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CONSTITUTIVE RELATION FOR 6061T6 ALUMINUM UNDER SHOCK LOADING CONDITIONS

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ABSTRACT

We publish for the first time a constitutive relation of Wallace and Straub based on plastic flow data extracted from 6061T6 Al shocks by Wallace. The data involves strain rates up to $10^7$ and shear stresses up to 5 kbar. We interpret the relation in terms of a dislocation-kinetics model.

INTRODUCTION

Meaningful dislocation modeling of the plastic front of shock waves has become possible in recent years with the achievement of accurate particle velocity and dislocation velocity measurements. This work has consisted mostly of fitting the shock velocity profile using a parameterized dislocation model and a computer hydrodynamics code. Recently, D.C. Wallace devised a rigorous thermodynamic scheme for weak shocks in solids. This scheme was superior to earlier efforts in that entropy contributions to the stresses were included, nonlinear elasticity was treated, and the anisotropy of the stress response was taken into account. He used this scheme in Ref. 4 to extract plastic constitutive information from the shock profiles of Johnson and Barker. This constitutive information depended only on the thermodynamics and elastic properties and not at all on any dislocation model. Wallace and Straub assembled the information into a constitutive relation. It is the purpose of this paper to publish the relation for the first time and interpret it in terms of a dislocation kinetics model. We explain the constitutive relation in terms of the Orowan equation and a dislocation generation model. We also explain why, in this situation, the plastic strain is a state variable to a good approximation.

THE CONSTITUTIVE RELATION

The open symbols of Fig. 1 show the constitutive data plotted in terms of $10^6(t' - t_0)/\dot{\psi}$ and $\psi\dot{\psi}$, where $\psi$ and $\dot{\psi}$ are the effective plastic strain and plastic strain rate, respectively. Also, $t'$ is the effective shear stress scaled according to a linear pressure and temperature dependence of the shear modulus of Aluminum, $G$. See Ref. 3 for the definitions of $\psi$ and $t'$ in terms of the tensor quantities. The
Fig. 1. The circles, squares, and triangles are data from 21, 37, and 89 kbar shocks, respectively. The line is an analytic fit.

The line is the constitutive relation of Wallace and Straub, given by Eq. (1) when 8.33 is added to the denominator of the r.h.s.:

$$10^6 (t'-t_o)/\dot{\psi} = c/\psi^{0.75}$$

where c equals 558. This 8.33 was used to interpolate to lower strain rates. Eq. (1) is the high strain rate asymptote which we use to study the high strain rate behavior. In Eq. (1), t' and t_o are in kbar and t is the scaled internal stress given by 1+22.5 \psi. The term 1 is due to the precipitation hardening of the alloy, which is taken as constant, and the other term is due to work hardening or dislocation accumulation. This expression was obtained by a straight line fit to the final shock stresses and strains. See Fig. 2. The fit is not perfect, which suggests that a better fit could be found. The data from the three shocks fits the constitutive relation remarkably well, which suggests that the results are fairly accurate and that a single dislocation process is in operation for most of the data.

DISLOCATION MODEL

In Fig. 2, t' is usually more than .5 kbar above the internal stress. This suggests that the dislocations are running freely in viscous glide. The situation resembles that found for diamond-lattice materials by Haasen and co-workers where initial dislocation densities were small, the over-stress large, and strong dislocation generation occurred. They found a behavior seen in Eq. (1): at constant strain rate, the over-stress, \tau = t' - t_o, fell with increasing plastic strain since lower dislocation velocities were needed when more dislocations were present.

Based on the above ideas, we will model \psi, \dot{\psi}, \tau, and the total dislocation density, N, by combining the Orowan Eq. (2), with an equation for dislocation generation, Eq. (3). Here, all dislocations are assumed mobile and contribute to \dot{\psi}:

\text{TBD}
Fig. 2. The squares are $t'$-data. The circles and triangles are $t$ and $F$ values. The $y$-axis scale applies to $F$, if taken to be unitless. The pluses show what $\psi$-values were used in the constitutive relation.

$$\dot{\psi} = b N V$$  
(2)

$$\Phi = \alpha N^6 \dot{\psi}$$  
(3)

