Fallout Model for the Robust Nuclear Earth Penetrator

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Modeling radioactive fallout from nuclear explosions requires a description of the radioactive cloud and base surge and an atmospheric transport model for the cloud dispersion. The atmospheric transport model is independent of the radioactive nature of the dust and I will stick to a simple model in this study.

The Radioactive Cloud Model

Radioactive Release

The first stage in understanding the fallout from a nuclear explosion is to estimate the amount of radioactivity released into the atmosphere. For external exposure to radiation, the main threat is from gamma-rays. The average gamma-ray activity[1] produced in a nuclear explosion has been calculated as 530 megacuries per kiloton of fission yield at one hour after the explosion, with an average photon energy of 0.7 MeV.

The data available on underground nuclear tests focuses on the fraction of the total activity found in "early" or "close-in" fallout (F_c), which measures only those particles that have been deposited in the first 24 hours[2]. The fraction of the total activity released into the atmosphere (f_{rel}) is greater than what appears in the early fallout ($f_{rel} > F_c$). The fraction F_c is dependant on the scaled depth of burst[3]. A summary of the activity release data available for U.S. and Soviet underground tests are shown in Tables 1, Table 2, and Figure 1.

Table 1: Activity Released from U.S. Underground Nuclear Tests.

Test	Yield (kt)	Depth of Burst (m)	Scaled Depth of Burst (m/kt$^{1/3}$)	Fraction of Total Activity in Early Fallout (F_c)
Jangle S[a]	1.2	0	0	0.50
Jangle U[a]	1.2	5.18	4.88	0.64
Teapot ESS[a]	1.2	20.4	19.2	0.46
Schooner[b]	30	111	35.8	0.48
Cabriolet[b]	2.3	51.8	39.3	0.028
Buggy[b,c]	1.08	41.1	40.1	0.038
Sedan[b,d]	100	194	41.7	0.18
Danny Boy[a]	0.43	33.5	44.4	0.04
Sulky[b]	0.088	27.4	61.6	0.001
Neptune[a]	0.115	30.5	62.7	0.005
Blanca[a]	19	255	95.4	0.0005

a) Release fraction from Knox-65, Table 1. b) Release fraction from Knox-69, Fig. 8. c) Multi-shot test, data shown is for a single test. d) The Sedan release fraction was approximated as 0.1 in Knox-65, but a more accurate value is listed in Knox-69, p.11.

Table 2: Activity Released from Soviet Underground Nuclear Tests[a]

Test	Yield (kt)	Depth of Burst (m)	Scaled Depth of Burst (m/kt$^{1/3}$)	Fraction of Total Activity in Early Fallout (F_c)
Chagan 1004	140	175	33.7	0.20

[1] Glasstone, §9.159.
[2] Glasstone, §2.28.
[3] The scaled depth can be found expressed as m/kt$^{1/3}$, m/kt$^{1/3.4}$, or m/kt$^{0.3}$ depending on the author. Izrael (p. 75) suggests that using a scaled depth expressed as m/kt$^{1/3}$ best corresponds to the experimental data for activity released into the atmosphere, so I will stick with this definition as much as possible.

Sary-Uzen 1003	1.1	48	46.5	0.035
Telkem-1 2308	0.24	31.4	50.5	0.002
Telkem-2[b] 2305-2307	0.24	31.4	50.5	0.003
Crystal	1.7	98	82.1	~0.01

a) Data obtained from Izrael, Table 4.1. b) Muti-shot test, data shown is for a single test.

