Technical Challenges of Exascale Computing

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1 ABSTRACT

JASON was tasked by DOE/NNSA to examine the technical challenges associated with exascale computing. This study examines the issues associated with implementing DOE/NNSA computational requirements on emerging exascale architectures. The study also examines the national security implications of failure to execute a DOE Exascale Computing Initiative in the 2020 time frame.


2 EXECUTIVE SUMMARY

The past thirty years have seen an exponential increase in computational capability that has made high performance computing (HPC) an important enabling technology for research and development in both the scientific and national security realms. In 2008, computational capability as measured by the LINPACK linear algebra benchmark reached the level of $10^{15}$ floating point operations per second (a petaflop). This represents a factor of 1000 increase in capability over the teraflop level ($10^{12}$ floating point operations per second) achieved in 1997. JASON was tasked by the DOE Office of Science and the ASC Program of NNSA to examine the technical challenges associated with exascale computing, that is, developing scientific and national security applications using computers that provide another factor of 1000 increase in capability in the 2020 time frame. DOE/NNSA posed the following questions to JASON:

1. What are the technical issues associated with mapping various types of applications with differing computation and communication platforms to future exascale architectures, and what are the technical challenges to building hardware that can respond to different application requirements?

2. In the past programming tools have been afterthoughts for high performance platforms. What are the challenges in designing such tools that can also be gracefully evolved as the hardware evolves?

3. What are the economic and national security impacts of failure to execute the DOE Exascale Computing Initiative (ECI)? What application capabilities will emerge in the absence of an initiative?
JASON’s assessment of these issues is summarized below and in a more detailed form in the main report.

2.1 Overview

While petascale computing was largely achieved through an evolutionary refinement of microprocessor technology, the achievement of exascale computing will require significant improvements in memory density, memory bandwidth and perhaps most critically, the energy costs of computation.

Much of the impressive increase in computing capability is associated with Moore’s law, the observation that the number of transistors on a processor has increased exponentially with a doubling time of roughly 18 months, as well as Dennard scaling which allowed for increases in processor clock speed. However, as of 2004, for a variety of technical reasons discussed in this report and elsewhere, serial performance of microprocessors has flattened. Clock speeds, for the most part, now hover around 2–6 gigahertz and there is no expectation that they will increase in the near future. The number of transistors continues to increase as per Moore’s law, but modern microprocessors are now laid out as parallel processors with multiple processing cores. It is projected that to achieve an exaflop (10\textsuperscript{18} floating point operations per second), an application developer will need to expose and manage 1 billion separate threads of control in their applications, an unprecedented level of parallelism.

At the same time, while the number of cores is increasing, the amount of total memory relative to the potential floating point capability is decreasing. This is largely due to the fact that memory density has not increased as quickly as floating point capability, with the result that maintaining the ratio of memory capacity
to floating point capability is becoming prohibitively expensive. Today, this is viewed as a matter of cost, but it will eventually limit the working set size of computations and underscores the need for new memory technologies that offer higher density. In addition, memory access times have decreased over time, but not as quickly as floating point capability has increased. These issues are well-known to processor architects as the “memory wall”.

Perhaps the most significant challenge for exascale computing is bounding the energy required for computation. DOE/NNSA has set a power budget for an exascale platform at roughly 20 megawatts. This is a typical power load for a modern large data center. Each picoJoule per second (a picowatt) expended by the hardware in communication or computation translates into 1 megawatt of required power at the exascale. For example, a computation requiring the delivery of an exaword of memory \(10^{18}\) 64 bit words per second (that is the delivery of one word for every floating point operation) would consume 1.3 gigawatts of power using today’s processors and memory. Although advances in device and circuit design will continue, by 2020 this number is expected to only decrease to 320 megawatts.

If these trends regarding memory size, memory bandwidth, and the associated energy costs continue to hold, two critical issues will emerge. First, it will only be possible to run applications that require a small working set size relative to computational volume. There are important DOE/NNSA applications that are in this class, but there are also a significant number that utilize working sets in excess of the projections for memory capacity of an exascale platform in the 2020 time frame. Second, only applications that can effectively cache the required memory will be able to run on an exascale platform within the required power envelope and also provide a reasonable percentage of peak computational
throughput. At present, many DOE/NNSA applications, including some that will be used for future stockpile stewardship investigations, require significant memory bandwidth to perform efficiently. The projected memory limitations will make it impossible to increase the spatial or model fidelity of current implementations of DOE/NNSA applications at a level commensurate to the thousand-fold increase in floating point performance envisioned by 2020.

An additional challenge will be insuring that an exascale platform will function in the presence of hardware failures. This is known as resilience. It is projected that an exascale platform may have as many as $10^8$ memory chips and $10^5 - 10^6$ processors. Data on resilience of high performance computers are relatively sparse. Without aggressive engineering however, crude projections show that such a machine will function without system interruption for merely tens of minutes.

The expected complexity of any future exascale platform as regards memory hierarchy, resilience, etc. will make it necessary to consider different approaches to software development. In the past, applications were largely written using explicit message passing directives to control execution in a way that was often very specific to the details of the hardware. Ideally, an application developer should make clear the type and amount of parallelism associated with a specific algorithm. The specific implementation on a given hardware platform should be left to a compiler aided by software that can optimize the generated code for maximal throughput. At present, compilers for traditional programming languages cannot infer such information without some specification of the underlying computation and communication patterns inherent in a particular algorithm. Promising research directions include domain-specific languages and compilers as well as auto-tuning software. A research effort is required to develop language constructs and tools to reason about massively parallel architectures in a way that does not
explicitly require detailed control of the hardware. In any case, software for future exascale platforms cannot be an afterthought, and the level of investment in software technology must be commensurate with the level of investment in hardware.

In order to attempt to address these issues, DOE/NNSA have initiated a set of “co-design” centers. These centers are meant to facilitate the communication of scientific problem requirements to hardware designers and thus influence future hardware designs. In turn, designers communicate hardware constraints to application developers so as to guide the development of future algorithms and software. JASON is concerned however that, as currently practiced, co-design may fail to give DOE/NNSA the leverage it needs. More focus is required to ensure that the hardware and software under development will target improvements in the performance of the dominant patterns of computation relevant to DOE/NNSA applications. In particular, there is a need to enhance the level of communication so that a true exchange of ideas takes place. In some cases, intellectual property concerns make such exchanges very difficult. Resolution of these issues is required if these centers are to contribute effectively.

2.2 Findings

Our findings regarding the technical challenges of exascale computing are as follows:

**Importance of leadership in HPC** US leadership in high performance computing is critical to many scientific, industrial and defense problems. In order to maintain this leadership, continued investment in HPC technology (both hardware and software) is required. It is important to note however, that maintenance of this leadership is not necessarily tied to the achievement of
exascale computing capability by 2020. Such leadership can be maintained and advanced with ongoing investments in research and development that focus on the basic technical challenges of achieving balanced HPC architecture.

Feasibility of an exascale platform by 2020 It is likely that a platform that achieves an exaflop of peak performance could be built in a 6–10 year time frame within the DOE/NNSA designated power envelope of 20 megawatts. However, such a platform would have limited memory capacity and memory bandwidth; owing to these limitations such an exascale platform may not meet many DOE/NNSA application requirements.

National security impacts To achieve DOE/NNSA mission needs, continued advances in computing capability are required and will be required for the foreseeable future. However, there is no particular threshold requirement for exascale capability in the 2020 time frame as regards those national security issues associated with the DOE/NNSA mission. For this reason, JASON does not foresee significant national security impacts associated with a failure to execute the DOE Exascale Computing Initiative by 2020.

Technical challenges The most serious technical challenge impeding the development of exascale computing in the near term is the development of power-efficient architectures that provide sufficient memory density and bandwidth for DOE/NNSA applications.

Focus on DOE/NNSA application requirements More focus is required to ensure that the hardware and software under development in support of an exascale capability will address performance improvements specific to the communication and computational patterns of DOE/NNSA applications.
**Focus on software tools**  More focus is also required to develop software that will facilitate development of and reasoning about applications on exascale platforms regardless of the details of the underlying parallel architecture.

**Co-design strategy**  The current co-design strategy is not optimally aligned with the goal of developing exascale capability responsive to DOE/NNSA application requirements. More focus is required to ensure that the hardware and software under development will target improvements in the performance of the dominant patterns of computation relevant to DOE/NNSA applications.

