Nanoflyer

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October 1997

JSR-97-115

Approved for public release; distribution unlimited.

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A recent proposal to use electrostatic forces to lift and propel a small airborne vehicle is examined. We show here that although this is permitted by the laws of physics, it is very inefficient, and is limited to low areal loads by the requirement to avoid electric breakdown. Electrostatic propulsion offers no special advantages which might justify the price of its inefficiency.
ELECTROSTATIC FLIGHT

A recent proposal by Dædalus Research suggests using electrostatic forces to lift and propel a small airborne vehicle. We show here that although this is permitted by the laws of physics, it is very inefficient, and is limited to low areal loads by the requirement to avoid electric breakdown. Electrostatic propulsion offers no special advantages which might justify the price of its inefficiency.

In electrostatic propulsion, just as in propulsion by a rotating airfoil, it is not necessary to exert thrust directly downward in the manner of a helicopter. It may be possible to employ a wing to provide lift and to make the thrust horizontal, only overcoming the drag induced by flow over the wing. Just as for a conventional aircraft, this reduces the thrust required to maintain altitude, in comparison to that required by a helicopter, by the lift-to-drag ratio $L/D$ of the vehicle. This is about 5 even for very small (cm-sized) wings, but may be less for the vehicle as a whole because of the necessity of moving a broad electrostatic propulsor broadside through the air.

The principle of electrostatic lift and propulsion is to fill a volume of air with a net negative charge density by introducing electrons from a field emitting cathode, and then to exert a force on this charged air by applying an electric field. The charged air fills the volume between two parallel flat electrodes, taken to be squares of side $s$ separated by a distance $b$. We will make the approximation that this can be treated electrically as if $b \ll s$, neglecting fringing fields.

Electrons are assumed to be injected at the cathode by field emission at numerous sharp points, and then to attach to $O_2$ molecules, which have an
electron affinity of 0.44 eV, and are the only major component of air with a positive electron affinity. We assume all the electrons are bound in the form of $O^-_2$, which (using the Saha equation) is found to be a good approximation at room temperature, but which will not be correct at higher temperature. We also assume that no other ions are present; if other ions are created the efficiency will be less than even the low values we calculate.

The electric field between the electrodes accelerates the $O^-_2$ towards the anode, and collisions transfer momentum to the neutral air molecules. The electrodes must be porous to admit a flow of neutral air through the cathode and to permit it to escape through the anode, at which the ions are neutralized. This flow of air provides the reaction force against which the electrostatic engine exerts its thrust, and is analogous to the downwash of a helicopter rotor or a propeller wake.

We first present a very simple rough estimate. Assume the charge density and electric field between the electrodes are uniform, which is correct in the limit in which the charge density is small (and the field imposed by the electrodes is large). Then the total charge contained between the electrodes is

$$Q = -n_i e s^2 b,$$  \hspace{1cm} (1)

where $n_i$ is the ion density and $-n_i e$ the net charge density. The magnitude of the thrust or force is

$$F = EQ,$$  \hspace{1cm} (2)

where $E = V/b$ is the electric field produced by the electrodes and $V$ is the potential drop.

The charge $Q$ produces an additional electric field which tends to push the ions and air out from between the electrodes, keeping the ions from reaching the anode by the most direct path and reducing the reaction force imparted to the air. This is undesirable, because it reduces the net charge
on which the imposed field acts, increases the frictional losses as ions are
pushed laterally through the neutral air, and reduces the net field acting on
the net charge. We can approximate this self-field

\[ E_s \sim \frac{Q}{(s/2)^2} \]  for \( b \leq s \)  \hspace{1cm} (3)

\( (b > s \) is an inefficient electrode geometry because the fringing fields are
dominant).

In order for the efficiency to approach its maximum value we must have

\[ E_s < E. \]  \hspace{1cm} (4)

Then

\[ Q < \frac{F^{1/2} s}{2}. \]  \hspace{1cm} (5)

In order to minimize \( E \) (which will turn out to be uncomfortably large) we
maximize \( Q \) by taking the inequality (5) to be an equality. Then

\[ E = 2 \left( \frac{F}{s^2} \right)^{1/2} \]  \hspace{1cm} (6)

and

\[ V = 2b \left( \frac{F}{s^2} \right)^{1/2}. \]  \hspace{1cm} (7)

There are two concerns: the power which must be expended to supply
the thrust \( F \) and the magnitude of the potential. The power expended is
largely that dissipated resistively at a rate \( jE \) per unit volume, where \( j \) is
the current density. The ratio of power to thrust

\[ \frac{P}{F} = \frac{jE s^2 b}{F} = \frac{n_i e E s^2 b v_{dr}}{F} = \frac{Q E v_{dr}}{F} = v_{dr}, \]  \hspace{1cm} (8)

where \( v_{dr} \) the ion-neutral drift velocity, given by

\[ v_{dr} = kE. \]  \hspace{1cm} (9)

The mobility of \( O_2^- \) in air \( k = 4.4 \text{ cm}^2/(\text{Volt s}) = 1.3 \times 10^3 \) cgs. Hence

\[ \frac{P}{F} = 2k \left( \frac{F}{s^2} \right)^{1/2}. \]  \hspace{1cm} (10)
For plausible values of $F = 10^3$ dyne and $s = 1$ cm, $P/F = 8 \times 10^4$ cm/s. We have ignored inefficiencies resulting from the significant value of $E_s$, viscous drag in air flow through the porous electrodes, etc., which may be significant.

