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YIELD OF THE HIROSHIMA BOMB

Classification changed to UNCLASSIFIED
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Per Jack C. B. Fischel, Acting Asst. Dir. 3-4-45

Work done by:
F. Reines
T. J. White

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Report written by:
F. Reines
By comparing condenser gauge records obtained at Hiroshima and Nagasaki, it is possible to estimate the yield of the Hiroshima gun. The result is that Hiroshima was $18-1/2 \pm 5$ kT based on an assumption of $23 \pm 2$ kT for Nagasaki.

The basis for the assumed yield for Nagasaki is as follows. The Bikini Able and Trinity models were the same. Ball of fire ratios give 23.8 kT for Trinity if the radio-chemical yield for Bikini Able, 22 kT, is taken as the reference figure (Sandstone Scientific Directors Report, Vol. 9). It is to be noticed that yield ratios from fireball measurements are considered much more reliable than the radio-chemical techniques as employed at Trinity. Hence, it seems reasonable to assume the Nagasaki yield to be $23 \pm 2$ kT.

The analysis presented here is based on the one usable record obtained from each of the Japanese drops. The comparison is made using the rather accurately known ($\pm 3\%$) durations of the positive phases rather than the poorly known peak overpressures. The essential feature of the analysis is the near duplication of height of gauge, distance to gauge, and height of burst for the two drops. Because of this circumstance, such poorly known theoretical factors as altitude effects should have little bearing on a comparison between these two cases.

The usual scaling law, in the absence of atmospheric effects, would require

$$\frac{t}{w^{1/3}} = f\left(\frac{R}{w^{1/3}}\right),$$

(1)
where $W$ is the yield of the bomb in kT, $\tau$ is the duration of the positive phase, and $R$ is the distance at which $\tau$ is measured. We require for our present purposes an approximation of $f$ over a reasonably small range of the variable $\frac{R}{W^{1/3}}$.

The rate of change of the duration of the positive phase with respect to $R$ is

$$\frac{d\tau}{dR} = \frac{1}{C} - \frac{1}{V},$$

where $C$ is sound velocity and $V$ the shock front velocity at $R$. For a given bomb we may then write

$$d\left(\frac{\tau}{W^{1/3}}\right) = (\frac{1}{C} - \frac{1}{V}) d\left(\frac{R}{W^{1/3}}\right).$$

The shock conditions give

$$\frac{C}{V} = \left(\frac{\gamma+1}{2}\right) \frac{P}{P_0} + 1\right)^{-1/2},$$

or approximately,

$$\frac{1}{V} = \frac{1}{C} \left(1 - \frac{\gamma+1}{4\gamma \frac{P}{P_0}}\right),$$

where $p$ is the excess pressure over $P_0$, the unshocked air pressure, and $\gamma = 1.4$.

At this large distance from the bomb a reasonable assumption for $p$ is

$$p = K\left(\frac{R}{W^{1/3}}\right)^{-1},$$

where $K$ is a constant over the range of $\frac{R}{W^{1/3}}$ considered.

Combining (3), (4) and (5),

$$d\left(\frac{\tau}{W^{1/3}}\right) = (\frac{\gamma+1}{4\gamma C \frac{P}{P_0}}) d\left(\frac{R}{W^{1/3}}\right).$$
where \( p_0, C \) may be taken as constant over the range of integration.

Then

\[
\frac{\tau}{w^{1/3}} - \frac{\tau_0}{w_0^{1/3}} = \frac{(\gamma+1)K}{4\gamma C p_0} \log \left( \frac{\frac{R}{w^{1/3}}}{\frac{R_0}{w_0^{1/3}}} \right),
\]

the constant of integration being split into two parts and so chosen that the function fits the following data for Nagasaki:

- \( C = 1000 \) ft/sec
- \( p_0 = 4.39 \) psi
- \( p = 0.15 \) (± 30%)
- \( R_0 = 36,000 \) ft (± 5%)
- \( W_0 = 23 \) kt (± 10%)
- \( \tau_0 = 1.25 \) sec (± 3%).

These figures and those used below for \( \tau \) and \( R \) for Hiroshima are taken from LAMS-428. Variations in \( C \) and \( p_0 \) are insignificant.