### 2.3 Recommendations

Our recommendations are as follows:

**Continued investment in exascale R&D is required**  Rather than target the development of an exascale platform in the 2020 time frame, DOE/NNSA should invest in research and development of a variety of technologies geared toward solving the challenging issues currently impeding the development of balanced exascale architecture: increased memory density, memory bandwidth, energy-efficient computation, and resilience.

**Establish intermediate platform targets**  DOE/NNSA should establish a set of intermediate platform targets in pursuit of balanced HPC architecture over a realistic time frame. An attractive set of intermediate targets are platforms that provide *sustained* computational floating point performance of 1, 10, and ultimately 100 petaflops, but optimized for DOE/NNSA computational requirements, with memory capacity and bandwidth targets that exceed current microprocessor vendor road maps, and with a maximum power consumption of 5 megawatts or less. A variety of technical approaches should
be supported in meeting this target with eventual down-select of the most promising approaches.

**Assess application requirements** Undertake a DOE/NNSA effort to characterize in a standard way the computational patterns and characteristics (i.e. memory capacity, memory bandwidth, floating point intensity and global communication) of the suite of DOE/NNSA applications.

**Enhance investment in software tools** Support development of software tools at a budgetary level commensurate with that provided for hardware development. In particular, support for tools like domain-specific languages and auto-tuning software is needed so that users can reason about programs in terms of scientific requirements as opposed to hardware idiosyncrasies.

**Improve the co-design strategy** Enhance the current co-design strategy so that it not only focuses on the optimization of existing codes, but also encourages hardware and software innovation in direct support of the dominant patterns of computation relevant to DOE/NNSA applications.
3 DOE/NNSA COMPUTING CHALLENGES

The past three decades have seen an exponential increase in computational capability that has made computing an essential part of the scientific enterprise. Shown in Figure 3-1 is the evolution of the number of floating point operations per second (flops) that can be achieved on a modern high performance computer. Two types of results are shown - the peak rate in which all functional units of the computer are active and the “maximum” rate that is measured using the LINPACK benchmark that measures the time it takes to factor a full \( N \times N \) matrix. It can be seen that, in 1993, peak speeds were on the order of \( 10^{10} \) floating operations per second (flops) or tens of gigaflops. In 1997, through the development efforts of the NNSA ASC program, a capability to compute at \( 10^{12} \) flops or a teraflop was demonstrated and ten years later, again via the efforts of the ASC program, a petaflop capability was demonstrated. What is perhaps more remarkable is that this increase was achieved largely by the development of increasingly capable microprocessors without a significant change in hardware architecture.

3.1 Study Charge from DOE/NNSA

A logical question is whether this trend can continue to provide an additional factor of 1000 in the 2017 time frame - ten years from the development of computers capable of peak speeds approaching a petaflop or petascale computation. This next level - \( 10^{18} \) floating point operations per second is known as exaflop or exascale computing. As will be detailed below, achieving this next level of capability will be very challenging for a number of reasons detailed in this report, and noted in many previous studies.
DOE and the NNSA tasked JASON to study the possibility of developing an exaflop computational capability. We quote below from the study charge as communicated by DOE/NNSA to JASON:

“This study will address the technical challenges associated with the development of scientific and national security applications for exascale computing. The study will examine several key areas where technology development will be required in order to deploy exascale computing in the near future:

1. Applications: It is likely that a future exascale platform will utilize a hierarchical memory and network topology. As a result, there may be barriers to optimal performance for certain types of scientific applications. What are the technical issues associated with mapping various types of applications with differing computation and communication platforms to future exascale archi-
tectures, and what are the technical challenges to building hardware that can respond to different application requirements?

2. Programming environments: The development of application codes for future exascale platforms will require the ability to map various computations optimally onto a hierarchical computing fabric. In the past, programming tools have been afterthoughts for high performance platforms. What are the challenges in designing such tools that can also be gracefully evolved as the hardware evolves?

3. What are the economic and national security impacts of failure to execute the DOE Exascale Computing Initiative (ECI)? What application capabilities will emerge in the absence of an initiative? ”

3.2 Projected Configuration of an Exascale Computer

The challenges in delivering a factor of 1000 over present day petascale computing are significant for several reasons. In Table 3.1, we list some of the requirements associated with exascale computing in relation to present day petascale computing. For reasons associated with powering and cooling of modern microprocessors, it is no longer possible to speed up processors by simply increasing the clock speed. This limitation is discussed further in Section 4. As a result, computational throughput is increased today by exposing parallel aspects of the program being executed and delegating the computation to individual processor cores. This is not a new development, and as shown in Table 3.1, today’s petascale systems already use this “multi-core” approach. What will be different however, when considering exascale computing, is that because the clock speed cannot be easily increased, the
<table>
<thead>
<tr>
<th>Attribute</th>
<th>Petascale (realized)</th>
<th>Exascale target</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak flops</td>
<td>$2 \times 10^{15}$ flops</td>
<td>$1 \times 10^{18}$ flops</td>
</tr>
<tr>
<td>Memory</td>
<td>0.3 Petabyte</td>
<td>50 Petabytes</td>
</tr>
<tr>
<td>Node performance</td>
<td>$1.25 \times 10^{11}$ flops</td>
<td>$2 \times 10^{12}$ flops</td>
</tr>
<tr>
<td>Node memory bandwidth</td>
<td>$2.5 \times 10^{10}$ bytes/sec</td>
<td>$1 \times 10^{12}$ bytes/sec</td>
</tr>
<tr>
<td>Node concurrency</td>
<td>12 cores</td>
<td>1000 cores</td>
</tr>
<tr>
<td>Number of nodes</td>
<td>$2 \times 10^{4}$ nodes</td>
<td>$1 \times 10^{6}$ nodes</td>
</tr>
<tr>
<td>Total concurrency</td>
<td>$2.25 \times 10^{5}$ threads</td>
<td>$1 \times 10^{9}$ threads</td>
</tr>
</tbody>
</table>

Table 3.1: Attributes of an exascale computer

throughput per core is not expected to increase significantly and so the only way to increase throughput is to increase the number of cores on a node.

It is projected that one can realistically build systems with roughly $10^5 - 10^6$ nodes, and so in order to provide a machine with a peak capability of an exaflop one would have to deploy $10^5$ nodes each performing at a computational rate of 10 teraflops or $10^6$ nodes performing at a rate of 1 teraflop. In order to accomplish this, it is anticipated that each node will contain on the order of 1000 processing cores, each responsible for one or several parallel threads of control of a given program. Each core can ideally perform about $10^9$ floating point operations and so, regardless of the number of nodes or cores, building an exascale machine in this way means exposing and managing $10^9$ parallel threads of control. Such a level of parallelism has never been previously contemplated.

Another issue that leads to additional challenges when one considers computing at this scale is the amount of available memory and the ability of a processor to read and write data in such a way as to deliver it to the processor functional units at a rate sufficient to prevent idling of the processor. It will be seen that, owing to issues associated with cost and efficiency, it has not been possible to build memories for high performance computers of a size and speed that scale.
with the growth of the floating point throughput of the processor. It is already the
case today that the flop to memory ratio (in terms of memory size) for a petascale
machine is roughly 6 to 1 and, as can be seen from Table 3.1, this ratio is predicted
to increase further in the absence of significant improvements in memory design.

Finally, the development of an exascale platform is ambitious because of the
projected power requirements. Modern data centers typically provide about 20
megawatts of power. The building of an exascale system using today’s technology
would require much more than this as discussed further below.

3.3 Overview of DOE Exascale Computing Initiative

In order to understand the needs for exascale computing, DOE and NNSA initiated
in 2008 a series of community workshops on a number of relevant areas such as
basic energy sciences, climate, materials, national security, etc. The goals of the
workshops were to

- Identify forefront scientific challenges,
- Identify those problems that could be solved by high performance comput-
ing at the extreme scale,
- Describe how high-performance computing capabilities could address is-
  sues at the frontiers associated with the relevant scientific challenges,
- Provide researchers an opportunity to influence the development of high
  performance computing as it pertains to their areas of research, and
- Provide input for planning the development of a future high-performance
  computing capability to be directed by the Advanced Scientific Comput-
All of the scientific communities participating in the workshops identified a set of grand challenge problems that could profit from increased computing capability, and all welcomed the possibility of a thousand-fold increase in computing capability over current state of the art. For example, climate researchers identified the issue of integrated model development, that is, the need to simulate the oceans and particularly sea-ice and ocean circulation in interaction with the atmospheric dynamics that drive climate change. Researchers in Basic Energy Sciences identified issues associated with materials science such as computation of excited states and charge transport, the dynamics of strongly correlated systems, and the need to bridge disparate time and length scales in modeling the behavior of materials.

