JASON

Technical Report
JSR-78-09

November 1978

JASON 1978 SONIC BOOM REPORT

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### REPORT DOCUMENTATION PAGE

**Title:** Jason 1978 Sonic Boom Report

**Performing Organization Name and Address:**
SRI International
1611 North Kent Street
Arlington, VA 22209

**Controlling Office Name and Address:**
Advanced Research Projects Agency
1400 Wilson Boulevard
Arlington, VA 22209

**Monitoring Agency Name and Address:**

**Distribution Statement:** Cleared for open publication; distribution unlimited.

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**Keywords:** Sonic Booms, SST, Thermosphere

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**Abstract:**
Press reports of "East Coast Mystery Booms" has led to a number of studies of the propagation of shock waves into the thermosphere. This report shows that SST (Concorde) generated booms (a) do not heat the thermosphere, (b) do not create thermospheric wind, and (c) do not cause chemical changes in the thermosphere.
EXECUTIVE SUMMARY

Press reports of "East Coast Mystery Booms" have led to a number of studies of the propagation of shock waves into the thermosphere. The energy, $E$, in an N-wave is proportional to the square of the relative overpressure, $s$,

$$E = L p_s^2$$

where $L$ is the length of the N-wave, $p$ the ambient pressure and $\Delta p$ the shock overpressure. The fractional energy loss due to the entropy jump across the shock traveling in the $x$ direction is proportional to the relative overpressure at every point along the travel path

$$\frac{1}{E} \frac{dE}{dx} = -\frac{s}{2\gamma L}$$

where $\gamma$ is the ratio of the heat capacities. Solving for the relative overpressure at high altitudes, we obtain a limiting strength of the shock of

$$\frac{3L}{H}$$

where $H$ is the scale height. Even though the N-wave lengthens as the shock propagates upwards, the increase in scale height insures that shock remains weak at all heights. For a typical Concorde Mach 2 boom having
an initial strength of 100 Pa, the turnover height of the boom is 160 km.
In reaching this altitude, 90% of the energy is lost by 100 km and 97% of
the energy is lost by 160 km. As a result of the shock remaining weak,
the change of ambient temperature is only about $10^{-2} K$ and the velocity
impacted to the thermosphere is about 3 cm/s. The boom is also lengthened
to about 4 km, so that the peak frequency at ground levels is about 0.1 Hz.

SST generated booms thus

1) do not heat the thermosphere;
2) do not create thermospheric wind;
3) do not cause chemical changes in the thermosphere.

General considerations of the propagation of infrasound suggest a
number of research priorities including:

1) construction and operation of directional infrasound
arrays to monitor natural and artificial infrasound
sources;
2) measurement of the source spectrum of a wide variety
of artificial sources;
3) coupled infrasonic and ionospheric observation to
study the interaction of near surface generated
disturbances with the ionosphere;
4) construction and operation of a directional array of
infrasonic sources to study propagation conditions
in the thermosphere.
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I OVERVIEW

Public attention was drawn to unexplained atmospheric sonic phenomena by a series of press reports of "Mysterious East Coast Booms" from early December 1977 through March 1978\(^1\),\(^2\). Numerous citizen reports of intense sounds in New Jersey, in the vicinity of Charleston, South Carolina and on the southwest tip of Nova Scotia prompted a number of independent investigations\(^3\),\(^4\),\(^5\). In addition to the citizen reports, certain of the events were recorded by acoustic arrays of the Lamont-Doherty Observatory\(^6\) and by seismic arrays of Lamont-Doherty Observatory, Western Observatory and Baptist College in Charleston where acoustic events are distinguished by vertical motion with no horizontal displacement. Citizens also noted lights in the sky, sometimes associated with the noises at times when there was no thunderstorm activity. Sound events have also been reported in other areas of the east coast such as the Tidewater region of Virginia, but almost all reports have come from Nova Scotia, New Jersey and South Carolina. Press reports of events in New Jersey and South Carolina ceased in late March, but citizen reports of booms in Nova Scotia continued through August 1978.

Public concern led Senator Harrison Williams (N.J.) to request the White House to conduct an investigation. Frank Press, Director of the Office of Science and Technology Policy, referred the matter to the
Department of Defense which in turn designated the Naval Research Laboratory (NRL) as the lead agency. The resulting NRL study concluded that a majority of the New Jersey and South Carolina booms could be explained by the supersonic flight of military aircraft in the "warning areas" off the east coast. The NRL study noted that unusual temperature and wind conditions could probably account for the amplification and propagation of the sonic carpet boom over the long distances required if the military aircraft were responsible. The NRL study also briefly noted that the booms in Nova Scotia could be attributed to Concorde flights.