We used the value $2.86 \times 10^{-8} \text{ cm}^{-1}$ for $b$, the Burgers vector length, and $\alpha$ is a parameter to be fitted. $V$ is the average dislocation velocity, which we take to be $D \dot{\psi}$, where $l$ and $D$ are parameters. The dot superscripts above denote differentiation with respect to time. Later we take $\alpha = 1$ as the most reasonable choice for freely moving dislocations. In this case, $D$ becomes equal to $b/B$, where $B$, the viscosity coefficient, was taken to be $2.66 \times 10^{-4}$ dyne-sec/cm$^2$, a pure Al value. See Fig. 6 of Gorman et. al.
We solved for $N$ from Eqs. 2 and 3 by eliminating the time differential between them and integrating on $N$ and $\psi$. The result, assuming the initial $N$ was zero, was:

$$ (2 - \delta)(a/b) \int_{-\delta}^{0} d\psi = N^{2-\delta}. \quad (4) $$

This result can be substituted into the Orowan Eqn. and with further algebra the following equations obtained:

$$ \tau = F_s [(2-\delta) \psi (2-\delta) \mu \psi]^{(2-\delta)/2}, \quad (5) $$

where $\mu = 1/[\epsilon(2-\delta) + \beta]$. Eq.(5) is of the same form as the constitutive Eq.(1) except for the factor $F_s$. Fitting the exponents of Eq.(5) with those of Eq.(1) results in the unique value $5/3$ for $\delta$ and the relation $\epsilon + 3 \beta = \epsilon$. Hence, no matter what the right velocity law is, i.e. the value of $\epsilon$, we learn that dislocation multiplication depends on the $5/\epsilon$ power of $N$. We note that the law found to work in Ge by Haasen and others was $N \tau V$. Our $N$-term differs, possibly because our source density might depend on the node density of the dislocation network, $N_{node}$, instead of the dislocation line length, $N$.

In order to assess the variation of the factor $F_s$, we calculated this quantity along the shock process using the $\tau/\psi$ data of Fig. 2. $\epsilon$ was set to 1. The results were not sensitive to varying $\epsilon$ by .5 or so. The Taylor factors drop out of the product $\tau \psi$. From the calculated values of $F_s$ can be seen that $F_s$ is usually around one for the points used to construct the constitutive relation, and otherwise varies from about 2 to .2 or so. This variation of about 10 over the shock process is much less than the variation of about 1000 of the ordinate of Fig. 1. Hence, $F_s$ is constant to a good approximation by comparison and the constitutive relation (1) holds to a good approximation except at the beginning and end of the shock. This means that $\psi$ is approximately a state variable for the loading paths encountered here. More generally, $N$ would be used instead.

As a check on our dislocation model, we should compare the dislocation densities at the end of the shock, calculated by Eq.(4), with those measured in the recovery experiments of Rhode et al., who found a density $N$, of about 1.5x10$^{16}$ after the passage of a 90 kbar shock, in 6061T6 Aluminum. Eq.(4) predicts densities of about 2.4x10$^{16}$, 6.1x10$^{16}$, and 3.8x10$^{16}$ for the 21,37, and 89kbar shocks, respectively. Considering that no parameters were adjusted, the present formulation gives tolerable densities, except for the 89kbar shock. We believe that the problem is due to using an unphysical model for the internal stress, i.e. the expression $1/2 \psi^{2/3}$ for $\tau$. We have done preliminary calculations using the model $A N^{2/3}$ for $\tau$, where $A = G b^2/(2\kappa)$; the $A$ successfully used by Haasen and others in modeling the diamond-lattice materials. It is based on the average stress between individual dislocations. Without adjusting any parameters, $N$ for the 89kbar shock was reduced at least an order of magnitude. Further work is underway.

In calculating $N$ from Eq. (4), we used a value for $\alpha$ obtained by equating $1.2 \times 10^{-16}/(f)^{1/2}$, where the $c$ is that of Eq.(1), with the collection of constants in Eq.(5). The square root of $f$=2.06, the Taylor factor, is necessary to account for polycrystalline effects.
We calculated dislocation velocities from \( V = (b/B)\), rather than from the Orowan equation because the dislocation densities were suspect. The highest velocity found was 3 times the shear velocity, which is reasonable.

In our model, temperature effects appear in the shear modulus scaling and in the temperature dependence of \( B \), which is known to be roughly linear near room temperature, the initial shock condition. In Eq.(5), \( D \) and thus its \( T \)-dependence, occur to the \( 1/4 \) power after \( \delta = 5/3 \) is used. This constitutes weak \( T \)-dependence. Since the temperature increases calculated in Ref. 4 were less than 32K for all points used in the constitutive relation (except the three end points of the 89kbar shock, where the increases were about 85K), we can only say that the dislocation model predicts only small temperature effects and those actually experienced were not really large enough to test them.

CONCLUSIONS

We have modeled shock loading conditions in which the driving stress exceeds the internal barrier or frictional stresses sufficiently to cause the dislocations to run freely and multiply rapidly. The internal stress or work hardening model used here is very special to the large overstress-small initial work hardening situation. For large overstresses and strong initial work-hardening, the present model should work if the \( \tau_0 \) model is generalized to include stored dislocations.

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