For NNSA, the main goal is the use of modern computation in support of stewardship of the nuclear stockpile. But within this broad area are fundamental issues such as the physics of nuclear fusion, the constitutive behavior of materials at high pressure and temperature, and the properties and chemistry of the actinide elements. All of these problems, which are key to ensuring confidence in the stockpile, require state of the art computational capability.

While all the relevant scientific communities could easily make the case that a factor of 1000 increase in capability would lead to increases in understanding, the challenge of achieving the level of parallelism as outlined above was typically not addressed. It was understood that significant changes in program and algorithm structure may be required in order to keep 1 billion threads of control active, and, indeed, the mapping of the computational patterns associated with various grand challenges is a research challenge in itself. DOE has therefore proposed the Exascale Computing Initiative that seeks to make investments in research and
development that would lead to the introduction of an exascale platform operating within a power budget of roughly 20 Megawatts in the 2020 time frame.

3.4 The 2008 DARPA Study

Several studies have already been carried out on the feasibility of exascale computing. Perhaps the most technically comprehensive of these studies is the 2008 Exascale Computing Study commissioned by the DARPA IPTO Division and the Air Force Office of Scientific Research (AFOSR). This study was led by Peter Kogge and was carried out by a group of experts in high performance computing [27].

The study examined the even more ambitious possibility of building an exascale system in the 2015 time frame and identified four major technical challenges:

Energy and power  A simple extrapolation of computing power requirements led to the conclusion that it would be impossible to achieve the deployment of an exascale platform using present day technology that would require only 20 Megawatts to operate by 2015. The main issue is the cost of moving data from processor to memory or processor to processor.

Memory and storage  The development of memory and storage technology follows a slower growth curve than that of processors with the result that memory capacity and speed have slowed significantly relative to processor capability. The projection to the exascale implies a system with a small fraction of memory capacity and bandwidth to computational capability.

Concurrency and locality  As mentioned above, it will be necessary to maintain something like a billion threads of control to achieve an exaflop. To do this,
it will also be necessary to make sure the requisite data is readily accessible to the computational units. Thus the data must be staged appropriately and the locality of the data must be maintained.

**Resiliency** Because an exascale system will involve on the order of $10^5$ or more processors, hardware failures will occur more frequently than in systems of smaller size. The challenge of resiliency involves understanding the “lifetime” of components and the assessment of the mean time to failure of the system. If such failures are frequent, it will be necessary to have strategies in place to route the computation around such failures so as to ensure the computations complete.

The 2008 DARPA study also emphasized the point that exascale computing is not only limited to the development of systems at the scale of a data center, but encompasses a more general program associated with the ability to further improve the performance of modern computing systems. The issues delineated by this study apply to all scales of computing. For example, there may be requirements in the future to develop at the smallest scale embedded or uniprocessor systems that use massive parallelism for a variety of tasks. This goal is known colloquially as the “teraflop laptop”. A goal that is intermediate between an exaflop computer and a teraflop laptop is the development of a system that provides a petaflop of peak performance but with power requirements sufficiently modest that it could be housed in a small departmental computing facility. At present, petascale systems consume upwards of several megawatts and so this again represents an ambitious goal.

This study examines the evolution of technology four years since the DARPA report, but with an emphasis on how applications might be mapped on the evolving massively parallel architecture. This report also emphasizes some of the soft-
ware challenges associated with programming exascale systems. Owing to the rather compressed time schedule associated with this study, it will not be possible to delve into these topics in as great a level of detail as the DARPA study. Remarkably, all of the observations made in the 2008 DARPA report as regards the state of computing technology remain valid today, and so interested readers should consult this study for a more complete technical assessment.

### 3.5 Overview of the Report

In Section 4, we discuss in more detail the nature of the hardware challenges associated with achieving exascale capability. Our discussion will focus on the evolution of modern microprocessors. We also discuss the emerging gap between processor and memory performance. We assess some of the processing and memory requirements associated with applications relevant to the DOE/NNSA mission. We then discuss the resilience of modern multiprocessor systems. Finally, we examine some of the requirements for archival storage of the datasets that can potentially be generated.

In Section 5 we examine some of the application requirements associated with DOE/NNSA mission needs. It is not possible to cover this area completely, but the brief discussions provided in this section on requirements for applications such as climate simulation and combustion do highlight some key research requirements connected with some of the hardware limitations discussed in Section 4. We also discuss briefly the application requirements of the applications used by NNSA in stockpile stewardship. A more extended discussion of the way in which high performance computing is used in stockpile stewardship is available in a separate (classified) appendix (Appendix C).
In Section 6, we discuss some technology developments aimed at overcoming some of the limitations associated with modern computing hardware. Again, we cannot be comprehensive in our coverage; among the areas discussed briefly are the development of new memory architecture and the use of photonics to aid in the efficient movement of data. We do discuss extensively the idea of using motifs of high performance computing as a way of understanding memory access patterns and communication overhead. These motifs (called “dwarfs”) may be a useful organizing principle for both processor architecture and software abstractions aimed at optimizing performance.

In Section 7, we discuss some of the issues regarding design of software for HPC applications. Traditionally, this has been done using the message passing interface (MPI) in which messages to processors are constructed and transmission and reception of these messages is actively managed by the programmer. It is projected that, given the need to manage 1 billion threads, such an approach may eventually be unworkable. Some alternatives include the use of domain specific languages as well as the use of software auto-tuners to optimize the use of computational resources.

Finally, in Section 8, we conclude with some observations about co-design, the proposed process by which hardware designers and application developers engage in collaboration to influence future hardware design options. We then conclude with some recommendations.
4 HARDWARE CHALLENGES FOR EXASCALE COMPUTING

In this section, we describe some of the evolutionary changes in hardware for high performance computing (HPC). We then describe how recent hardware trends pose challenges associated with developing hardware for exascale computation.

4.1 Evolution of Moore’s Law

The exponential increase in computing capability has been enabled by two technological trends: Moore’s law [31] and Dennard scaling [18]. Moore’s law refers to the observation by Gordon Moore that the number of transistors on a microprocessor essentially doubles every 18–24 months. Shown in Figure 4-1 is the number of transistors associated with various processors manufactured between 1971 and 2011 demonstrating that this remarkable trend has held up for more than thirty years.

Dennard scaling refers to the ability to increase clock speed while decreasing processor feature size. It was realized by Dennard and others in 1995 that it was possible to reduce the feature size of a microprocessor by a factor of two (thus quadrupling the number of transistors on a chip) while also decreasing the processor voltage by a factor of two. This also had the salutary effect of reducing energy utilization by a factor of 8, since the energy scales with the capacitance of a device and the square of the voltage. It was then possible to double the speed of the processor by doubling the clock rate. The power consumption for the processor would remain the same as that for a processor with the lower clock rate and larger feature size, and so one would gain increased speed for the same processor.
Figure 4-1: Number of transistors on a microprocessor as a function of time. The trend follows an exponential with a doubling time of roughly 24 months [31].

size. In this sense, it is possible to increase processor capability by a factor of 8 for the same total power. The evolution of processor voltages with time is shown in Figure 4-2. It can be seen that starting in 1995, rail voltages were decreased from about 5 volts over ten years down to about 1 volt or slightly below [27].

However, two physical issues eventually limited the ability to continue Dennard scaling. First, while total power is preserved as one halves the feature size and doubles the clock rate, the power density (power per unit area) increases by a factor of four. At a power density of 100 watt/cm², thermal management of the processor via air cooling becomes increasingly difficult, and one must then resort to more exotic (and more expensive) cooling technologies. As a result, pro-
Figure 4-2: Evolution of processor voltages over time for a variety of microprocessor types. [27]

Processor manufacturers have not increased clock speeds past 3–6 gigahertz so as to preserve the use of air cooling. The evolution of processor clock rates is shown in Figure 4-3. As can be seen in the Figure, these have plateaued and are not expected to increase.