In its study, NRL examined many alternative explanations for the booms, including: nuclear explosions, military research and development activities, military or civilian use of high explosives, ship disasters, USSR ship operations, geophysical exploration, antipodal events, missile launches and re-entries, low altitude satellites, explosive freezing of supercooled water vapor, meteorites, winter lightning, direct seismic generation and the explosion of biogenic or abiogenic methane. NRL concluded that all man-made sources could be excluded with the exception of military aircraft off Nova Scotia. NRL considered that natural sources were highly improbable; a conclusion that has been disputed by Chalton and MacDonald in a comprehensive analysis of about 600 separate events.

Noting that the reports of the east coast booms approximately coincided with the initiation of Concorde service into New York's J.F. Kennedy Airport in late November 1977, Jeremy Stone of the Federation of American Scientists suggested that Concorde was responsible for the New Jersey and
Charleston events as well as for the Nova Scotia booms. If Concorde were the source of the South Carolina booms, then the shock would have to propagate 1200 km, the distance from the last supersonic segment of the flight. Liszka has observed at Kiruna, Sweden, Concorde flights at distances of 3000 to 4500 km with a rms amplitude of about 0.1 Pa in the frequency band around 2 Hz. Liszka interprets the observed signal as a focused boom in which the signals from a finite segment of the flight path arrive simultaneously at a particular point on the earth's surface.

Stimulated by Stone's speculation, Garwin carried out a preliminary analysis of the propagation of a linear, lossless wave into the tenuous and hot thermosphere. A Concorde sonic boom has a leading shock with an overpressure \( \Delta p \) of about 100 Pa near ground level; that is, the strength (relative overpressure) which we designate by \( s = \Delta p/p \), is about \( 10^{-3} \). With Garwin's approximations, such a shock becomes very strong at 180 km altitude, since:

\[
\frac{s_{180}}{s_g} = \left( \frac{P_g}{P_{180}} \right) = 7 \times 10^3
\]

where \( p \) is the ambient pressure and where the energy \( E \) in the shock is proportional to \((\Delta p/p)^2\). The corresponding temperature rise inside the shock would be \( 10^4 \, \text{°K} \) with a resulting kinetic temperature corresponding to a particle velocity well above the escape velocity. Garwin tentatively concluded that the Concorde generated shocks could significantly perturb the thermosphere by depositing energy and momentum leading to thermospheric escape and high thermospheric winds. Further, the shock
would propagate through the thermosphere at an average velocity slightly less than the velocity of the aircraft at the point from which the shock was launched. Thus the acoustic disturbance could travel through the atmosphere at mean velocities well above the velocity of sound at sea level. The booms refracted by the thermosphere, called hyperbooms by Garwin, could travel horizontally for distances of hundreds of thousands of kilometers and through focusing might produce the observed east coast booms.

Gardner and Rogers have carried out a detailed analysis of the propagation of a weak shock in the atmosphere taking explicit account of the non-linear character of even a weak shock, the density and thermal structure of the atmosphere and attenuation due to linear dissipation. Gardner and Rogers find that the non-linearities of a weak shock lead to a substantial energy loss as the shock propagates upward in the thermally stratified atmosphere. They conclude that:

1. \( \Delta p/p \) never exceeds 0.2 in the thermosphere so that the shock remains weak throughout its path. A weak shock could neither significantly perturb the temperature or wind field of the thermosphere.

2. A Concorde boom generated at 17.5 km height with the airplane traveling at Mach 2 will reach the ground at a horizontal distance of 320 km from the airplane.

3. The sound received at this distance will have a small overpressure of less than 0.5 Pa and the energy will peak at about 0.2 Hz.
The analysis presented in this report carried out independently of the NRL work uses a much simpler model of sonic boom propagation to arrive at conclusions similar to those of Gardner and Rogers. In this analysis the fractional energy loss due to the entropy jump for a shock propagating in the $x$ direction is

$$\frac{1}{E} \frac{dE}{dx} = \frac{(\Delta p/p)}{2 \gamma L}$$

where $L$ is the sonic boom length and $\gamma$ the ratio of the heat capacities. The fractional energy loss is thus proportional to the overpressure at every point along the travel path. The relative overpressure $\Delta p/p$ stays well below the limit of $2\gamma L/H_s$ at all altitudes, where $H_s$ is the scale height of the atmosphere. This limit stays well below 0.5 at all altitudes (even though the sonic boom lengthens as it travels to high altitudes) because the scale height of the atmosphere also increases at high altitudes. Thus, the large scale height in the thermosphere insures that the shock will remain small.

