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IMPACT FUSION WITH A SEGMENTED RAIL GUN

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INTRODUCTION

The rail gun offers an attractive alternative to the traveling wave rifle for the magnetic acceleration of macroscopic (0.05 g) bullets for impact fusion. For power generation it is essential for the rail gun to be energy efficient. In this paper we review the basic rail gun equations and indicate how energy efficiency can be designed in. We set as our preliminary goal the delivery of E = 1 megajoule in dt = 10 nanoseconds, with a specific energy of 20 MJ/g (i.e. a bullet mass of 0.05 g); these values are taken from the requirements being considered for heavy-ion fusion. Using these numbers, we can solve immediately for the final particle velocity u_f from $E = 1/2\ m u_f^2$ to get $u_f = 200$ km/sec. For a delivery time of 10 nanoseconds, this velocity implies that the projectile length is about 2 mm. Impact fusion is feasible because of the coincidence that a bullet with all dimensions roughly 2 mm has the required mass.

BASIC RAIL GUN EQUATIONS

We begin by reviewing the basic equations which have been derived by J.P. Barber and others. We shall derive these equations in the simplest possible form, in order to emphasize the scaling laws and the physics contained in them. The power delivered by a current I and voltage V, ignoring losses, is:

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$$P = IV = d/dt(LI^2/2) + Fu$$

where L is the inductance, F is the force on the "armature" (i.e. the bullet), and u is the bullet velocity. Setting $V = d/dt(LI)$ and simplifying, we get the basic rail gun equation:

$$F = 1/2\ L'I^2$$

where L′ is the inductance per unit length, and is very close to L′ = 0.6 microhenries/meter for typical rail gun geometries. For simplicity we shall now assume constant acceleration, i.e. constant current I. The work done on the bullet is $E = Fz = LI^2/2$ where $L = L'z$. Note that the work done on the bullet is equal to the energy stored in the magnetic field of the rails; this inductive energy can be recovered, in principle. Note also that for fixed E and L′ the length of the rail gun "z" is proportional to I^{-2}. The required voltage can be calculated from $V = d(flux)/dt = IL'u$. The values for the current, maximum voltage, and acceleration time t for two lengths of rail-gun are given in the table below:

length z	=	100 meters	1 km
current	=	180 kA	60 kA
maximum voltage	=	23 kV + IR	7 kV + IR
time	=	0.001 sec	0.01 sec

The IR term in the voltage refers to the drop from resistive losses, and is discussed below. The required current can be supplied by capacitors or by high-voltage homopolar generators (see the paper at this conference by R.L. Garwin).

EFFICIENCY

For application to impact fusion, it is essential to keep the resistive losses to a minimum. To the extent that the resistances in the rail gun are independent of velocity u, the most difficult regime is the low velocity one where the power being transferred into bullet kinetic energy $P = Fu$ is low. On the other hand, designs which are inefficient at low velocities may be considerably more efficient at high velocities, for the same reason.

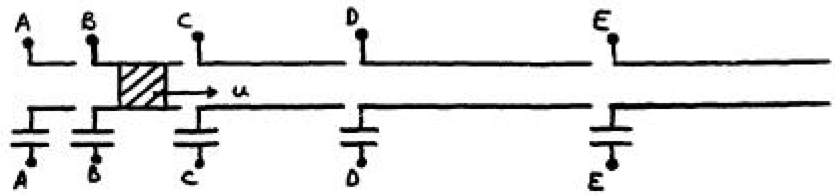
The energy lost to resistive heating of the bullet or driving plasma is $Q = I^2Rt = 2ERt/L'z = 4ER/L'u_f$. The "inefficiency factor" for the bullet $Q/E = 4R/L'u_f$ is independent of accelerator length, and depends only on the final velocity to be achieved, u_f. The resistance of a copper bullet with dimensions of 2 mm will be about 10^{-5} ohm; although resistive loss is not a problem with the bullet, the copper may be heated beyond its melting point. The resistance of a driving plasma can be calculated from Spitzer's formula, and is approximately $\rho = 65 \ln(H)/T^{3/2}$ where H depends on the temperature and density; $\ln(H) \approx 3$ for the densities and temperatures of interest. (Black body emission alone will prevent the temperature from rising above 10^{+5} K). Substituting values, and assuming that the plasma occupies a 2 mm cube, the resistance of the plasma is $R = 3 \times 10^{-3}$ ohm, and for the plasma $Q/E = 30R = 0.1$, an acceptable value. The resistive voltage drop across the plasma will be $V = IR = (60kA)(0.003) = 180$ volts.