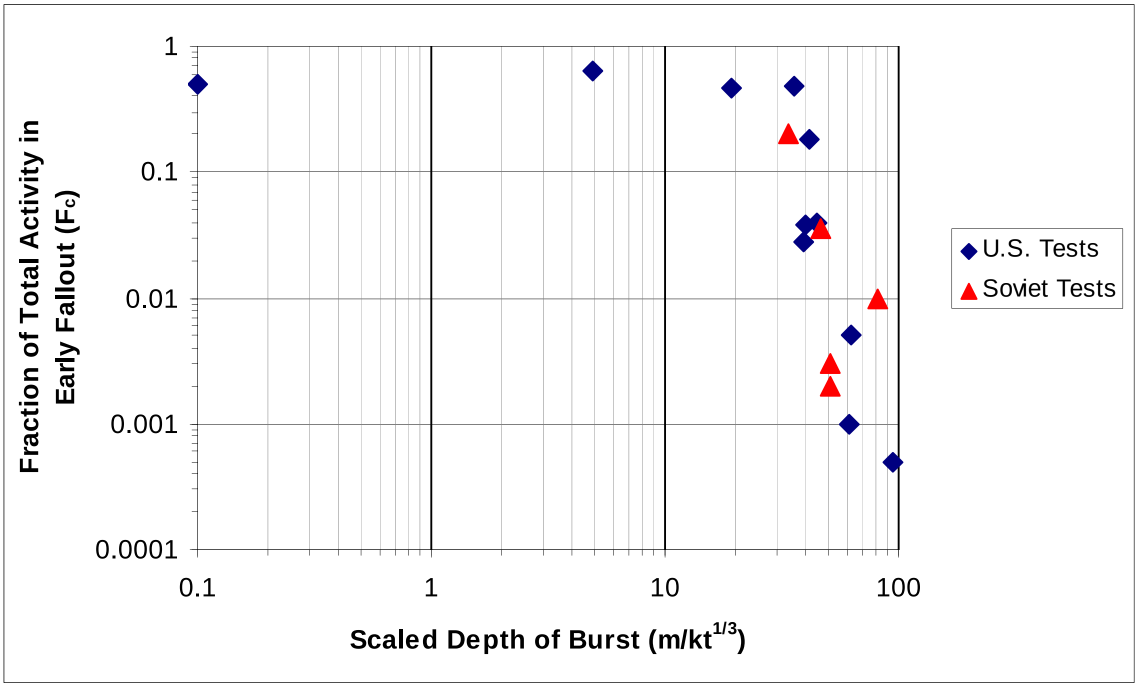

Figure 1: The fraction of the total activity that appears as early fallout (F_c) in underground nuclear explosions depends upon the scaled depth of burst.

About 50% of the activity produced in a surface burst is found in the early fallout[4], as indicated in Fig. 1. For scaled depths smaller than ~30 m/kt$^{1/3}$, the above data indicate there is no significant reduction in the activity released into the atmosphere. In fact[5], there is some indication that at the scaled depth where the base surge radius is greatest (about 9 m/kt$^{1/3}$) the fraction F_c is maximized at 75% of the total activity.

"A shallow subsurface burst, in which part of the fireball emerges from the ground, is essentially similar to a surface burst."[6] So for most purposes a scaled depth smaller than 1.5 m/kt$^{0.3}$ can be considered a surface burst[7], which is consistent with the data above.

The current conception of a RNEP has a yield of hundreds of kilotons or larger and could penetrate only a few meters underground. So a 320 kt weapon which is able to penetrate to a depth of 5 meters has a scaled depth of 0.73 m/kt$^{1/3}$, and a 1200 kt weapon at the same depth has a scaled depth of 0.47 m/kt$^{1/3}$. So for the purpose of determining the fallout of the RNEP, it is appropriate to treat the weapon as a surface burst. A

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[4] See Knox-65, p. 334. Also Glasstone (§9.59) states "For land surface bursts the early fallout fraction, which depends on the nature of the surface material, has been estimated to range from 40 to 70 percent. . . . The remainder will contribute to the delayed fallout, most of which undergoes substantial radioactive decay. . . ."
[5] Knox-65, p. 334.
[6] Glasstone, §9.51.
[7] Glasstone, footnote on p. 70.

conservative estimate is that 50% of the activity from the RNEP would be released into the atmosphere, which will underestimate the fallout since $f_{rel} > F_c$. A better estimate is described later in this paper.

The total activity released into the atmosphere (referenced to one hour after the explosion) is given by

$$A_{rel} = (530 \times 10^6 \text{ Ci/kt}) W f_{fis} f_{rel} . \qquad (1)$$

Assuming only 50% of the RNEP weapon yield comes from fission reactions ($f_{fis} = 0.5$) and $f_{rel} = 0.5$, a 1200 kt weapon will release 1.59 x 10^5 MCi into the atmosphere.