The results obtained here and by Gardner and Rogers are consistent with Grover's$^{10}$ observations of Concorde generated shocks. Using an infrasonic array, Grover measured at a horizontal distance of 300 km from the Concorde flight path a pressure pulse of 0.4 to 0.8 Pa in the frequency range of 0.3 to 0.5 Hz.
Liszka's observations of pressure amplitude of 0.1 Pa at a distance of 4500 km and at a frequency of about 2 Hz can also be interpreted in terms of the calculations presented here. After the first bounce off the thermosphere, the ground pressure (Δp/p) at a horizontal distance of 300 km is about $10^{-5}$. The ground-reflected shock strength is smaller by a factor of 100 compared to the initial shock having a pressure amplitude of 100 Pa. With the small over-pressure, non-linear weakening will be small and the infrasound wave will propagate a distance $R$ with only geometrical reduction in strength proportional to $R^{-\frac{3}{2}}$, provided the attenuation is negligible. For infrasonic waves attenuation at heights below 100 km is less than about 0.001 db/km at 1 Hz. For distances of 4500 km, the geometrical spreading reduces the pressure amplitude by a factor of four and focusing can be called on to overcome dissipation losses above 100 km. Thus a measured rms amplitude of 0.1 Pa at 4500 km distance is consistent with an amplitude of 0.4 to 0.8 Pa at 300 km even though the longer propagation involves some 14 bounces off the thermosphere.
II PROPAGATION OF SONIC BOOMS IN A STRATIFIED ATMOSPHERE

We show here that upward-going sonic booms will not significantly affect thermospheric temperatures or winds, and that, although these upward-going booms may be refracted downward by the thermosphere, they lose so much energy during their upward travel that they cannot result in large sonic booms on the ground.

For simplicity of illustration we consider vertically traveling planar sonic booms. We thus avoid the necessity for considering either geometrical spreading or the change in direction due to refraction that a non-vertical wavefront would experience. We can draw all significant conclusions from this simple case. Our basic problem is to solve for the sonic boom strength as a function of altitude. We know that rather close to the aircraft altitude, the strength (relative overpressure) is about $10^{-3}$. If the boom reaches a strength of unity, a number of important effects would occur; but if our vertically traveling sonic boom always has a strength well below unity as it travels into the rarefied upper atmosphere, then no sonic boom at any angle will ever become strong (ignoring cusps and caustics), and we can easily calculate sonic boom effects.
A. Sonic Boom Characteristics

A supersonic aircraft creates a complicated shock-wave structure that propagates away from the aircraft’s flight path. At moderate distances from the aircraft, the structure is reasonably well approximated by a symmetric N-wave as shown in Fig. 1.

![Diagram of pressure waveform](attachment:image.jpg)

**Figure 1** PROPAGATION OF AN IDEAL N-WAVE

The true sonic boom will differ from this idealized form, but the differences will not affect our results (see Appendix A).

Once we have established the pressure waveform, we may calculate various useful quantities from elementary shock-wave theory (see Appendix A):
\( h \) = altitude (distance along direction of sonic boom travel)
\( u \) = speed of first shock discontinuity
\( c \) = speed of sound = \((\gamma R_m T_1)\)^{\frac{1}{2}}
\( T_a \) = ambient temperature before sonic boom passage
\( p_a \) = ambient pressure before and after sonic boom passage
\( \Delta p \) = overpressure just behind the first shock discontinuity
\( s \) = relative overpressure \(\Delta p/p_a\) (called the sonic boom strength)
\( \gamma \) = ratio of specific heats for the atmosphere as an ideal gas
\( \Delta T \) = increase in temperature just behind the first shock discontinuity
\( \Delta T_a \) = increase in ambient temperature after the passage of the entire sonic boom
\( E \) = total energy (per unit area) in the sonic boom wave front.

Within the boom the temperature rise is first order in the strength

\[
\frac{\Delta T}{T_a} \approx \frac{\gamma - 1}{\gamma} s
\]

while the heating of the air behind the boom is third order:

\[
\frac{\Delta T_a}{T_a} \approx \frac{\gamma^2 - 1}{12 \gamma^3} s^3
\]

The energy in the sonic boom is given by

\[
E = \frac{\gamma + 1}{6 \gamma^2} L p_a s^2 .
\]
As the sonic boom travels, energy is lost in heating the atmosphere. The energy loss relation is

\[ \frac{1}{E} \frac{dE}{dh} = \frac{s}{2\gamma L} \quad (4) \]

or twice as large if the ideal N wave shape is maintained (see App. A).

