NUCLEAR INERTIA FOR FISSION IN A GENERALIZED CRANKING MODEL

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SUBMITTED TO: to be presented at the Nuclear Dynamics Workshop III to be held at Copper Mountain, CO, on March 4-9, 1984.

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cranking model [1] has been widely used to calculate the associated with collective degrees of freedom. After the ring correlations, theoretical results obtained with the or nuclear rotations and γ-vibrations were in relatively ith experimental data. Calculations of β-vibrational iner-
erformed in the cranking model for fission deformations. lts were several times the irrotational values [2] and gave ment with experimental spontaneous-fission lifetimes [3,4], study a renormalization factor of 0.8 was required [4]. pointed out by many authors (see ref. 5), the Inglis crank-
ses two serious deficiencies. First, problems arise when cle potential contains momentum-dependent terms. Second, in ge pairing strength the inertia approaches zero instead of a onal) limit.
approaches to the cranking model which did not lead to such ults were developed by Migdal [6], Belyaev [7] and Thouless They showed that these deficiencies of the cranking model k of self-consistency, since the reaction of the mean field α motion is neglected in the Inglis model. In ref. 5 we ents and developed a generalized cranking model for station-
ton. Here we show how to develop a time-dependent formal-
to β-vibrations and fission [9]. th the time-dependent equation for the generalized density

\[ \frac{\partial \rho}{\partial \tau} = -i \left[ H, \rho \right] \] (1)

ed that the Hamiltonian \( H \) and consequently the generalized depend on the collective variable \( \varepsilon \). Furthermore, we notion is adiabatic, which permits the replacement
Choosing the basis so that
\[ [\mathcal{K}_0, \mathcal{R}_0] = 0 \],
we then obtain to lowest order in the collective variable the equation
\[ i\hbar \dot{\mathcal{R}}_0 = [\mathcal{K}_0, \mathcal{R}_1] + [\mathcal{K}_1, \mathcal{R}_0] \]
for the generalized density matrix. Here \( \mathcal{K} \) and \( \mathcal{R} \) symbolize the matrices
\[
\mathcal{K} = \begin{pmatrix} h - \Delta \end{pmatrix} \quad \text{and} \quad \mathcal{R} = \begin{pmatrix} \rho & K \\ -K & 1 - \rho \end{pmatrix}.
\]
The usual cranking-model approximation consists of neglecting the \( \mathcal{K}_1 \) term in eq. (3). We obtain \( \dot{\mathcal{R}}_0 \), which appears on the left-hand side of eq. (3), by differentiating eq. (2) with respect to time.

From this point onwards we proceed analogously to ref. 5 and evaluate the first-order correction to the generalized density matrix \( \mathcal{R}_1 \). Its trace with the generalized collective momentum operator then yields the nuclear inertia \( B \). However, in contrast to the stationary formalism, the time-dependent formalism leads to an additional pairing-vibration coupling term [3] because of the implicit dependence of the pairing gap on the collective variable.

Keeping the \( \mathcal{K}_1 \) term in eq. (3) gives rise to two additional contribution to the inertia that are proportional to \( h_1 \) and \( \Delta_1 \). The \( h_1 \) contribution arise when the potential contains momentum-dependent terms. In the stationary case one obtains
\[ h_1 \propto (1 - m/m^*) , \]
where \( m^* \) is the effective mass. This can lead to a considerable change in the inertia [5]. We expect a similar relationship to also hold in the time-dependent case [10]. The additional \( \Delta_1 \) term, for which an explicit expression is obtained from the continuity equation [6], keeps the nuclear inertia finite in the limit of large pairing strength.

To demonstrate the effect of the \( \Delta_1 \) contribution on the inertia, we now specialize to the harmonic-oscillator potential. In the limit of zero temperature and a constant pairing gap, we obtain for the inertia
The equation provided is:

\[ h^2 \sum_{p,q} |\langle p| \Delta \epsilon | q \rangle |^2 \frac{E_p E_q - \hbar p \hbar q + \Delta^2}{2E_p E_q(E_p + E_q)} + h^2 \sum_{p} \frac{1}{8E_p} (\hbar p' \Delta - \hbar p \Delta')^2, \quad (5) \]

\[ \Delta \epsilon \] denotes differentiation with respect to \( \epsilon \). Note the plus sign of \( \Delta^2 \) in the first term, which arises from the \( \Delta_1 \) contribution.

In Fig. 1, we show the first term of the inertia for \( \beta \)-vibrations as a function of pairing strength, calculated with respect to Nilsson's spherical parameter \( \epsilon \) [2,4,5]. The pairing-vibration coupling term is considered here, since it vanishes for large pairing strength. The Inglis cranking inertia approaches zero for large pairing, while the inertia containing the \( \Delta_1 \) contribution remains finite and close to

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**Figure 1.** Dependence of the \( \beta \)-vibrational inertia upon the pairing gap \( \Delta \) in a harmonic-oscillator potential at the equilibrium deformation. The solid curve gives the present result calculated in the generator model with 15 oscillator shells, the long-dashed curve corresponding result calculated in the Inglis cranking model and the dash-dotted curve gives the irrotational result.
the limiting irrotational value. The deviation arises from the slow convergence of the cranking inertia with increasing basis size [5].

For a harmonic-oscillator potential with an effective mass, relation (4) holds, and the reaction of the pairing field to the collective motion is given by

$$ \Delta_1 \alpha Y_{20} . $$

For a more realistic modified-harmonic-oscillator potential we expect similar results. In particular, we expect that the proper inclusion of the effective-mass term $h_1$ for $\beta$-vibrational inertias may account for the renormalization factor of 0.8 that was originally needed to reproduce experimental spontaneous-fission lifetimes [4].

REFERENCES

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