Cloud Formation

As mentioned above, the RNEP can be treated effectively as a surface burst. This means that it should form a familiar mushroom cloud, which consists of a stem and a cap. Data on cloud dimensions as a function of yield can be found in Glasstone, and is recreated here in Figures 2 and 3. According to Glasstone[8], these curves "may be taken to be representative of the average altitudes to which nuclear clouds from surface (or low air) bursts of various yields might be expected to rise in the mid-latitudes . . . "

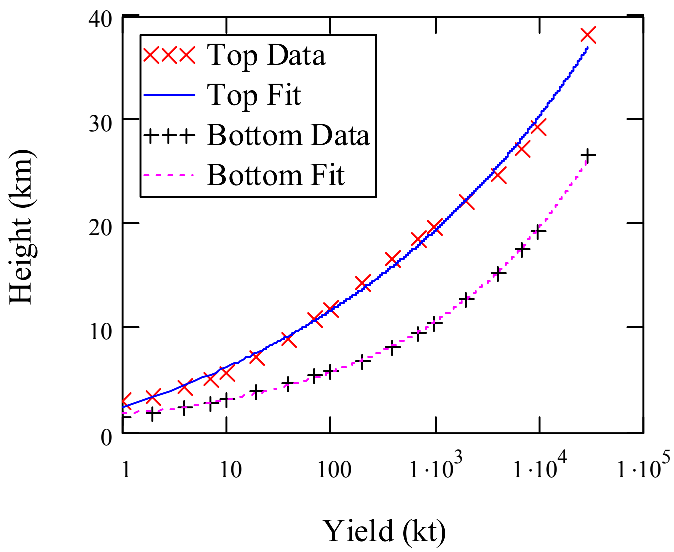

Figure 2: Height of the bottom and top of the stabilized mushroom cloud for a surface burst**.** Data are extracted from Glasstone Figure 9.96.

[8] Glasstone §9.96.

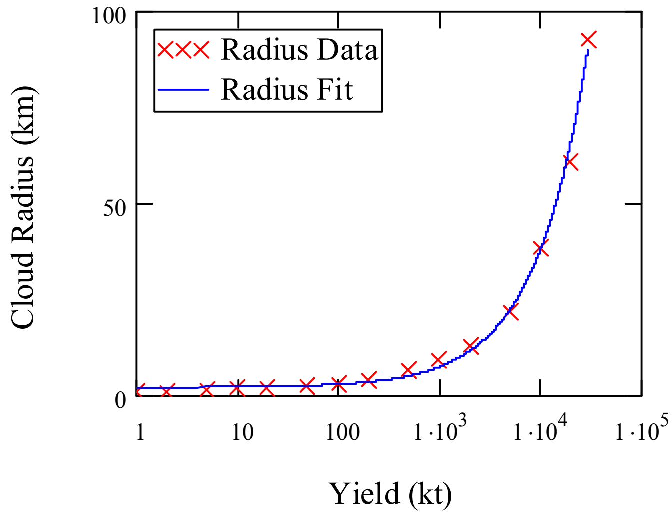

Figure 3: Radius of the stabilized mushroom cap for surface bursts. Data are extracted from Glasstone Figure 2.16.

In order to automate the fallout calculation process, the cloud dimensions were fit to a power law of the yield (W). This is not intended to be a theoretical based fit, but is merely a simple method to read the data off the graphs shown above.

Height of the cloud top (km): $h_{top} = 9.1W^{0.15} - 6.7$. (2a)

Height of the cloud bottom (km): $h_{bottom} = 1.6W^{0.27} + 0.13$. (2b)

Radius of cloud cap (km): $R_{cloud} = 0.017W^{0.83} + 2.0$. (2c)

Information on the stem is a little more uncertain. According to Glasstone (§2.17), for yields below 20 kt the stem radius is about half the cloud radius. As the yield increases, this ratio decreases so that for yields in the megaton range the stem radius is 0.1-0.2 times the cloud radius.

Applying these formulas for a 1200 kt burst produces a cloud with a radius of 8.1 km that lies between 11.1 km and 20.0 km above the earth. I will assume that the stem radius is one-tenth the cloud radius (0.81 km).

Activity Distribution versus Particle Size

The mushroom cloud is comprised of dust particles that have incorporated the radioactive fission products from the nuclear detonation. The typical density of dust particles is 2500 kg/m^3. The distribution of activity versus particle sizes takes on a log-normal distribution[9], where $F(r_1, r_2)$ is the fraction of the total activity that is on particles ranging in size from radius[10] r_1 to r_2 .

$$F(r_1, r_2) = \frac{1}{\sigma\sqrt{2\pi}} \int_{\log(r_1)}^{\log(r_2)} Exp\left(-\frac{(\log(r) - \log(r_0))^2}{2\sigma^2} \right) d(\log(r)) = \int_{\log(r_1)}^{\log(r_2)} N(r) d(\log(r)) \qquad (3)$$

The parameters σ and r_0 depend upon the nature of the soil, but not on the yield of the weapon[11]. For a surface burst in Nevada type soils it has been reported in Izrael[12] that r_0 = 56.5 microns and σ = 0.732.