The air behind the sonic boom will not only be heated (by \( \Delta T_a \)) but will also be given momentum. This momentum transfer will result in atmospheric winds, with speeds given by

\[ v = \frac{c}{\gamma(\gamma - 1)} \frac{\Delta T_a}{T_a} \quad (5) \]

The first shock travels slightly faster than an acoustic wave:

\[ u \equiv c \left( 1 + \frac{\gamma + 1}{4\gamma} s \right) \quad (6) \]

resulting in a lengthening of the boom while it travels

\[ \frac{dL}{dh} = \frac{u-c}{c} = \frac{\gamma + 1}{4\gamma} s \quad (7) \]

The above simple relations, plus a model for the atmosphere, are all that is needed to establish the results of this section.
B. No Lengthening; Isothermal Atmosphere

A simple example will illustrate the physics of the situation. Let us ignore the changes in $L$ with distance, taking $L$ as a constant $L_0$. Furthermore, let us consider an isothermal atmosphere. The sound speed $c_o$ is a constant in such an atmosphere. We need the pressure as a function of altitude in order to make use of (3). We obtain this through the sound-speed relation

$$c^2 = \gamma p_o / \rho$$

(8)

and the hydrostatic relation

$$\frac{\partial p}{\partial h} = -c g$$

(9)

resulting in the solution

$$p_a = p_o e^{-h/H_s} \quad H_s = \frac{\gamma g}{c_o^2}$$

(10)

That is, the well-known exponential atmosphere. Let $p_o$, $s_o$ be the ambient pressure and sonic-boom strength at the aircraft altitude. Then (3), (4), and (10) yield a differential equation for the energy $E$ as a function of altitude:

$$\frac{1}{E} \frac{dE}{dh} = -\frac{1}{2\gamma L_o} s_o e^{h/2H_s} \left( \frac{E}{E_o} \right)^2$$

(11)
The solution to this equation can be easily found, and yields immediately the energy and strength as a function of altitude:

\[ E = E_0 \left[ 1 + \frac{H_s}{2\gamma L_o} s_0 \left( \frac{h}{2H_s} \right) \right]^{-2} \]  \hspace{1cm} (12)

\[ s = \frac{s_0 e^{h/2H_s}}{1 + \frac{H_s}{2\gamma L_o} s_0 \left[ e^{h/2H_s} - 1 \right]} \]  \hspace{1cm} (13)

The solution for the strength shows a strong saturation phenomenon, since at high altitude, regardless of the initial strength at aircraft altitude, we have

\[ s \rightarrow \frac{2\gamma L_o}{H_s} \Rightarrow s_{\text{max}} \]  \hspace{1cm} (14)

Typical values for \( L_o \) (~300 meters) and \( H_s \) (~10 km) will yield a saturation value \( s_{\text{max}} \sim 0.1 \).

We could at this point, easily calculate all the atmospheric effects of interest within this simple model. We would find that a strength of \(<0.1\) never leads to significant heating or winds in the upper atmosphere Eqs. (1), (2) and (5), and that a boom with \( s_o \sim 10^{-3} \) loses more than 95% of its energy by the time it has reached a height of 140 km Eq. (12).
C. Lengthening

However, the lengthening of the boom Eq. (7) is not negligible. If we allow the boom to lengthen in our calculations, then as the altitude increases the saturation value of the strength also increases. As a result the boom strength may continually increase; with typical initial parameters we find that the boom would become a strong shock (s ~ 1) at 170 km altitude.

The increase to a strong shock occurs because of the rapid decrease in ambient pressure in the assumed isothermal atmosphere. We find that if a more accurate model of the atmosphere that includes the thermosphere is used, the boom does not become strong.

D. Atmospheric Model

The temperature in the atmosphere (Fig. 2a) begins to increase rather suddenly at an altitude of about 100 km. We model this increase by parameterizing the sound speed as a function of altitude:

\[
c = c_0 = \begin{cases} 
300 \text{ m/sec} & \text{if } h < 100 \text{ km} \\
300 \text{ m/sec} & \text{if } h \geq 100 \text{ km}
\end{cases}
\]
so that the sound speed increases linearly to $2c_0$ at an altitude of 160 km. In the case of a sonic boom from a Mach 2 aircraft, the upper turning point of the approximate ray trace will therefore be at an altitude of 160 km.