The second issue is associated with transistor leakage current. As a transistor shrinks in size, the oxide layer used to form the insulating layer also shrinks and this creates a larger sub-threshold leakage current, with the result that the switching characteristics of the transistor become unreliable. Because of this, as processor sizes have decreased, it has not been possible to further decrease power supply voltages. This trend is also shown in Figure 4-2. The overall effect of these trends is shown in Figure 4-4 which shows number of transistors, clock rate and power plotted together. There is clear knee in the clock and power curves owing to the issues described above. As a result of these issues, the overall instruction level parallelism of the processor has also flattened since it is now no longer possible to execute an instruction in a shorter time.
While it has been possible to continue increasing the number of transistors on a microprocessor as per Moore’s law (as clearly borne out in Figure 4-1, as of 2004 this is now done in a different way. Processor manufacturers now lay out multiple processor elements on the chip as shown in Figure 4-5. These elements are called processor cores and can independently execute the full instruction set of the microprocessor. The cores can communicate via message passing over a network or through shared on-chip memory. Typically, the cores possess their own local memory hierarchy in the form of cache memory, but will also have to perform read and write operations to off-chip memory which is typically Dynamic Random Access Memory (DRAM). It is anticipated that in the absence of new processor technology, the future development of modern microprocessors will proceed through an increase in the number of cores. At present, the cores on mainstream microprocessors are identical, but it is envisioned that heterogeneous microprocessors will become available in which various cores perform specific functions such as I/O, security, etc.
Figure 4-4: Evolution of Moore’s law. The top curve shows the total number of transistors on a microprocessor as a function of time and the trend continues to follow Moore’s law. The second curve shows that clock speed has flattened. The third curve indicates that this was done to keep power levels low enough for cooling purposes. The final curve shows instruction level parallelism because clock speeds have flattened, instruction level parallelism has also flattened [34].

### 4.2 Evolution of Memory Size and Memory Bandwidth

While the number of cores on each processor is increasing, the amount of total memory relative to the available computational capability is decreasing. This is a curious measure, but relates to how much memory is available for a given performance level. In the past, memory technology scaled in a similar way to processor capability, and so it was possible to provision one byte or more of avail-
able memory for each flop of processing capability. At around the same time as the transition to multi-core architecture, the ratio of available bytes to flops began decreasing. This is shown in Figure 4-6. Prior to 2004, the memory size was comparable to the number of flops and the ratio of bytes to floating point capability as measured in flops hovered around and in some cases exceeded one. As of 2004, the ratio dropped (note that the figure uses a log scale) and is now edging close to 0.1 for heterogeneous (i.e. multi-core) processor architectures. If this trend persists (and with current technology it is expected to worsen), it will have an important impact on the type of applications which can be run on machines that use traditional DRAM. We discuss this further in Section 5.

Memory capacity using traditional DRAM technology turns out to be a matter of cost. As shown in Figure 4-7, the cost of memory has not been decreasing as rapidly as the cost of floating point performance. From the point of view of
Figure 4-6: Evolution of the ratio of total memory capacity to floating point performance. The vertical axis units are bytes/flop. Prior to 2004 it was possible to provision 1 byte per flop. Recently this has dropped to less than 0.10 byte per flop [27].

the amount of memory resident on single processor, enormous advances have still been made. Today, DRAM costs about $5 per gigabyte and so the cost of provisioning say a 32 gigabyte memory for a personal computer is not prohibitive. However, if one desires a ratio of one byte per flop for an exascale machine, this will require an exabyte of memory and at today’s costs this is will be $5 B. It is anticipated that while memory costs will decrease, extrapolations using the JEDEC memory roadmap still indicate a cost of perhaps $1 per gigabyte or more in the 2020 time-frame [53]. As a result, even in 2020 an exabyte of memory will cost on the order of $1B. Typical budgets for DOE/NNSA high end computer systems are on the order of $100–200M, and so using current memory technology, it will only be possible to provision roughly 200 petabytes of memory at best. More
Figure 4-7: Evolution of memory cost as a function of time. Also shown is the evolution of floating point cost for comparison. It is seen that floating point costs have decreased far more rapidly [48].

realistically, a memory size of 50 petabytes or so is envisioned.

One might argue that improvements in technology will lead to higher memory densities and so the 4 GB DRAM of today would evolve into the 1 Terabyte memory of 2020. Indeed, memory density has increased over time as shown in Figure 4-8. However, the rate of increase of memory density has never been as rapid as that of Moore’s law for the number of transistors on a processor. As shown in the Figure, memory density increased at a rate of 1.33 MBit/chip/year from 1987 through about the year 2000. Afterwards however, the rate slowed to 0.66 Mbit/chip/year. Today, it is possible to purchase 4Gb on one memory chip. If current rates of increase hold, it will not be possible to have a terabit memory chip until perhaps 2034.
Memory bandwidth, or the ability to move data from memory to processor registers is also a key concern. The memory system associated with a modern microprocessor is provisioned hierarchically. For processors, one wants rapid instruction execution, while for a memory one wants high density to maximize data available to the processor. As a result, as processors have become more capable and can execute instructions more rapidly, a performance gap has developed in that the access times for memory, while decreasing, have not decreased as rapidly as processor cycle times. This is shown in Figure 4-9. In 1970 the access times for DRAM and processor cycle times were comparable, but by 1990, as chip designers discovered the benefits of decreasing feature size, lowering voltages, and increasing clock speed, processor cycle times dropped dramatically. As a result, the ratio of DRAM access time to processor cycle time began to increase. By 2007, the ratio increased to several hundred to one. Processor designers call this
problem the “memory wall” or the “von Neumann bottleneck”, since it was von Neumann who first developed the idea of an independent processor that retrieved its instructions and data from a separate memory.

Processor designers addressed this issue by designing hierarchical memories to mask the memory latency. In addition to the main memory in the form of DRAM, modern processors possess memory caches which can store data from DRAM so that future requests for that data are readily available. Cache memories located on-chip are typically built from static RAM or SRAM. This type of memory is constructed from transistors and is very fast, but it has the lowest data density. It is also more susceptible to radiation induced upsets and so must also be designed with error correction logic. In contrast, DRAM cells use capacitors for storage and transistors to move the charge onto an accessed bit-line. This
results in a very dense memory, but compromises must be made here too. For example, memory access produces not just one desired word, but a whole line of memory which can be a thousand or more words. For these and other reasons, memory access times from DRAM have not decreased as rapidly. This is not to say that one could not build a faster DRAM. Indeed there are several promising approaches such as embedding the DRAM in the processor, or altering the DRAM core to tailor the amount of data that is retrieved. However, all these concepts impose a penalty on the die size, and, typically, vendors have not embraced these ideas because providing high memory density is the dominant driver in commodity computers.

Given the hierarchical nature of processor memory, it is preferable to find a piece of required data in the cache where it can be accessed more rapidly. Data is transferred to cache from main memory in blocks of fixed size called cache lines. When the processor needs to read or write a location in main memory, it first checks to see if the memory address or one associated with the cache line is available. If the data is present this is called a cache hit. If not, it is a cache miss; a new entry is allocated in the cache, and the required data is read in from main memory. The proportion of successful cache accesses is called the cache hit rate, and this is a measure of effectiveness for the way in which a particular algorithm utilizes the processor. Processors use caches for both instruction, data and also address translation.

Cache misses are characterized by how the miss occurs:

**Compulsory misses** These occur whenever a new piece of memory must be referenced. The size of the cache or the way the cached data is associated with main memory will make no difference here. An increased cache block size can help in this case, as there will then be a higher probability the required
data is cached, but this is algorithm dependent. The best approach is to prefetch the data into the cache. This requires active management of the cache which is typically left to the compiler. Even prefetching has limits if a large amount of data is required.

**Capacity misses** Capacity misses occur because the cache is simply too small. If one maps the capacity miss rate vs the cache size one can get a feel for the temporal locality of a piece of data. Note that modern caches are typically always full and so reading a new line requires evicting an old line of data.

**Conflict misses** This is a miss that occurred because the required data was already evicted from the cache. Some of this can also be dealt with through the provision of larger caches, but because this entails a penalty in terms of silicon area and performance relative to the provision of functional units, it is not generally cost effective to increase the cache size.

