(1) hot bubble rise, particle-in-cell calculations for 
(2) nuclear explosion, rise of hot gas bubble calculations

RISE THROUGH THE ATMOSPHERE OF A HOT BUBBLE
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ABSTRACT

The Particle-in-Cell method for multidimensional fluid-dynamics calculations has been used on the IBM Stretch Computer to study the motion of a hot gas bubble created by a nuclear explosion in the earth's atmosphere. In the example, a total energy of $8.24 \times 10^{20}$ ergs was deposited at an altitude of 95 km. After 2.85 sec the originally-heated air was moving upwards with a mass-averaged velocity of 3.5 km/sec and was still accelerating somewhat, probably to an asymptotic value of 4.0 km/sec. Along the cylindrical axis at 2.85 sec, the upwards velocities varied from 0.0 km/sec at the bottom to 5.8 km/sec at the top; most of the central material was moving at velocities greater than 4.0 km/sec. This report also includes details concerning bubble shape, energy histories, and shock configurations.
INTRODUCTION

The deposit of considerable energy into a spherical section of the earth's atmosphere would be followed by subsequent non-spherical air movements, with the initially heated region elongating vertically and rising. We have studied in detail a particular example and present the results because of several features which apply generally to this class of phenomena. The study was purely theoretical; the fluid-dynamics equations were solved numerically by a high-speed computer, and no direct comparisons with experiments are presented.

THE SPECIFIC PROBLEM

The model is a simplified one of the conditions which might be found shortly after a high-altitude nuclear explosion. The specific example was suggested by C. Longmire. At initial time, t = 0, the disturbed air was a heated sphere with center at 95 km above ground and radius of 2.7 km. It was assumed that no motion had occurred but that the sphere of air had been heated to a uniform specific internal energy of 800 (km/sec)$^2$ and had a radial velocity whose magnitude was given by

$$4.25 \left[ 1 + \frac{Z - Z_c}{6 \text{ km}} \right] \frac{\text{km}}{\text{sec}}$$
Here $Z$ is the height above ground, with $Z_c$ being the center height. The atmospheric density variation was taken to be

$$\rho = 1.2 \times 10^{-9} \exp \left( - \frac{Z - Z_c}{6 \text{ km}} \right) \frac{\text{gm}}{\text{cm}^3} \cdot$$

Finally, the equation of state of the air was taken as polytropic, with the specific heat ratio in the heated sphere being $\gamma = 1.6667$, while that outside was $\gamma = 1.3333$. These values were chosen so as to account for dissociation and ionization. The acceleration of gravity was considered negligible except insofar as it created the initial density distribution.

THE METHOD OF SOLUTION

Since the problem involves two space dimensions ($R$ and $Z$ in cylindrical coordinates) and results in considerable distortion of the air, we have used the Particle-in-Cell (PIC) method as currently written for the IBM Stretch Computer in a Group T-3 code called DUX. The basic technique has been described in detail in IA-2301* and the references mentioned there. DUX, however, uses several slight modifications, particularly of importance in the treatment of the energy equation. Details are available from the authors; thus we give only a brief description here.

The computing process employs basically an Eulerian viewpoint.

Thus, a cylindrical region of space about the originally-heated sphere was chosen to be large enough to contain the region of interest.

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*F. Harlow, IA-2301, April 1959.
throughout all times of significance. In this case, the cylindrical region extended from 12 km below the center to 38.4 km above, and had a radius of 19.2 km. A slice through the cylindrical region was then subdivided into a finite-difference mesh of cells whose sides were 0.6 km long. The air was represented by particles in the cells, whose motions gave the configuration changes. One-hundred per cell were used in the originally-heated air, while four per cell were outside.

A PIC-method calculation proceeds through a sequence of finite time steps in each of which the fields of velocity, energy, position, etc. are changed by small amounts. The results are observed through summary prints given by the computer and by configuration pictures given through the SC-4020 Microfilm Recorder. From the summary prints there can be obtained the time histories of such functionals as the totals of momentum, kinetic and internal energies, and volumes. Detailed profiles of the local densities of mass, energy, etc. can also be obtained from the computer whenever desired. From all of the available data, the conclusions of significance can then be drawn.

RESULTS OF THE CALCULATION

The configurations are shown in Fig. 1 a-g, in which the unretouched dot positions are shown for a sequence of times. In the last picture, for which the time is \( t = 2.85 \) sec, it can be seen that the shock pulse has broken away from the "piston" of initially-heated air, and is followed by a rarefaction. Vertical stratification of the air is caused partly by
shock-rarefaction interactions, but may also arise because of the initially-coarse resolution by the finite-difference mesh (whose cells are not shown).

Energy histories are shown in Fig. 2. Noteworthy is the nearly-constant behavior of external kinetic energy (Curve C). The vertical momentum of the initially-heated air is shown as a function of time in Fig. 3. Division of this by the total mass in the bubble \((1.02 \times 10^8 \text{ gm})\) gives the mass-average velocity which can be read from the same figure.

Finally, Fig. 4 shows a profile of the vertical velocity along the cylindrical axis at a time \(t = 2.85\) sec. The datum points are cell-wise values (which have the PIC-method fluctuations to be expected for this situation), while the line is an eye-drawn mean.

While this example refers to a specific initial configuration, the qualitative nature of the results would apply to a variety of similar problems. PIC-method calculations would give details in cases where quantitative details are required.
Figure 1a. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 0.0$ sec.
Figure 1b. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 0.43$ sec.
Figure 1c. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 0.87$ sec.
Figure 1d. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 1.31$ sec.
Figure 1e. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 1.75$ sec.
Figure 1f. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. $t = 2.41$ sec.
Figure 1g. Sequence of particle configurations showing the air motions about the initially-heated bubble. The squares are four kilometers on a side. \( t = 2.85 \) sec.
Figure 2. Energy histories for the bubble and surrounding air. A and B refer to the initially-heated air and are, respectively, the kinetic energy \( (x10^{-18}) \) and internal energy \( (x10^{-19}) \). C and D are, respectively, the kinetic energy \( (x10^{-19}) \) and internal energy \( (x10^{-19}) \) of the not-initially-heated air.
Figure 3. History of vertical momentum ($\times 10^{-7}$) and mass-averaged velocity of the initially-heated air.
Figure 4. Velocity profile along cylindrical axis for a time $t = 2.85$ sec. Dots show cell values while line is eye-drawn mean.