The pressure and density in our model atmosphere may again be found by the use of the sound-speed relation

$$c^2 = \frac{YP}{\rho} \quad (16)$$

and the hydrostatic relation

$$a_h p = -pg \quad (17)$$

These equations immediately lead to a local scale height

$$H_s^{-1} = -\frac{1}{p} a_h p = \frac{YP}{c^2} \quad (18)$$

which may be integrated to obtain the pressure as a function of altitude (Fig. 2b).

E. Sonic Boom Properties as a Function of Altitude

Given our model atmosphere Eqs. (15) and (18) and our propagation of a sonic boom Eqs. (3), (4) and (7) we may easily numerically
Figure 2 TEMPERATURE, PRESSURE AND DENSITY DISTRIBUTION IN THE ATMOSPHERE
Integrate to find the sonic boom starting at an altitude of 17 km, with strength \( s_0 = 10^{-3} \) and length \( L_0 = 30 \) meters, believing these to be characteristic of a supersonic aircraft. (One effect not mentioned previously was taken into account: that the increase of sound speed with altitude also causes a lengthening of the boom, contributing to lowering its strength.)

Figure 3 shows the strength of the boom as a function of altitude. The boom reaches a strength of little more than 0.1 at 160 km altitude. Note that an extrapolation of the isothermal atmosphere (valid below 100 km) would have given a strong shock in this region. Thus the thermospheric properties just manage to avoid the production of strong shocks.

From this result we may calculate thermospheric heating and winds.

Inside the shock \[ \Delta T = \frac{\gamma - 1}{\gamma} s T_a \leq 30^\circ \]

Behind the shock \[ \Delta T_a = \frac{\gamma - 1}{12\gamma^3} s^3 T_a \leq 0.03^\circ \]

Wind \[ v = \frac{c}{\gamma(\gamma-1)} \frac{\Delta T_a}{T_a} \leq 3 \text{ cm/sec} \]

These values are clearly negligible compared to other fluctuations in the atmosphere.
Figure 3  VARIATION OF SHOCK STRENGTH WITH ALTITUDE
Figure 4 shows the calculated energy in the sonic boom as a function of altitude. We see that 90% of the energy is lost before reaching 160 km altitude.

Figure 5 shows the lengthening of the boom. At 160 km altitude the boom reaches a length of about 4 km.

We may describe the history of an upward-going sonic boom, following Figures 4 and 5, by approximating the true refractive path (with an upper turning point at about 160 km) by a vertically upward path to 160 km followed by a vertically downward path to the ground from 160 km. In that approximation only 3% of the initial energy is remaining at the upper turning point, and the boom has lengthened to 4 km. One can show that in the downward-going leg of the path, only about 30% of the energy at 160 km is lost, and the length does not change significantly; the smallness of these changes is due to the rapid decrease in the strength as the boom enters denser atmosphere. Hence the signal reaching the ground after refracting off the thermosphere is approximately 50 times less energetic than the sonic boom that travelled directly downward from the aircraft, and it is about 4 km in length, giving peak frequencies in the infrasonic region (~0.1 Hz).

The loss in energy and the lengthening results in a strength at the ground of
Figure 4  DECA Y OF ENER GY OF A WEAK SHOCK PROPAGATING VERTICALLY IN THE ATMOSPHERE
Figure 5 LENGTHENING OF A WEAK SHOCK IN THE ATMOSPHERE
If this signal bounces off the ground and undergoes another thermospheric refraction, its strength will remain so small that no significant energy loss or lengthening will occur.

F. Other Effects

A more extensive treatment of this problem would consider the correct direction-changing path of an angled sonic boom that spreads in three dimensions. Such a treatment would also consider the caustic that forms after the wavefront has turned over. In addition, viscosity in the air could be included. After this work was completed an extensive paper by Gardner and Rogers (Naval Research Laboratory) was received that took all these effects into account. They reached conclusions that are qualitatively and even quantitatively similar to those we have derived with our simplified model.
III RESEARCH NEEDS

Widespread interest in the "East Coast Mystery Booms" has focused attention on the problem of understanding the propagation of low frequency sound in a strongly stratified, inhomogeneous atmosphere having winds of various strengths at different levels. In the United States the subject received considerable attention in the 1950s and 1960s, first because of an interest in detecting nuclear explosions in the atmosphere and second because of concern over the effects of flying supersonic transports, both American and the Concorde. More recently the subject of infrasound has been largely neglected. At the start of the East Coast events there was only one low frequency acoustic array in operation, the Lamont-Doherty facility. At Lamont, Donn operates two tripartite arrays of capacitor microphones. The larger array is on a 600 to 900 meter spacing and has been in operation since 1967 while the smaller array is on 60-80 meter spacing and was put in operation in July, 1977. During March and April, 1978, in order to supplement existing observational facilities, Mitre operated three two-microphone correlators at Bedford, Massachusetts, Atlantic City, New Jersey and McLean, Virginia. The study of the East Coast Booms was aided by the existence of several seismic arrays where atmospheric pressure disturbances were recorded only along the vertical axis of the seismograph. While useful, seismometers, because of their poor performance in the 0.01 to 5 Hz region and because of the impedance mismatch...
across the air-ground interface, are less than ideal instruments for observing atmospheric disturbances.