A cache miss from an instruction cache will cause the most delay as the thread of execution must halt until the instruction is retrieved. A cache miss from a data cache may not be as detrimental as instructions that do not depend on the needed data can be executed instead until the required data arrives. This “out of order” execution strategy was successful in several processors but is now considered to be of limited utility because processor speeds have grown faster than memory access speeds.

### 4.3 Memory Access Patterns of DOE/NNSA Applications

From the discussion above, it can be seen that the overall performance of a given application will depend on the amount of memory required to perform the operations associated with a given algorithm as well as the access pattern associated
with that memory. As a first step, it is important to understand what percentage of operations are related to memory access in a given DOE/NNSA application. The suite of possible applications is of course very large, but a representative sample has been studied as briefed to JASON in [32]. The percentage of various types of operations executed in these codes has been examined and is shown in Figure 4-10. DOE/NNSA applications are generally viewed as floating point intensive, as a large part of the workload pertains to the numerical solution of partial differential equations (PDEs). Remarkably, the number of floating point operations in the execution of this particular suite of DOE/NNSA applications is only approximately 10% of the total. Memory operations account for 40%, integer operations account for another 40%, and branch operations for the remaining 10%. Of the integer operations which account for 40% of the total number of instructions, roughly 40% of these arise from the calculation of the addresses of floating point data. Only 10% of integer instructions are used for actual integer computation.

A much more detailed look is provided by the (very busy) graph shown in Figure 4-11 as developed by J. Vetter and his colleagues [49]. In this Figure, the
resource attributes of a number of applications relevant to the DOE/NNSA workload are plotted as percentages. Overall, these statistics validate the distribution of operations shown in Figure 4-10. As can be seen, roughly 35% of the operations of most of the codes are memory operations. While some of codes do exhibit a significant percentage of floating point intensity, the majority do not. The strong floating point peaks originate from codes such as HPL which implements the high performance LINPACK benchmark [36]. This code performs an LU decomposition of a random matrix, and is tuned to minimize memory references while maximizing floating point throughput.

Another view of this distribution of memory vs. floating point operations is seen in Figure 4-12. Here we plot the results of memory operations, floating point operations etc. by examining the distribution of the percentages for each performance attribute that has been recorded in [49]. As can be seen, the median
fraction for memory operations is at roughly 35%, indicating half of the surveyed applications exhibit memory access intensities at 35% or more. The situation for floating point operations is even more striking. The median for both floating point operations and floating point single instruction multiple data (SIMD) operations is quite low. Thus the applications associated with the DOE/NNSA workload require significant memory references relative to floating point and so their performance is very sensitive to the characteristics of the memory system such as memory bandwidth. This occurs for several reasons. Compilers are not always able to optimize floating point throughput without assistance from the programmer. In addition work is required on the organization of the data flow of the applications so as to understand if alternate approaches can lead to better performance. We discuss this further in Section 4.4.

Because of the hierarchical nature of the memory system on a modern microprocessor, there is a significant performance penalty if a required piece of data is not available in the various caches of the microprocessor. In this case, the data, in the form of a cache line, must be requested from main memory, and if no other productive work can be performed while this access takes place, the processor will stall. An interesting study of this issue was undertaken by Murphy et al. [33], who examined the the implications of the working set size on the design of supercomputer memory hierarchies. In particular, they compared the working set sizes of the applications in the Standard Performance Evaluation Corporation (SPEC) floating point (FP) benchmark suite to that of a set of key DOE/NNSA applications run at Sandia National Laboratories. The type of computations performed by these applications are quite typical of the workload associated with DOE Science applications as well as NNSA stewardship applications. They include

36
• LAMPPS – a classical molecular dynamics code designed to simulate atomic or molecular systems,

• CTH – a multi-material large deformation shock physics code used at Sandia to perform simulations of high strain rate mechanics, and

• sPPM – a simplified benchmark code that solves gas dynamics problems in 3D by means of the Piecewise Parabolic Method [11].

The methodology used in this study is to extract an instruction stream of about four billion instructions from each of these codes. Care was taken to ensure that the instructions were associated with the core computational aspects of each of the applications. To determine the working set miss rate of a given application, the authors simulated a 128 MB fully associative cache using a least recently used (LRU) cache eviction strategy. During each load or store operation in the instruction stream, the cache list is searched for the requested word address. If the entry is found (a hit) a hit counter for that block is incremented. That entry is then promoted to the head of the cache list so that it becomes the most recently used item. By varying the working set size, that is, the list of cache entries, it is possible to examine the memory requirements for a given application. Note that this approach measures what is called the temporal working set size miss rate; miss rates here refers only to temporal locality as opposed to spatial locality. It is possible that more sophisticated caching or prefetch schemes that take spatial locality into account could reduce miss rates. As the working set size increases, the probability that the required datum will be found increases, and the miss rate decreases. But it will eventually plateau at a level where further increases in working set size (up to 128 million blocks) will not reduce the miss rate, and when this point is reached one is measuring the compulsory miss rate. In this case, the required data for this maximum working set size must be fetched from main memory.
Figure 4-13: A measure of the bytes required per flop for a variety of Sandia applications. Note that the vertical axis label of bytes/flop now refers to required memory bandwidth [33].

Once the miss rate and flop rate are measured, it is possible to infer a memory bandwidth requirement by dividing the cache miss rate by the number of flops required to perform a given computation. This byte to flop ratio\(^1\) is indicative of the memory bandwidth required. For example, if this rate is less than one, then less than one byte per flop must be accessed from main memory. This is a favorable situation for a modern microprocessor, because it indicates the computation has high arithmetic intensity; there is significant use (and reuse) of the bytes accessed from memory into the caches. In this case, we can expect the floating point units to perform at near optimum rate. On the other hand, if this ratio is greater than one then it implies that the computation will be limited by the bandwidth associated with access from main memory. The results for the Sandia applications are shown in Figure 4-13. It can be seen that even with a cache size of 128M words, the byte

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\(^1\)Not to be confused with the memory capacity to flop ratio discussed earlier.
to flop ratio never goes below one. For applications that perform regular memory accesses like sPPM or CTH, we see that, eventually, we hit a plateau of 3-4 bytes per flop. But this only happens at a cache size of 100 kB or so. Modern level 1 caches are typically 64 kBytes in size. Level 2 caches are generally bigger, up to several megabytes. On a typical multi-core processor, however, only a level 1 cache is available to each core. Access to caches at level 2 and above takes place via a shared memory bus.

We emphasize that this analysis is not definitive for two reasons. First, it does not completely take into account the role of prefetching of data from memory to a level 2 cache. Secondly, the binaries that are run in this analysis are not hand-optimized, and it may be possible to improve this byte to flop ratio by applying techniques to manage cache affinity. The authors in reference [33] also applied this analysis to the SPEC-FP benchmark and showed that the byte/flop ratios for these applications are smaller than those for the Sandia suite. One can conclude, therefore, that the DOE/NNSA workload may require greater memory bandwidth than conventional floating point intensive applications, but this requires further systematic assessment. A more quantitative view of the memory bandwidth issues is presented in the next section.

### 4.4 The Roof-Line Model

A useful framework for understanding the performance of scientific applications on a given hardware platform is the “roof-line” model [54]. The essence of this idea is that applications with sufficiently high byte to flop ratios, as discussed above, will be limited in the floating point rate they achieve by the memory bandwidth of the system. Figure 4-14 shows the roof-line model for a generic computer that is characteristic of today’s commodity platforms, with a peak floating point
Figure 4-14: Roof-line model for a generic 2012 commodity computer, with 100 Gflop/sec peak floating point performance, and 20 Gbyte/sec memory bandwidth. The solid curve represents the maximum achievable floating point performance of an application as a function of its arithmetic intensity (flops-to-byte ratio). Below 5 flop/byte, an application is memory bandwidth-limited, as shown by the sloped line. Above 5 flop/byte, an application can achieve the peak floating point performance provided by the CPU. The dashed line shows a hypothetical scientific application at an arithmetic intensity of 0.5.

capability of 100 Gflop/sec, and a peak memory bandwidth (between DRAM and CPU) of 20 GB/sec. Plotted in the figure is the maximum achievable floating point performance as a function of the arithmetic intensity (flop-to-byte ratio, where “byte” refers to a byte of memory read from DRAM) of an application running on this hardware.² At high arithmetic intensity (> 5), there is sufficient memory bandwidth to keep the processor fed, so the peak floating point rate of the CPU is achievable. At low arithmetic intensity (< 5) there is not enough memory bandwidth to keep the floating point units busy, so the maximum achievable floating

²Note, the following discussion uses the metric of flops to bytes rather than bytes to flop. The former is the right metric to use when there is good reuse of cache data.
Figure 4-15: Roofline model for a generic commodity computer in 2020 (solid line) and 2012 (dashed line). Memory bandwidth is assumed to double every 3 years, while floating point performance is assumed to double every 1.5 years. Note that the ridge point moves from 5 flop/byte to 32 flop/byte.