[9] For general information on aerosol particle size statistics see Hinds Chapter 4. Application of the log-normal distribution for nuclear explosions is discussed in Glasstone (§9.165), Izrael (p 8,141), Knox-65, and Garcia (Chapter III, Appendix A).
[10] Strictly speaking, the particle radius is for an "equivalent" spherical particle, since the actual particles can be irregular in shape.
[11] Izrael p 8.

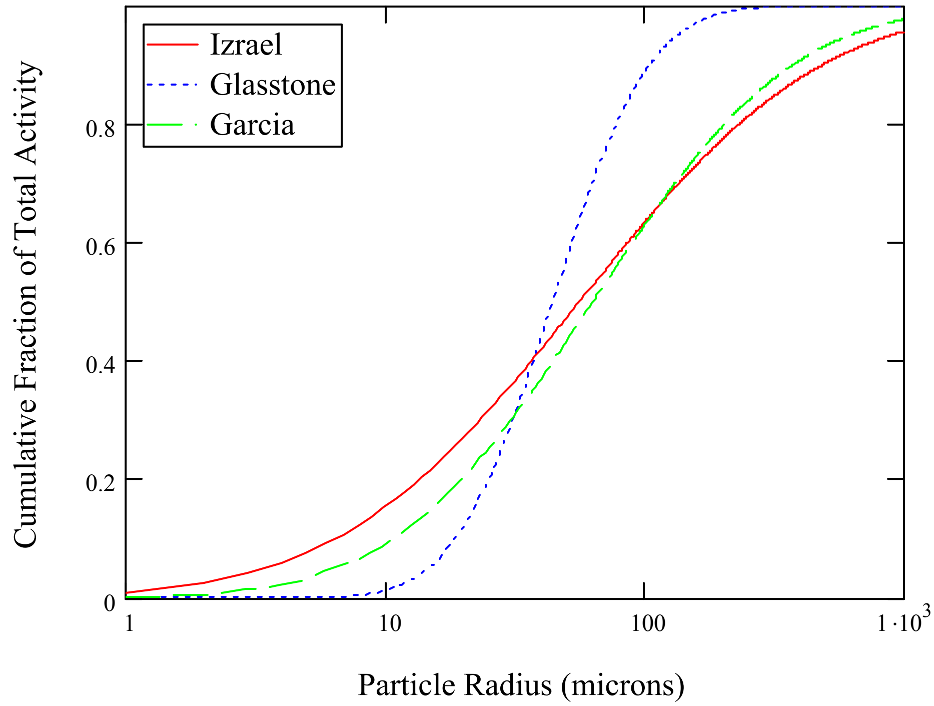

Figure 4: Activity-Particle distributions from surface nuclear explosions found in the literature. This study uses the distribution parameters mentioned in Izrael.

Glasstone[13] uses a distribution with r_0 = 44.6 microns and σ = 0.292, and Garcia[14] uses a distribution with r_0 = 63.9 microns and σ = 0.602. These three different distributions are compared in Figure 4. The parameters reported in Izrael will provide a more conservative estimate of the radiation dose received on the ground, and for this reason they will be used in this study. In general, a broad distribution will result in fewer particles reaching the ground within a given timeframe (24 hours for example) and an increased concentration of activity close to ground zero.

Distribution of Particles within the Stem and Cloud

It is also necessary to describe how the particles are distributed within the cloud and the stem. It is estimated that the fraction of released activity found in the main cloud[15] (f_{cloud}) is 0.9, so the fraction found in the stem (f_{stem}) is 0.1. I was unable to find a clear description of how the particles are distributed throughout the volume of the cloud (and stem), so it is necessary to create an idealized model. The simplest model is to have all the particles evenly distributed throughout the cloud (and stem) so the activity-density (a) is a constant. However, in order to make it easier to accommodate for cloud diffusion a Gaussian distribution of particles is used in the horizontal, where r is the distance from the vertical axis of the cloud.