The recent investigation by the Lamont6,12,13 and Kiruna7,14,15 groups have indicated that numerous artificial sources can be detected at large distances, for example rocket launches, the operations of SSTs, paper mills, hydroelectric plants and offshore-oil drilling equipment. The indicated richness of these sounds suggest that a number of properties of the atmosphere, for example stratospheric and thermospheric winds, could be monitored on a routine basis using netted acoustic arrays. Such observations are possible because of a fundamental property of the atmosphere; infrasonic waves can propagate over long distances because the attenuation is very small in the 0.01 to 5 Hz region, a frequency band where natural atmospheric noise is also very low.

The general considerations presented above suggest a number of research priorities:

A. Observational

1. The construction and operation of several netted infrasonic arrays operating in the 0.01 to 5 Hz region to monitor Concorde and other artificial sounds of infrasound along the East Coast. The existing Lamont facility needs to be supplemented by other arrays in order to understand better the Concorde signals. As noted by Gardner and Rogers9 an array on a base length of one kilometer has poor directional
and fluctuations in the E layer reflection height of as much as 7 km.

3. Formation of incoherent echoes, both infrasonic (2 Hz) and ultrasonic (0.1 Hz) report that fluctuations of atmospheric turbulence lead to reflections with acoustic detection.

4. Correlation of source spectrum with acoustic spectrum. The source spectrum and the wave spectrum of a wave shock, the attenuation of weak reverberant echoes are needed in order to study the non-linear acoustic behavior of the atmosphere. These spectra are needed in order to study the non-linear acoustic behavior of the atmosphere. These spectra are needed in order to study the non-linear acoustic behavior of the atmosphere.

5. Sources of infrasonic noise. The infrasonic data are not very well understood. Data on the infrasonic spectrum of industrial sources is known and the source spectrum in the infrasonic region of the condenser is presented. The spectra of industrial sources. The near field spectra of industrial sources.

6. As well as to plan the location of future arrays.

7. Essential if we are to understand the origin of the natural noise day, season, and weather as well as array location. Such data are needed for the determination of the background noise levels in the infrasonic frequency region as a function of time of day.

8. Arrays should be employed to determine the background noise, which are applied to the atmospheric pressure wave. Both fixed and portable techniques developed in this field would be expected to be applicable to the infrasonic spectrum. A few hundred dollars and sophisticated data processing could commercialize available capacitor microphone are relatively inex-

Resolution of sound waves in the few centimeters of a meters range.
The interaction of weak shocks with the weakly ionized portion of the ionosphere should be investigated both to understand the nature of the coupling, the possible contribution of the neutral-ion interaction to dissipation at high altitudes and the possible use of ionospheric observation to detect weak shocks launched at ground level or in the atmosphere. Liszka and Olsson also observed a short-duration E-layer echo at 130 km 50 sec before the arrival of the infrasonic waves, suggesting that the supersonic aircraft may have generated another mode of propagation, possibly gravity waves.

4. Artificial noise-generating arrays. An array of infrasound sources producing a narrow beam of high intensity could provide a powerful tool for probing the physical conditions in the high atmosphere. Such a facility would be particularly useful in investigating the dissipation mechanisms in the thermosphere. Recent calculations of the dissipation assume that classical viscosity and heat conduction are the principal mechanisms for dissipation. For the thermosphere, inhomogeneities in the density and temperature fields on temporal and spatial scales smaller than those of the transmitted wave can act as an effective eddy viscosity which could be much larger than the classical viscosity. A sound array coupled to an ionosonde facility would be a powerful tool for investigating the neutral-ion interactions noted above.
B. Theoretical

1. Origin of background noise in the infrasound region. Natural infrasonic noise is generally attributed to turbulent fluctuations in the wind field. The relations between these fluctuations and ambient weather conditions have not been adequately explored. An understanding of the nature and origin of the natural background noise is essential to selecting quiet sites for infrasonic arrays.

2. Attenuation of weak shocks in the high atmosphere. The concepts of classical viscosity and heat conduction are probably no longer appropriate when the collision frequency in the ambient atmosphere is of the same order as the infrasound frequencies. The physical nature of the attenuation in the atmosphere above about 100 km needs to be understood in order to interpret long-range propagation of infrasound.