Point rate is limited by the memory bandwidth, as indicated by the sloped line. The point on this figure where the memory bandwidth-limited floating point rate meets the peak floating point rate is known as the “ridge point,” and occurs at an arithmetic intensity of 5 for this generic processor. In other words, to reach peak floating point performance on this hardware requires an application that can perform 5 floating point operations for every byte of memory read from DRAM. Scientific applications of interest to DOE or NNSA span a range in arithmetic intensity, but rarely have intensities above 1 [33]. The optimistic value of 0.5 is shown in the figure as a vertical line. For such an application on this hardware, the maximum achievable floating point performance is merely 10 Gflop/sec, a factor of 10 below the peak performance of the CPU of 100 Gflop/sec.
There is every indication that the trend of CPU performance doubling every 18 months (via Moore’s Law) [31] will continue; as indicated previously, this performance increase is realized today by an increase in the number of processor cores instead of an increase in the performance of an individual core, but this detail is not relevant to the current discussion. The trend for memory bandwidth improvement, however, has been the subject of much less focus. Over the past decade, memory bandwidth to CPU has doubled approximately every 3 years [38], and current indications are that the growth rate will decrease absent new developments.

Making the optimistic assumption that memory bandwidth trends will continue (at a doubling every 3 years), and assuming that CPU performance will continue to follow Moore’s Law, we plot in Figure 4-15 the roof-line model for a commodity system in 2020. For reference, we include the roof-line model for today’s hardware as the dashed line. By 2020, CPU performance gains will have outpaced memory bandwidth gains to such a degree that to reach peak floating point performance on our representative system will require something like 32 flops per byte in arithmetic intensity! Our reference application with an arithmetic intensity of 0.5 will reach a mere 63 Gflop/sec in floating point performance on a CPU capable of a peak floating point rate of 4 teraflop/sec. This corresponds to an efficiency of only 1.5%!

Yelick and her colleagues at Berkeley have applied the roof-line model to DOE/NNSA applications [57]. In Figure 4-16, we show the roof-line curve for the Intel Xeon 550, a commercial processor used in workstations and servers. The Xeon processor has a peak speed of about 256 gigaflops per second using single precision arithmetic. The memory system for this processor is such that it requires an arithmetic intensity of about 4 flops per byte in order to realize this peak per-
Figure 4-16: Roofline results for the Xeon processor [57].

formance. The colored regions correspond to several types of applications that use the same computational patterns as DOE/NNSA applications. For example “SpMV” in the Figure stands for a sparse matrix vector multiply, an operation that is relevant for example to finite element analyses. As can be seen, the efficiency for this pattern is quite low, ranging from 2 gigaflops at the low end to 8 gigaflops at the high end; the memory bandwidth achieved is quite low, with a flops to byte ratio less than one in all cases. This is to be contrasted with the DGEMM application corresponding to a full matrix-matrix multiply. This corresponds to a different computational pattern, and the algorithms for matrix multiplication of full matrices allow for significant reuse of cached data. This type of calculation is often used to characterize the peak floating point performance of a modern processor. The other colored regions represent applications that have computational patterns that do not perform optimally given the roof-line limits, presumably because traditional memory caching strategies are inadequate.
In Figure 4-17 we show the same roof-line curve but for a modern graphics processing unit (GPU), the NVIDIA Fermi C2050. A GPU can provide significant performance improvements over a traditional processor provided the flop to byte ratio is sufficiently high. For example, the dense matrix-matrix multiply is accelerated by almost a factor of four over the Xeon processor. In contrast, applications like the sparse matrix-vector multiple do not show appreciable speed-up. Again, the issue is to be able to either cache memory effectively or apply an application-specific prefetching strategy that can hide the latency of the required memory accesses.

Extrapolating current trends, it is clear that commodity hardware is changing in a way that will continually reduce the efficiency of existing science applications. The question, of course, is “What can be done?” The roof-line model shown in Figure 4-15 suggests possible solutions. Since processors are currently
limited in terms of memory bandwidth, and will certainly be so in 2020, one could imagine attempting to influence vendors to design higher bandwidth CPU–DRAM interconnects. One difficulty faced here is that memory bandwidth is approximately proportional to the number of leads coming from the CPU package, and we are currently facing physical constraints in increasing this number. Another difficulty with increasing memory bandwidth, as we discuss below, is that the energy cost to move data (the dominant energy cost in scientific calculations) at exascale may greatly exceed a reasonable power budget.

Another possible solution suggested by Figure 4-15 is to increase the arithmetic intensity of DOE/NNSA science applications. This can be achieved in one of two ways. First, by optimizing and tuning a code, one can sometimes increase the flops-to-byte ratio without changing the underlying algorithm. This typically takes the form of structuring memory accesses to increase the cache hit rate, and usually leads to modest increases in arithmetic intensity, although, to our knowledge, a thorough study of the potential for this approach has not been undertaken for DOE/NNSA applications. In Section 7, we describe some techniques for automating and simplifying this process. The second method for increasing the flops-to-byte ratio is to modify the underlying algorithm itself. This may be possible in some cases, but the path forward is not clear for all applications associated with the DOE/NNSA workload. This is clearly a research priority. It is interesting to note that the development of capable hardware can make it possible to use algorithms which were previously not thought to be suitable. For example, while the idea of the fast Fourier transform (FFT) was understood some time ago (possibly by Gauss), it was only the invention of the digital computer which made it a revolutionary advance as pointed out by Cooley and Tukey [13].
Although our discussion of hardware so far has been kept rather simplified, it should still be clear that the scaling trends do not favor scientific applications if the flops to byte ratios are indeed as low as indicated in the previous section.

4.5 Energy Costs of Computation

An additional major technical issue in realizing exascale computing is the cost of energy for computation. It is not hard to see that energy cost is a potentially significant issue. For example, the IBM BG/P computer recently installed at the Lawrence Livermore Laboratory achieves a LINPACK benchmark speed of 16 petaflops and uses roughly 8 megawatts of power. If one envisions achieving an exaflop by simply scaling up this technology, such a computer would require 400 megawatts to operate. At current rates for power this would cost $400M per year assuming it were possible to deliver 400 megawatts to the data center housing the computer.

Shown in Figure 4-18 are the energy costs of various basic operations as measured in picoJoules on a 64 bit word. The gray curve represents the energy costs today. The blue curve represents projected energy costs for these operations in 2020. For example, a double precision floating point operation today requires about 25 pJ of energy. By 2020, as feature sizes for microprocessors shrink further, it is expected that such an operation will require only 4 pJ. The costs for register access are even less, and the costs for accessing an 8 kB SRAM are comparable. This is because all such operations take place close to the functional units of the processor, and as mentioned earlier, SRAM memory is built from transistors and is designed for rapid access, but has low density relative to DRAM.
Other operations that require communication over the processor must factor in the cost of signaling plus the cost of performing the operation. For example to communicate a 64 bit word over a distance of 1 mm across the chip costs roughly 8 pJ per mm of distance. This is due to the resistance that must be overcome in performing the signaling. Note too that while a factor of six reduction in energy use is projected for floating point operations by 2020, a more modest factor of two improvement is projected for communication across the chip.

Finally, there is a very large disparity between the energy costs of floating point computation and off-chip memory access. Today, a DRAM access for 64 bits requires 1.2 nJ and this may decrease by a factor of 4 to 320 pJ by 2020. The ratio of energy costs between memory access and floating point today is about 50 to 1 today but is projected to increase to 80 to 1 by 2020.
While the energy costs of individual operations seem quite modest, they become substantial when one contemplates building an exascale computer. A power utilization rate of 1 pJ per second translates into a utilization rate of 1 MW per second if one simply scales up to the exascale. Because the power budget for most modern data-centers is on the order of 20 MW the provision of hardware resources for an exascale machine is constrained not only by technological issues but also by energy utilization.