This value of $P/F$ should be compared to the value found for more conventional propulsion by rotating airfoils, which is $\approx [F/(\rho_{air}s^2)]^{1/2} \approx 900$ cm/s. Electrostatic propulsion demands nearly 100 times as much power, a ratio which is independent of the areal loading $F/s^2$. The reason for this inefficiency is the large power dissipated in friction between the ions and the neutral air, whose relative drift velocity is $v_{dr}$.

The required electric fields are large. For the previous parameters $E \approx 20,000$ Volts/cm. This is uncomfortably close to the breakdown field of 30,000 Volts/cm for air. In the neighborhood of the field-emitting points the field is substantially larger, and at least corona discharge may occur.

In the thin capacitor limit ($b \ll s$) an exact calculation, allowing for the effect of the self-field of the space charge, is straightforward. Poisson's equation for the electrostatic potential is

$$\nabla^2 \phi = \frac{d^2 \phi}{dz^2} = -4\pi n_i e, \quad (11)$$

where $z$ measures the distance from the cathode. The conservation of charge implies that the current density $j$ is constant, and $j$ is related to the electric field by the ion mobility:

$$j = E(z)n_i(z)e k \quad (12)$$

Eliminating $n_i(z)e$ gives

$$\frac{4\pi j}{kE(z)} = -\frac{d^2 \phi}{dz^2} = \frac{dE(z)}{dz}. \quad (13)$$

Upon integration

$$E(z) = \left[\frac{8\pi j}{k}(z - z_0)\right]^{1/2}. \quad (14)$$
The constant of integration \( z_0 \) depends on the charge density. The maximum possible charge (and hence the smallest electric field for a given surface loading \( F/s^2 \)) is found if \( z_0 = 0 \), so that the field drops to 0 at the cathode (the charge density \(-n_i(z)e\) becomes infinite there, by Equation 12). The surface loading is found from the difference in electric stresses at the two electrodes:

\[
\frac{F}{s^2} = \frac{E^2(b) - E^2(0)}{8\pi} = \frac{jb}{k}.
\]  

(15)

The power dissipated is

\[
\frac{P}{s^2} = \int jE \, dz = j \int E \, dz = jV.
\]  

(16)

The ratio of power to surface load is

\[
\frac{P}{F} = \frac{V k}{b} = \frac{2}{3} kE(b).
\]  

(17)

The field \( E(b) \) is 2.5 times the rough estimate of Equation 6; requiring \( E(b) < 30,000 \) Volts/cm to avoid breakdown limits the surface loading to 400 dyne/cm\(^2\). Equation (14) shows that \( E(b) = 3V/(2b) \); that is, the ionic space charge increases the field at the anode by 50% compared to a uniform field with the same potential. At this surface loading the ratio of power to thrust is \( 9 \times 10^4 \) cm/s, close to the rough estimate of Equation 10. The ratio \( P/F \) for a rotating airfoil with the same surface loading is \( \approx 550 \) cm/s. Electrostatic propulsion demands about 150 times more power than a rotating airfoil, a ratio more pessimistic than the rough estimate, but again independent of the areal loading.

Another way of looking at the inefficiency of electrostatic propulsion is to estimate the endurance of such an aircraft. This depends on the power supply, which we will take to be a battery with the highest estimated specific energy of any battery, \( \mathcal{E} = 400 \text{ Wh/kg} = 1.44 \times 10^{10} \text{ erg/gm} \) for lithium thionyl chloride. Fuel cells may provide a few times higher specific energy, but are not available off the shelf. Hydrocarbon fuels have even higher high energy content (roughly an order of magnitude greater than that of the best
batteries), but micro-scale heat engines and generators do not exist, and may have low efficiency if they are built. Taking a fraction $f_b$ of the vehicle's mass $M$ as battery and equating the energy available to the power expended in an endurance time $t$ yields

$$f_b\varepsilon M = \frac{gM(P/F)t}{L/D},$$

(18)

where $L/D = 1$ if the thrust is exerted directly downward, as in a helicopter. Substituting $f_b = 0.5$, $L/D = 1$ and taking the previously estimated $P/F = 9 \times 10^4$ cm/sec yields

$$t = \frac{f_b\varepsilon(L/D)}{g(P/F)} = 85 \text{ sec.}$$

(19)

The numerical value depends on a number of parameters whose values can only be estimated (for example, it is not possible to draw the energy of a battery nearly as fast as required). It is still apparent that the endurance time is hundreds of times shorter than that of more conventionally powered aircraft, and is too short to be useful.
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