3. Coupling of weak shocks with ionized particles. The observation of E layer height variations associated with the upward propagation of a weak shock suggest a strong coupling. The detailed physics of the coupling needs examination as does the possible contribution of weak plasma effects to the attenuation.

4. Artificial generation of gravity waves in the atmosphere. The present report has examined the infrasound part of the spectrum. Artificial sources of gravity waves may exist, for example Concorde,
and these artificial waves could perturb the ionosphere. A variety of potential artificial sources should be examined in terms of source strength so as to determine whether observations below 0.01 Hz could yield information about artificial sources and about the propagation of gravity waves in the atmosphere.
REFERENCES

A. **Fundamental Equations**

Consider a sonic boom consisting of a shock discontinuity followed by a continuous waveform of length $L$ (Fig. A-1). The shock is travelling into ambient atmosphere at speed $u$.

![Diagram](image)

**Figure A-1 CONDITIONS IN A PROPAGATING WEAK SHOCK**

Just after the shock (i.e. at $x$ small but positive) we define the pressure, density, and temperature by $p_o = p(o); \rho_o = \rho(o); T_o = T(o)$. The velocity of the gas $v(x)$ is non-zero within the shock, and again we define $v_o = v(o)$. 

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The full set of relations between quantities on either side of the shock discontinuity are the following:

Mass conservation
\[ \rho_a u = \rho_o (u - v_o) \]  

Momentum conservation
\[ \rho_a uv_o = p_o - p_a \]  

Energy conservation
\[ \rho_a u \left| c_v (T_o - T_a) + \frac{1}{2} v_o^2 \right| = p_o v_o \]  

Equation of state
\[ p_o = \rho_o R M T_o \]  

Within the sonic boom these same conservation laws yield:

\[ \rho_a u = \rho(x) \left[ - v(x) \right] \]  

\[ \rho_a \frac{d}{dx} v(x) = \frac{d}{dx} p(x) \]  

\[ \rho_a \frac{d}{dx} \left| c_v (T - T_a) + \frac{1}{2} v^2 \right| = \frac{d}{dx} (p v) \]  

\[ p(x) = \rho(x) R M T(x) \]  

Equations (1) - (4) yield a number of relations by simple algebra, which we present in terms of the shock strength

\[ s = (p_o - p_a)/p_a \]  

and the ambient sound speed

\[ c = \left( \gamma p_a/\rho_a \right)^{1/2} \]
One can show that the shock speed is

\[ u = c \left( 1 + \frac{\gamma + 1}{2\gamma} s \right)^{\frac{1}{2}} \quad (11) \]

which yields for \( s << 1 \)

\[ u = c \left( 1 + \frac{\gamma + 1}{4\gamma} s \right) \quad (II-6) \]

The temperature just behind the shock is

\[ T_0 = T_a \left( 1 + s \right) \left( 1 + \frac{\gamma - 1}{2\gamma} s \right) \left( 1 + \frac{\gamma + 1}{2\gamma} s \right)^{-1} \quad (12) \]

which yields for \( s << 1 \)

\[ \frac{\Delta T}{T_a} \approx \frac{\gamma - 1}{\gamma} s \quad . \quad (II-1) \]

The entropy change across the shock is

\[ \Delta S = c_v \log \left( \frac{T_0}{T_a} \right) + R_m \log \left( \frac{a}{c_o} \right) \]

which yields

\[ \Delta S = c_v \log \left[ \left( 1 + s \right) \left( 1 + \frac{\gamma - 1}{2\gamma} s \right)^{\gamma} \left( 1 + \frac{\gamma + 1}{2\gamma} s \right)^{-\gamma} \right] \quad (13) \]
B. Relations within the Sonic Boom

The exact manner in which the various thermodynamic variables change as a function of distance behind the shock is a complicated question of fluid dynamics. However, we may make some general remarks:

- The integral of the absolute overpressure must ultimately be close to zero as the gas is assumed to return to velocity much less than the sound speed after the shock has passed. Thus some region of negative overpressure is a necessity.

- In regions of smooth pressure behavior, the thermodynamic changes will be isentropic. However, a second shock behind the first will also have an entropy jump. Because of the distortion of the shape of the shock, the entropy jumps in the second shock will be less than that of the first shock.

We must determine the total energy in a unit area of wavefront, and the energy left behind the wavefront in the ambient atmosphere.