$$a_{cloud} = \frac{A_{rel} f_{cloud}}{h_{top} - h_{bottom}} \cdot \frac{1}{2\pi \sigma_{cloud}^2} \exp\left(-\frac{r^2}{2\sigma_{cloud}^2}\right) \quad (4a)$$

$$a_{stem} = \frac{A_{rel} f_{stem}}{h_{bottom}} \cdot \frac{1}{2\pi \sigma_{stem}^2} \exp\left(-\frac{r^2}{2\sigma_{stem}^2}\right) \quad (4b)$$

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[12] Izrael p 8.
[13] These parameters are a fit to the information provided in Glasstone, Figure 9.164.
[14] These parameters are a fit to the data provided in Garcia, Appendix A
[15] Glasstone §9.61.

Assume that $R_{cloud} = 2\sigma_{cloud}$ and $R_{stem} = 2\sigma_{stem}$, then 86% of the activity lies within the visible cloud and stem. The above equations are referenced to one hour after the explosion.

Radioactive Decay

The radioactive contaminates will of course decay over time, which must be accounted for when determining the effects of fallout. For a single radioactive isotope the decay is exponential, but with more than 300 isotopes present in the fission products[16] it is simpler to approximate the net activity using a single decay model. The activity changes with time according to

$$A(t) = A(t_0)(t/t_0)^{-n}. \qquad (5)$$

Using a reference time (t_0) of one hour, the total activity at any given time can be computed by setting $A(t_0) = A_{rel}$. The exponent (n) varies somewhat[17], but a reasonable average is n=1.2. This decay model also accounts for the effects of fractionation[18], where the noble gases krypton and xenon escape from dust particles altering their radioactive content.

Atmospheric Transport Model

With a good description of the radioactive cloud now at hand, the next step is to describe how the cloud is transported through the atmosphere and the particles deposit themselves on the ground. Atmospheric transport and particle fallout does not depend upon the radioactive nature of the cloud. The model that follows is intended to be relatively simple to understand, but much more sophisticated models exist[19].

The transport model consists of an altitude dependant horizontal wind and a description of the horizontal diffusion. The fallout then occurs by gravitational settling using a standard atmospheric model.

Particle Fallout

The gravitational settling can be determined by calculating the altitude and particle size dependant terminal velocity (v_z). The model that describes the variation of the physical properties of the atmosphere with altitude is shown in Table 3. The temperature variation is approximated as a piece-wise linear model, and all other quantities are then derived from this using the ideal gas law[20].

Altitude Range (h)	Temperature (Kelvin)	Density of Air (kg/m^3)
0 – 11 km	218.15 K – (6.5 K/km) h	$1.225\left(1-\frac{6.5\ K/km}{288.15\ K}\cdot h\right)^{4.26}$

[16] Glasstone §1.62.
[17] Glasstone Figs. 9.16a and b, and §9.146 suggest this approximation is valid (within 25%) from 30 minutes to 200 days after the explosion. Izrael p. 13, also suggests that n=1.2 is a reasonable average.
[18] Glasstone §9.146.
[19] See, for example, the EPA's Support Center for Regulatory Air Models website www.epa.gov/scram001.
[20] The ideal gas law in this case is written as $P = (\rho_{air}/M_0)RT$, which relates the pressure (P), density (ρ_{air}), and temperature (T). The ideal gas constant R = 8.3142 Nm/mol K, and the mean molecular weight of air M_0 = 0.0289644 kg/mol. The dependence of the pressure and density on altitude (h) can be found by solving a single differential equation: $dP = -g\rho\, dh$. The gravitational constant g=9.80665 m/s^2. It is also assumed that the pressure at sea level is 101.325 kPa.

11 – 20 km	216.65 K	$0.364 \cdot \exp\left(-\dfrac{h - 11\,km}{6.34\,km}\right)$
20 – 32 km	216.65 K + (1 K/km)(h-20 km)	$0.088\left(1 + (1\,K/km) \cdot \left(\dfrac{h - 20\,km}{216.65\,km}\right)\right)^{-35.2}$

Table 3: The temperature and density of air equations from the U.S. Standard Atmosphere, 1976 (NOAA).