4.6 Memory Bandwidth and Energy

The memory bandwidth issue is largely one of energy, and, to a lesser extent, pin constraints. Accessing DRAM today requires about 30 pJ/bit, so a 200 GB/sec (1.6 Tb/sec) memory system on a single GPU node consumes about 50 W. This is divided three ways between row activation (bit-line energy), column access (on-chip communication from the sense amps to the pins of the DRAM), and I/O energy (off-chip communication).

There are techniques that can reduce all three components of energy. Row activation energy is high because each row access reads a very large (8K-bit) page. Reducing the page size will linearly reduce this component of energy. For example, using a 256 B page size will save a factor of 32 on row activation with little downside. Column access energy can be reduced by more efficient on-chip communication. On-chip communication energy is currently about 200 fJ/bit/mm and circuits have been demonstrated operating at 20 pJ/bit/mm. Finally off-chip communication using the single transmission line (STL) signaling standard used in DRAM takes about 20 pJ/bit (2/3 of the total energy) and signaling systems with 1-2 pJ/bit have been demonstrated. Based on expected improvements, one can expect total access energy for commodity memory to drop to the 5-10 pJ/bit
Table 4.2: Some point designs for an exascale computer. These estimates use the energy costs of DRAM access and floating point computation and assume the processing of the full memory capacity of the machine.

With today’s DRAM costs and a power budget of 50–75 W for DRAM access, one is limited to 200–300 GB/sec bandwidth. With a drop to 10 pJ/bit in 2020 the energy limit on bandwidth will be 600–900 GB/sec. Pin bandwidth is also a limiter here. One can place 512-1K channels per chip and run them up to 20Gb/sec (perhaps 40Gb/sec by 2020) so the pin bandwidth limit is 1.2–2.4 TB/sec today and is expected to be 2.4–4.8 TB/sec by 2020. With both today’s technology and expected scaling, energy is a bigger limiter than pin bandwidth. Also, one can overcome the pin limit by splitting a processing chip into several smaller chips. To first approximation, the pin bandwidth per chip, which is limited by the escape pattern, under the package remains constant.

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4.7 Some Point Designs for Exascale Computers

Using the energy and cost estimates discussed above, it is possible to make rough estimates of the power requirements for exascale platforms. Table 4.2 uses the memory cost figures in Section 4.2 and the energy figures in the previous section to compute rough power costs and memory costs for various configurations of an exascale system.

One can, in fact, design an exascale system today that has no memory, but provides enough floating point units for an exaflop. Such a machine would have little utility, but can be considered an (extreme) bounding case. Using the estimates above, such a machine would still require 25 MW to power and so could not be built today within a power budget of 20 MW, but could be built in 2020 and would be comfortably within the power budget. If we then ask that the machine provide about 1 byte per flop, we find that cost is an issue as the memory cost alone would be $1 B. Power will also be an issue as the cost of communicating an exabyte of memory to match the flop rate would require an additional 40 MW in 2020. If an application can efficiently reuse the data accessed from main memory, then the power requirements are lessened, since in the same period we do not have to access memory as frequently. In this case, we get close to the power requirements of 20 MW while maintaining a byte of memory per flop. However, memory costs for such a system are still prohibitive.

If one reduces the memory capacity so that the memory to flop ratio is 0.1 (a memory of 100 Petabytes) then memory costs become more reasonable although still quite high. Such a system would also dissipate less power in that a factor of 10 less energy is required to access the entire memory. If the accessed memory can be productively cached, and a ratio of 1 byte to 1 flop can still produce
reasonably sustained performance from the floating point units, then this system is well within our power requirements of 20 MW. However, if we look at current implementations of DOE/NNSA applications as described in Section 4.3 we see that the byte to flop requirements are more like 3-5 bytes per flop. In this case, the energy requirements do increase and eventually exceed our 20 MW limit.

Our crude estimates are roughly in line with more sophisticated estimates outlined in the 2008 DARPA report [27]. In Figure 4-19 we show the energy per flop as measured in present day architectures as well as the extrapolation to future years. Recall that one must be able to bring the total cost for an flop of computing to 20 pJ in order to stay within the 20 MW power envelope. The dots indicate current measurements of various types of present day architectures:
**Heavyweight** These architectures use high power CMOS; an example is the Cray Red Storm machine which uses Opteron processors,

**Lightweight** A typical lightweight architecture would be the IBM BlueGene machines which use low power CMOS,

**Heterogeneous** This type of architecture is typified by the Los Alamos Roadrunner machine which uses accelerators based on IBM’s cell processor to accelerate floating point intensive tasks. Another example would be hybrid machines that use GPU’s attached to general purpose compute nodes.

As can be seen the heterogeneous machines have been the most power efficient to date as they can make use of hardware optimized for throughput of data to the floating point units. It is anticipated that these types of architectures are the most promising in terms of projections of future power usage, but it can be seen that even for these, the achievement of energy utilization of 20 pJ per flop may not occur until 2024.

### 4.8 Resilience

In this section, we examine some of the salient issues associated with the resilience of HPC systems. Recent reports show that hardware errors are the dominant cause of system failures in modern HPC machines [27, 42]. As an example, Figure 4-20 shows a breakdown of root causes for system failures (both soft and hard) seen in the LANL HPC systems observed by [42] over a period of a few years. For these systems, hardware errors account for roughly 2/3 of all system failures. The components that tend to fail most in HPC machines are RAM and CPUs [27]. For example, failure predictions for BlueGene/L anticipated roughly 60% of system failures would be caused by RAM, and approximately 25% would be caused by
either CPUs or I/O ASICs [27]. Although RAM failures tend to outweigh CPU failures in the statistics reported in the literature, we note that there is a selection effect at work, since ECC-enabled RAM can detect and report errors, while CPUs are generally not designed to do so.

Hardware errors can be broadly categorized into three classes [12]: transient, intermittent, and permanent. Transient errors are those that are due to the environment. At sea level, neutrons induced by cosmic rays are thought to be the primary culprit, but alpha particles emitted by radioactive impurities in the electronics packaging may also play a role. Intermittent errors are those caused by marginal or failing hardware. In a typical scenario, a key parameter of a device, through aging, drifts in value and eventually exceeds the device’s built-in margin. One well-known example in the electronics industry is the negative bias temperature instability, in which the threshold voltage of a MOSFET increases with time. A more pedestrian, though very relevant, failure mode is the intermittency of contacts at solder joints. Permanent errors are, just as the name implies, irreversible.
physical changes that render a device inoperable and may be the eventual fate of some intermittent errors, although they can also be caused by some extreme environmental conditions.

Although there is a rich literature on the classification, detection, and causes of hardware errors, it is still not clear exactly what causes the errors seen in HPC-class machines. Although attempts have been made to assess the extant data, raw failure data is sparse. In addition, when failures are recorded, the root cause is often not known (although see [30] for an exception to this rule).

At the component level, however, some trends can be discerned. Transient failures in CPUs are thought to be caused primarily by cosmic ray-induced neutrons which, when they strike a silicon nucleus, produce charged secondary fragments. Those secondary fragments in turn generate excess electron-hole pairs. If those electron-hole pairs are near a p–n junction, the electric field will separate them, leading to a change in voltage. If the voltage is above threshold, an error results [47]. Since the cosmic ray-induced neutron flux is approximately constant over time-scales of months, the error rate should scale with the quantity of silicon in a chip. The scaling with voltage is more complex, but it has been shown that the failure rate increases with decreasing voltage [23]. Since the area of processor chips is now roughly constant with time, and core voltages appear to be leveling off, the transient failure rate for CPUs should be approximately constant across generations. This result is in fact consistent with the rule of thumb presented in [27] that the system failure rate scales with the total number of sockets. An interesting consequence is that as the number of cores per CPU chip is increasing, the failure rate per core should in fact decrease over time. As with CPUs, the failure rates of RAM modules should scale with the quantity of silicon. Some reports

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3 see, e.g., the Computer Failure Data Repository http://cfdr.usenix.org
indicate that the failure rate per DIMM has not increased across generations [41].