Since the Hiroshima gauge gave a peak overpressure of 0.089, it is assumed that \( p \) lies between 0.1 and 0.2 psi. The ± 3% taken for \( \tau_0 \) (and \( \tau \)) is based on an estimate of error caused by a drop of one minute at 15 ft/sec of a gauge with a time constant of 30 sec. The fact that the pressure-time curve went off scale at Nagasaki should not make less certain the duration of the positive phase since the break in the pressure-time record was sharp and when the record came back on scale it was smooth.
Agnew states the figure \( \pm 2\% \) given in IA-1023 for \( R_0 \) (and \( R \)) is in error and is disregarded here. In the method outlined in LAMS-377 for the computation of \( R_0 \) and \( R \), the RMS of the errors involved is 5\%.

The constant \( K \) in (5) is taken as the value which fits the Nagasaki data. Substituting \( \tau = 1.25 \) sec at 40,000 ft for Hiroshima in (7) yields \( \frac{W}{W_0} = 0.805 \), and, therefore, for 23 kT at Nagasaki, we obtain a yield of 18 1/2 kT for Hiroshima.

Letting \( \Delta W \) be percentage change, we find from (7) that

\[
\Delta W = \left[ \frac{K_1}{3} - \frac{\tau}{3W^{1/3}} \right]^{-1} \left\{ \frac{K_1}{3} - \frac{\tau}{3W^{1/3}} \right\} \Delta W_0 - \frac{\tau}{W^{1/3}} \Delta \tau + \\
+ \frac{\tau_0}{W_0^{1/3}} \Delta \tau_0 + K_1 \Delta R + \left[ \frac{\tau}{W^{1/3}} - \frac{\tau_0}{W_0^{1/3}} - K_1 \right] \Delta R_0
\]

\[
+ \left[ \frac{\tau}{W^{1/3}} - \frac{\tau_0}{W_0^{1/3}} \right] \Delta P \right\}
\]

where \( K_1 = \frac{(7+1)R_0}{4 \tau_0 W_0^{1/3}} \).

Therefore,

\[
\text{RMS } \Delta W = 10.45 \left\{ 9.15 \times 10^{-3} (\Delta W_0)^2 + 0.223 (\Delta \tau)^2 + 0.193 (\Delta \tau_0)^2 + 0.0344 (\Delta R)^2 + 0.0233 (\Delta R_0)^2 + 1.08 \times 10^{-3} (\Delta P)^2 \right\}^{1/2}.
\]
With the percentage changes allowed above,

\[ \text{RMS} \cdot \Delta W = 28\% \]

and finally for Hiroshima \( W = 18-1/2 \pm 5 \text{ kT} \).

It is interesting that a calculation of yields for Hiroshima and Nagasaki, based on the observed durations of the positive phases, and reported by Wieboldt in LAMS-428, also yields the ratio

\[ \frac{W}{W_0} = \frac{18}{23} \]

The absolute values given by Wieboldt are significantly higher than those given here, but since he attempts to give a yield using only theory starting with the calculation of the explosion and following the blast through an inhomogeneous atmosphere to the gauge, the ratio is expected to be more reliable.

Obviously it is desirable to fix the Hiroshima yield more closely. None the less, the number obtained here has the virtue of being based on a measurement which is not subject to the vagaries of blast damage as was Penney's estimate. A city is not the ideal blast gauge.

It is to be compared with the calculation of the Hiroshima yield of \( 15 \pm 5 \text{ kT} \) (T. Taylor, private communication). Further, for use in the correlation of damage with yield, it is seen that insofar as the peak overpressure is the damage criterion, the uncertainty in blast damage radius is given by

\[ \frac{\Delta R}{R} = \frac{\Delta W^{1/3}}{W^{1/3}} = \frac{1}{3} \frac{\Delta W}{W} = \pm \frac{1}{3} \frac{5}{10} \approx 0.1, \]

a figure compatible with the accuracy to which it is possible to assess blast damage.
The present estimate could be improved by a more accurate determination of a reference point than that provided by Nagasaki. This improvement would, of course, be limited by the errors in the Hiroshima data and so the error ± 5 kT could not be reduced by more than \( \frac{1}{\sqrt{2}} \) and would still be at least ± 3.5 kT.