Resistive loss in the rails is a more serious problem, especially for the longer rail guns. The DC resistance for two 1-km rails with a cross-section of 2 mm x 2 mm is $R_{DC} = 8.5$ ohms. When the bullet is at a position z the resistance is $R_{DC}(z/z_m)$ if skin-depth effects are ignored (the true resistance will be greater). The energy lost to resistive heating in the rails during constant acceleration a is

$$Q = \int I^2 R dt = \frac{I^2 R_{DC}}{z_m} \int z dt = \frac{I^2 R_{DC} a}{2z_m} \int t^2 dt = \frac{I^2 R_{DC} t}{3}$$

From the above result, we can define an average or effective resistance $R = R_{DC}/3 = 2.8$ ohms. We find for the rails, $Q/E = 30R = 85$, and an efficiency $E/(Q + E) = 1.2\%$. One cannot arbitrarily reduce the resistance by increasing the rail cross-section, for to do so reduces L′. A smaller L′ implies a smaller force on the bullet, and hence a longer rail gun to achieve the same terminal velocity.

To improve the efficiency, it is necessary to divide the rail into segments, with current flowing only in the segment carrying the bullet. A schematic diagram is shown in the figure:



The current behind the bullet will penetrate to an average depth given by

$$s = \sqrt{\frac{\rho \Delta t}{\mu_o}} .$$

where Δt is the time since passage of the bullet, ρ is the resistivity, and μ_o is the permittivity ($= 1.3 \times 10^{-6}$ in MKS units, for non-magnetic materials). For a large number of segments N, we can assume that the skin depth s is less than the thickness of the rails, and derive the following formula (see Appendix) for the effective resistance of the segmented system:

$$R = \frac{Q}{I^2 t} = \frac{8\ell}{3h} \sqrt{\frac{\rho \mu_o}{tN}}$$

For z = 1000 meters, $\rho = 1.7 \times 10^{-8}$ ohm-meters (Cu), and the height of the rails h = 2 mm, this gives $R = 2/\sqrt{N}$ ohms. Thus for N segments, we have $Q/E \simeq 65/\sqrt{N}$. For N = 1000, the efficiency $e = E/(Q + E) = 30\%$. Additional gains are possible by increasing N, the limit looking like a lumped delay line with propagation velocity matching the bullet velocity and pulse length not much longer than the bullet.

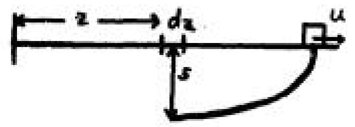
For the 1000 segment, 1 km rail gun, the maximum IR voltage drop from the resistance in the rails is 60kA x $2/\sqrt{1000}$ = 4 kV, comparable to the back EMF required of 7kV (see the table on page 2).

The segmented rail gun can be fed by power supplies along its length, represented by capacitors in the figure. No switches to close the circuit are shown, because the switching is automatically provided by the bullet. Since the resonance time of the L-C circuit is approximately equal to the transit time of the bullet, it may be possible to use the automatic switching of the bullet to open the circuit at a time of near-zero current flow.

APPENDIX

Effective Resistance of a Rail Gun Segment

Consider one element dz of segment of length ℓ_i. Let resistive heating during passage of bullet = dQ. Skin depth = $s(t) = \sqrt{\frac{t\rho}{\mu_o}}$.



$$\frac{dQ}{dz} = \int_o^{t_2 - t_1} I^2 \frac{dR}{dz}\, dt = \frac{I^2 \rho}{h} \int \frac{dt}{s} = \frac{I^2 \rho}{h} \sqrt{\frac{\mu_o}{\rho}} \int_o^{\Delta t} \frac{dt}{\sqrt{t}} = \frac{2I^2}{h} \sqrt{\rho \mu_o \Delta t}$$

where h = height of rail.

Now integrate this over segment of length ℓ_i. Assume velocity u_i = constant.

$$\Delta Q_i = \frac{2I^2}{h} \sqrt{\rho\mu_o} \int \sqrt{\frac{\ell_i - z}{u_i}}\, dz = \frac{4}{3} \frac{I^2}{h} \sqrt{\frac{\rho\mu_o}{u_i}}\, \ell_i^{3/2} = \frac{4}{3} \frac{I^2 \ell_i}{h} \sqrt{\rho\mu_o t_i}$$

(a) Suppose lengths are chosen such that $t_i = t/N$.

$$Q = \Sigma \Delta Q_i = \frac{4}{3} \frac{I^2 \ell}{h} \sqrt{\frac{\rho \mu_o t}{N}}$$

so

$$R = \frac{4}{3} \frac{\ell}{h} \sqrt{\frac{\rho \mu_o}{tN}}$$

For 2 rails, double this answer.

(b) Suppose lengths are chosen such that $\ell_i = \ell/N$.

$$Q = \Sigma \Delta Q_i = \frac{4}{3} \frac{I^2}{h} \sqrt{\rho \mu_o} \frac{\ell}{N} \Sigma \sqrt{t_i}$$

This sum can be done by approximating it with an integral, and the answer is the same as for (a).