The viscosity of air (η) depends only upon the temperature, which varies with altitude. The viscosity is calculated using Sutherland's formula[21],

$$\eta = \left(1.458 \cdot 10^{-6} \ \tfrac{\text{kg}}{\text{m}\cdot\text{s}\cdot\text{K}^{1/2}}\right) \cdot \frac{T^{3/2}}{110.4 + T} \,. \qquad (6)$$

The primary forces governing the fall of the particle are the drag force ($F_d = \frac{1}{2} c_D \rho_{air} \pi r_{part}^2 v^2$), the buoyancy ($F_b = \frac{4}{3}\pi r_{part}^3 \rho_{air} g$), and the gravitational acceleration ($F_g = mg$). The particles are assumed to be spherical (with radius r_{part}), and the drag coefficient (c_D) is approximated[22] as

$$c_D = \frac{24}{\text{Re}}(1 + 0.15\,\text{Re}^{0.687}). \qquad (7)$$

The Reynolds number (Re) is a dimensionless number used to characterize how air flows around an object, and is defined as $\text{Re} = 2\rho_{air} v\, r_{part} / \eta$. Balancing the forces gives

$$m \cdot \frac{dv}{dt} = F_g - F_b - F_d = \tfrac{4}{3}\pi r_{part}^3 \left(\rho_{obj} - \rho_{air}\right) g - 6\pi \eta r_{part} \left(1 + 0.15\,\text{Re}^{0.687}\right) \text{v} \,. \qquad (8)$$

The radioactive dust particles are small enough that the time it takes for them to reach terminal velocity can be ignored. Since the density and viscosity of air vary with altitude, so does the terminal velocity. The terminal velocity (v_{term}) is occurs when $\frac{dv}{dt} = 0$, which must be calculated numerically from

$$v_{term}(1 + 0.15\,\text{Re}^{0.687}) = \tfrac{2}{9}(\rho_{obj} - \rho_{air}) g\, r_{part}^2 / \eta \,. \qquad (9)$$

It is worth noting that the Hazard Prediction and Assessment Capability (HPAC) program, the Defense Threat Reduction Agency's program for simulating nuclear weapon fallout, uses a slightly different method for calculating the terminal velocity. HPAC uses the program SCIPUFF for modeling the behavior of the radioactive cloud in the atmosphere. SCIPUFF[23] assumes that the viscosity of air is constant (η=1.6x10^{-6} kg/m s) and the altitude dependence of the terminal velocity is entirely determined by the density of air. This assumption makes a significant difference[24] in the calculation of the terminal velocity as shown in Figure 5.

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[21] NOAA, Equation 51.

[22] Hinds, p.44. This approximation agrees to within 4% of the experimental value for Re < 800, and is within 7% for Re < 1000.

[23] R.I. Sykes, *et. al*, "PC-SCIPUFF Version 1.2PD: Technical Documentation", Titan Research & Technology Division, Titan Corp., (ARAP Report No. 718, Sept. 1998), §4.2.1. This manual can be found at www.titan.com/products-services/336/index.html.

[24] SCIPPUFF uses a drag coefficient $c_D = \frac{24}{\text{Re}}(1 + 0.158\,\text{Re}^{2/3})$, but this is not significantly different than what is used in this study.

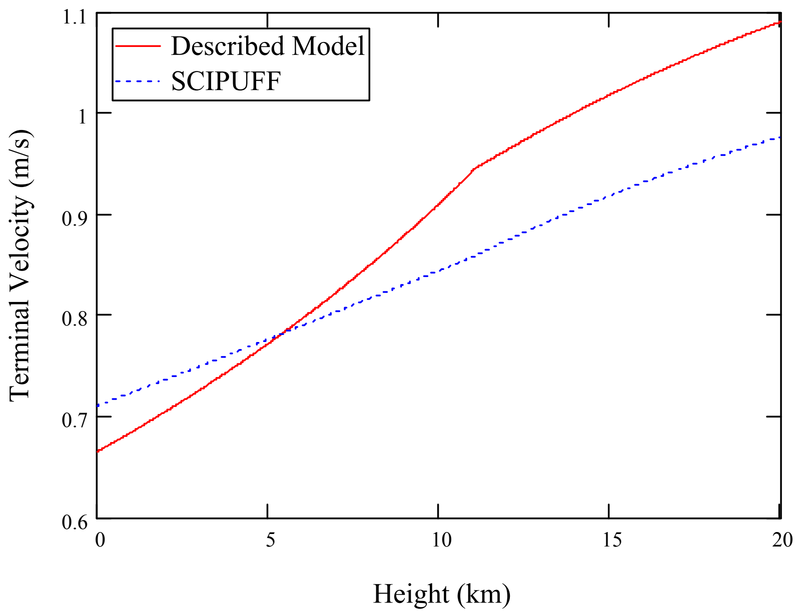

Figure 5: A comparison of the terminal velocity used in this model with that used by SCIPUFF at the mean particle radius (r_0 = 56.5 microns). This could cause a significant difference in fallout patterns if high winds exist at either high or low altitudes.

