geq j^{-1}$ character. An interesting counterexample to these two cases is provided by $^{35}$Cl(p,n)$^{35}$Ar. A spectrum for this reaction at 120 MeV is displayed in Fig. 10. The vertical bars in this spectrum represent the corresponding beta decay strengths in the same manner as in Figs. 3,5-8. Also, the dashed vertical line is meant to represent the Fermi strength according to the relative normalization of Eq. (3). Clearly, if the supposedly "universal" ratio of Eq. (3) is applied to this case, the Fermi cross section is considerably overestimated relative to the GT cross sections. In other words, this simple comparison seems to indicate that the Fermi cross section is much smaller than that predicted by Eq. (3). Since this should be a $d_{3/2}d_{3/2}^{-1}$ transition, this reduction is consistent with the calculated effect presented in Fig. 1. However, this comparison is complicated by the fact that the GT transitions for this case are all very weak. In fact, shell model calculations by Brown and Wildenthal indicate that all but one of these transitions can be attributed largely to $l$-forbidden amplitudes of the type $l_{s_{1/2}}d_{3/2}^{-1}$. Simple comparisons of the sort just made may therefore be very misleading. More data for Fermi transitions of $j\leq j^{-1}$ character are clearly desirable.

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REFERENCES


Fig. 10 Cross section spectrum for $^{35}\text{Cl}(p,n)^{35}\text{Ar}$ at $0^\circ$ and 120 MeV. The vertical bars represent the GT transition strengths for analogous beta decays. An estimate of the Fermi cross section based on the relationship of Eq. (4) is indicated by the dashed vertical line and goes off scale.