The energy per unit volume at a distance \( x \) behind the shock is

\[
E = \left[ c_v (T - T_a) + \frac{1}{2} \rho \right] \rho(x) \tag{14}
\]
Using energy flux conservation we have

$$
\varepsilon = \frac{1}{\rho_a u} p(x)v(x)\rho(x) \quad (15)
$$

Mass flux conservation yields

$$
\varepsilon = \frac{p(x)\rho(x)}{\rho_a} - p(x) \quad (16)
$$

Now we assume isentropy over the decay of the pressure so that

$$
\gamma(x) = \frac{p_o(p/p_0)^{1/\gamma}}
$$

$$
\varepsilon = \frac{\gamma(x) - 1}{\rho_a (p/p_o)} \frac{1}{\gamma} \left[ p(x) \right]^\gamma - p(x) \quad (17)
$$

To find the total energy in the wave we must integrate \( \varepsilon \) over the length of the waveform: Thus we must know \( p(x) \). We know that the integral of \( p(x) - p(a) \) must be very close to zero. Let us assume the usual N-wave form of the pressure

$$
p(x) = p_a \left[ 1 + \frac{2}{L} \left( \frac{L}{2} - x \right) \right] \quad 0 < x < L \quad (18)
$$

One can then show that

$$
E = \int_0^L \varepsilon \, dx = p_a L \left\{ \gamma \left[ \frac{2\gamma+1}{2\gamma} \right] \frac{2\gamma+1}{2\gamma} \frac{2\gamma+1}{2\gamma} s \left[ \left( 1 + s \right)^\frac{2\gamma+1}{2\gamma} - \left( 1 - s \right)^\frac{2\gamma+1}{2\gamma} \right] \right\} \left\{ \frac{2(2\gamma+1)}{(1 + s)^{\frac{2\gamma+1}{2\gamma}}(1 + s)^{\frac{2\gamma+1}{2\gamma}}} - 1 \right\} \quad (19)
$$

For \( s << 1 \) we find

$$
E = \frac{\gamma+1}{6\gamma^2} L \rho_a u^2\varepsilon^2
$$
C. Relations after Sonic Boom Passage

An entropy jump across the first shock followed by isentropic expansion back to ambient pressure, will leave the atmosphere heated behind the shock. The second shock in an ideal N wave will generate as much entropy as the first, since the underpressure in the following shock wave equals the overpressure in the leading wave. In an actual case, the shock lengthens as a result of momentum transfer and the second shock will be weaker. In this report we consider the weakening of the shock resulting only from entropy jump at the first shock, recognizing this approximation may underestimate the fractional energy loss by as much as a factor of two but probably much less.

\[
\Delta S = c_v \log \left( \frac{T_1}{T_a} \right)^\gamma
\]  

so that

\[
\frac{T_1}{T_a} = \left(1 + s\right)^{1/\gamma} \left(1 + \frac{\gamma-1}{2\gamma} s\right) \left(1 + \frac{\gamma+1}{2\gamma} s\right)^{-1}
\]  

For \( s \ll 1 \) we have

\[
\frac{\Delta T}{T_a} = \frac{\gamma^2 - 1}{12\gamma^3} s^3 = 0.03s^3
\]

so that even moderate-strength shocks heat the atmosphere very little.
The energy loss in travelling a distance \( L \) is
\[
\Delta E = c_v \Delta T \rho_{\text{a}} L,
\]
and using our formulas for \( \Delta T, \ E, \) noting that \( c_v = R_m/(\gamma - 1) \) we find
the fractional energy loss in traveling a distance \( L \) is
\[
\frac{\Delta E}{E} = \frac{s}{2\gamma} \quad (22)
\]
Hence we may immediately write the energy loss equation
\[
\frac{1}{E} \frac{\partial}{\partial t} E = -\frac{s}{2\gamma L} \quad (11-4)
\]
or
\[
\frac{1}{E} \frac{\partial E}{\partial t} = \frac{s}{\gamma L}
\]
for a \( N \) wave maintaining its ideal shape.

The momentum transferred to the atmosphere is related to the energy
deposition. A wave travelling at nearly acoustic speeds (i.e. \( s \ll 1 \)) would
have the relation

\[
\text{Momentum transferred} = \frac{\text{Energy deposited}}{c} \quad (23)
\]

Thus the velocity imparted to the atmosphere after the passage of the
boom would be

\[
v = \frac{\text{Energy deposited/unit mass}}{c} = \frac{c \Delta T}{c} = \frac{c \Delta T}{c} \quad (24)
\]
This immediately yields

\[ v \approx \frac{c}{\gamma (\gamma - 1)} \frac{\Delta T_a}{T_a} \]  

(II-5)
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