This model assumes that there is no vertical turbulence. So the time it takes a particle fall to the ground (t_f) is

$$t_f(h_0, r_{part}) = \int_0^{h_0} \frac{dh}{v_{term}(h, r_{part})}, \qquad (10)$$

where the functional dependence on the particle size (r_{part}) and initial height (h_0) is shown.

Horizontal Transport

The actual radioactive fallout pattern depends upon the local wind conditions at the time of the explosion. For this study, the wind velocity $v_x(h)$ is only in one direction and depends only on the altitude (h). The downwind distance (D_x) that a particle of size r_{part} travels is then

$$D_x(h_0, r_{part}) = \int_0^{h_0} \frac{v_x(h)}{v_z(h, r_{part})} dh. \qquad (11)$$

Improving the Estimated Released Activity

Earlier it was estimated that the activity released into the atmosphere is equal to the activity accounted for in the early fallout. This of course ignores those particles which remain aloft after 24 hours. Applying the fallout model to the Jangle S explosion should provide a better estimate of the released activity. Let $r_{\min}(h)$ be the minimum particle size that will fall within 24 hours from a height h. The fraction of activity remaining in the cloud (δ_{cloud}) and stem (δ_{stem}) after 24 hours (ignoring decay) is

$$\delta_{cloud} = \frac{1}{h_{top} - h_{bottom}} \int_{h_{bottom}}^{h_{top}} F(-\infty, r_{\min}(h))\, dh \qquad (12a)$$

$$\delta_{stem} = \frac{1}{h_{bottom}} \int_{0}^{h_{bottom}} F(-\infty, r_{\min}(h))\, dh \qquad (12b)$$

For a 1.2 kt nuclear explosion, the mushroom cloud extends from 1.8 km to 2.6 km above the ground. It then follows that after 24 hours 14% of the activity released into the cloud remains airborne ($\delta_{cloud} = 0.14$) and 8.3% of the activity released into the stem remains airborne ($\delta_{stem} = 0.083$). As mentioned above, it is estimated that 90% of the released activity is found in the cloud and 10% is within the stem. Thus after 24 hours approximately 13.5% of the released activity remains airborne.

For the Jangle S explosion 50% of the total activity was found in the early fallout. Based on the above calculation, 58% of the total activity was released into the atmosphere ($f_{rel} = 0.58$)[25].

Horizontal Diffusion

As the radioactive cloud moves through the atmosphere, it will slowly expand in size. The result is that the concentration of particles per unit area decreases as the fall out time (t_f) increases. This phenomenon is incorporated into the model through the standard deviation (σ). The dispersion of dust particles is dominated by eddy diffusion in the atmosphere, which is characterized by the diffusion coefficient (K) which is taken to be approximately 10 m^2/s. This suggests[26] that the initial standard deviation ($\sigma_0 = \sigma_{cloud}$ or $\sigma_0 = \sigma_{stem}$) in Equations 4a and 4b should be replaced by a time dependant standard deviation $\sigma(t)$.

$$\sigma(t) = \begin{cases} \sqrt{\sigma_0^2 + 2K\,t} & t < t_f \\ \\ \sqrt{\sigma_0^2 + 2K\,t_f} & t > t_f \end{cases} \qquad (13)$$

Where a horizontal slice stops expanding when it hits the ground at time t_f .

Calculating the Radioactive Dose

Consider the cloud as being composed entirely of particles with radius r_{part} , which represents a fraction $N(r_{part})\, d(\log(r_{part}))$ of the total activity in the cloud. The cloud is then divided into many horizontal slices of thickness *dh*. As a slice falls to the ground it broadens but otherwise remains intact. The activity concentration on the ground (i.e. Ci/km^2) at time t associated with a single slice is

$$\alpha_{cloud}(h_0, r_{part}, \vec{r}) \left(\frac{t}{t_0} \right)^{-1.2} \Theta\left[t - t_f\right] dh\; d(\log(r_{part}))\,, \qquad (14a)$$

where

$$\alpha_{cloud}(h_0, r_{part}, \vec{r}) = \left(\frac{A_{rel} f_{cloud}}{h_{top} - h_{bottom}} \right) \frac{1}{2\pi\sigma^2(t_f)} e^{-r^2/2\sigma^2(t_f)} \cdot N(r_{part})\,. \qquad (14b)$$

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[25] $f_{rel} = \dfrac{F_c}{1 - (f_{cloud}\delta_{cloud} + f_{stem}\delta_{stem})} = \dfrac{0.50}{1 - (0.9 \times 0.14 + 0.1 \times 0.083)} = 0.58$

[26] Also see Knox-65, p 341 where he uses a similar formulation for disks of uniform distribution.

The dependence upon the initial height (h_0) comes from the fall time $t_f(h_0,r_{part})$. The Heavyside step function[27], $\Theta[t-t_f]$, insures that the activity concentration is zero before the slice reaches the ground. A similar function (α_{stem}) can be written for the stem.

Estimating that the average photon energy is 0.7 MeV at one hour after the explosion, then the dose rate at one meter above the ground[28] due to a uniform concentration of 10^6 Ci/km^2 is 2.05 rads/hr. This corresponds to a conversion factor (γ) of $2.05\text{x}10^{-6}$ (rads/hr)/(Ci/km^2).

The health risk for someone exposed to radioactive fallout depends upon the total dose $Q(\bar{x},T)$ they receive. Here it is assumed that the person is located at point $\bar{x}=(x,y)$ on the ground and is completely exposed to the fallout radiation from the time of the explosion up until a time T after the explosion.

$$Q(\bar{x},T)=\gamma\int d(\log(r_{part}))\left[\left(\int_{h_{bottom}}^{h_{top}} dh\ \alpha_{cloud}(h,r_{part},\bar{r})+\int_0^{h_{bottom}} dh\ \alpha_{stem}(h,r_{part},\bar{r})\right)\cdot I_T(h,r_{part})\right] \qquad (15)$$

$$I_T(h,r_{part})=\int_0^T dt\left(\frac{t}{t_0}\right)^{-1.2}\Theta[t-t_f(h,r_{part})] \qquad (16)$$

In the integrand, the radial vector $\bar{r}=\bar{x}-D_x(h,r_{part})\hat{x}$. The time integral I_T is zero if time T is less than the fall time ($t_f(h,r_{part})$), otherwise

$$I_{T>t_f}(h,r_{part})=\int_0^T dt\left(\frac{t}{t_0}\right)^{-1.2}\Theta[t-t_f(h,r_{part})]=5t_0\left(\left(\frac{t_0}{t_f(h,r_{part})}\right)^{0.2}-\left(\frac{t_0}{T}\right)^{0.2}\right). \qquad (17)$$

The remaining integration in Equation 15, over the height (h) and the particle size ($\log(r_{part})$), must be performed numerically.

It is also useful to know the dose rate q (i.e. rads/hr) at a particular point on the ground. Here it is assumed that the radioactive particles have stopped falling in the area in order to simplify the calculation.

$$q(\bar{x},t)\approx\gamma\left(\frac{t}{t_0}\right)^{-1.2}\int d(\log(r_{part}))\left[\int_{h_{bottom}}^{h_{top}}\alpha_{cloud}(h,r_{part},\bar{r})dh+\int_0^{h_{bottom}}\alpha_{stem}(h,r_{part},\bar{r})dh\right] \qquad (18)$$

This is written as an approximation because, from a mathematical perspective, a horizontal Gaussian particle distribution combined with a log-normal activity distribution will always have some particles falling in a given area. The unit-time reference dose rate (q_1) used by Glasstone, is then defined using

$$q(\bar{x},t)=q_1(\bar{x})\left(\frac{t}{t_0}\right)^{-1.2}. \qquad (19)$$

So $q_1(\bar{x})$ is the dose rate at point $\bar{x}$ on the ground, referenced to time t_0. This can be used to compare the model developed in this paper with the model found in Glasstone.

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[27] In general, $\Theta[x]=\begin{cases}1 & x>0\\ 0 & x<0\end{cases}$.

[28] The conversion factor for various heights and average photon energies can be computed using Glasstone, Fig 9.155.

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