Adopting the [27] rule of thumb that the system failure rate scales with the socket count, we can predict future system-level failure rate trends. For clarity of exposition, we will explore two hypothetical exascale systems: one in which RAM causes all system failures, and one in which CPUs cause all system failures. The reason for the simplification is that memory density and CPU performance scale differently with time as indicated previously. In addition, combining the two in a mathematical model is not particularly insightful. We will take the LANL Roadrunner computer as our reference system, with a birth date of 2008, and with a peak performance we define as 1 petaflop (factors of two are irrelevant here). Transistor count doubles every 2 years for CPUs which, as pointed out previously, means core count per CPU doubles every 2 years. Adopting 2020 as our target date for an exascale system, we find that in the 12 years from 2008 the core count per CPU should increase by a factor of $2^{12/2} = 64$. Requiring floating point performance that is $1000 \times$ that of Roadrunner will thus require $1000/64 \approx 16$ times as many CPUs and, consequently, 16 times as many CPU sockets. The growth of socket count for high performance computers over time is shown in Figure 4-21. The failure rate of such a system would be 16 times higher than that of one of today’s petascale machines. For example, if a current petascale machine has a reasonable mean time to system interrupt (MTSI) of 64 hours, an exascale machine would have an MTSI of merely 4 hours!

We turn now to an exascale machine with $1000 \times$ the memory capacity of Roadrunner (even though such a system may be infeasible for other reasons, as described in Section 6). Recent trends show memory density doubling every three years [3]. By 2020, memory density will have increased by a factor of $2^{12/3} = 16$. Requiring $1000 \times$ the memory capacity of Roadrunner will require the number of
memory modules, and hence sockets, to increase by a factor of 64! For a current petascale machine with a reasonable mean time to system interrupt (MTSI) of 64 hours, an exascale machine would have an MTSI of merely 1 hour! Clearly resiliency will be a challenge for exascale.

Issues of resiliency have already had to be addressed for petascale systems, and so there are many strategies already in place to deal with hardware errors. The simplest is perhaps global checkpoint/restart. However, if writing the full memory footprint to disk and then reading it take longer than the MTSI, such a scheme is not a workable solution. The problem could be potentially resolved by placing some NVRAM with fast I/O on each node so one doesn’t have to perform slower disk I/O. Alternatively, it could be handled with a more efficient checkpointing scheme [9]. Modular redundancy at varying levels (e.g., double, triple) has been used effectively, although it incurs a significant penalty in efficiency if used globally. There are some programming tools that allow the user to specify reliability levels for different parts of the computation [24]. It is important to
continue work on software-based mitigation measures to deal with intermittent system failures. However, it is also important that a systematic investigation be undertaken to uncover the root causes of the hardware failures observed in HPC-class machines, and work with vendors to increase hardware reliability.

We describe some potential hardware-based mitigation measures for transient errors. If further DOE-supported investigation reveals that, as expected, cosmic ray-induced neutrons are the dominant source of transient errors, there are avenues to be explored. The cosmic ray flux varies by a factor of $\sim 2$ from the Earth’s equator to the poles [58]. Locating a compute cluster in Miami instead of Albuquerque, although perhaps an unpopular decision, would reduce the transient error rate by approximately one third. The cosmic ray flux also varies with altitude (as ASC Q showed). Locating installations at low altitudes or underground would decrease the error rate. Neutron shielding (e.g., concrete, polyethylene) could also be placed around a compute cluster to reduce neutron flux exposure. Vendors typically perform simulations and experiments to characterize hardware errors [45]. DOE could attempt to influence vendors early in the design phase to produce more error-resistant hardware. This could be realized by adding ECC to chip components or, in some cases, modifying the chip design.

### 4.9 Storage

We describe here some of the issues associated with archival storage for an exascale machine. There are several challenges here. First, check-pointing of the data to archival storage has to be done in a different way than currently performed. The time taken to checkpoint may exceed the MTSI of the machine and so some sort of intermediate buffering (a burst buffer) must be considered. Second, the very size of the required disk array will further exacerbate the power utilization issues
discussed previously. As a result, the archival facility may have to be operated in a rather different way with many of the disks spun down most of the time. Finally, there is the issue of data loss and retention due to disk failure.

4.9.1 Density

Currently, the capacity of hard disk drives ranges from less than 300 GB (for high-end “Enterprise Class” storage) to 3 TB (for consumer-grade storage), with a relatively slow growth in areal density of 20–25% per year [16]. Previously, the storage industry had enjoyed some years of more than 100% growth in areal density per year, which accompanied the introduction of the Giant magnetoresistance (GMR) read head, but subsequent gains have been more difficult. The industry has moved from longitudinal to orthogonal recording, as well as to harder magnetic materials. There is a significant challenge in writing the magnetic media since the field required to write the materials with a smaller domain size is difficult to contain and could affect adjacent domains. A proposal to do what is called shingled recording [2] is expected to yield an increase in density by a factor of 2–4, but at the cost of making the hard disk drive no longer a simple random access device. The expected density gain from shingled recording is also expected to be short-lived [16].

Increases in areal density require harder magnetic materials in order avoid thermal instability. Charap’s recognition of the super-paramagnetic effect [8] was the cause of the most recent large technological shift from longitudinal to orthogonal recording. The next large step in recording density is expected to result from Heat-Assisted Magnetic Recording (HAMR) which uses a laser to heat a small area in order to reduce its coercivity. An alternative to HAMR that is being investigated is Microwave-Assisted Magnetic Recording (MAMR), which instead of a
laser uses microwaves. HAMR will be required to go beyond more than about 1.3 Tb/in$^2$ with the step beyond HAMR and MAMR being bit-patterned media, both lithographic and self-organizing are being considered, but this is not expected until after 2016 [16]. It is believed that areal densities of more than 10 Tb/in$^2$ may be achieved before bit-patterned media will be required.

Thin film magnetic domains have both a length and a width on the platter surface; the width is related to the track density, which in turn is a function of the physical dimensions and the magnetic properties of the read/write head and of the control system. Currently, track density is increasing faster than bit density, resulting in a more square bit aspect ratio. Track density is currently approximately 400,000 tracks per inch (TPI) as of 2012, and is expected to exceed 700,000 TPI by 2016. There is some concern that engineering difficulties may limit areal density to below the atomic densities of 10–50 Tb/in$^2$. For example, a head fly height of 4 nm is required for more than 1 Tb/in$^2$ [16]. But thermal effects related to super-paramagnetism are of much more immediate concern.

In the 2016 time frame, it is expected that a 12 TB hard disk drive will be shipped in a 3.5 inch form factor, and 4–5 TB hard disk drive in a 2.5 inch form factor. A 12 TB hard disk would require 4 platters in a 3.5 inch form factor. It is unlikely that the number of platters will increase significantly since the form factor is now a standard and more platters require more energy. If the trend in areal density continues, then we can expect 24–30 TB hard disk drives to be available by 2020 in a 3.5 inch form factor.

In order to build a petabyte ($10^{15}$ bytes) of non-redundant storage using the current generation of 3 TB hard disk drives, about 333 would be required. An exabyte ($10^{18}$ bytes) would require about 333,000 such hard disk drives, and 100 exabyte file system would require about 3,333,000 such disk drives. The expected
gains in areal density would reduce this by about a factor of ten by 2020. Performance increases more slowly - approximately as the square root of the areal density (it is a function of the bit density and the track pitch).

4.9.2 Power

The power profile of hard disk drives is dominated by the energy required to keep the platters spinning (called rotation), and secondarily by the energy required to move the read/write head (called seek).

The power consumption of current hard disk drives is given in Table 4.3 for standard 3.5 inch hard disk drives and in Table 4.4 for smaller, lower power mobile hard disk drives. These tables include pure-write workloads, a streaming video workload, a mixed read/write workload, and when the disk drive is idle (this excludes explicit sleep states). A typical consumer-grade hard disk drive such as the 2 TB Western Digital WD20EFRX consumes approximately 4.8 W while writing, 3.6 W under a light read workload, and 3.5 W while idly spinning. An “Enterprise Class” hard disk drive such as the Hitachi Ultrastar 15K300 can store only 300 GB but spins at 15,000 RPM and consumes approximately four times the power.

It is unlikely that the platters will spin faster than 15,000 RPM purely for energy reasons, and it is also unlikely that they will spin slower than 5,400 RPM for performance reasons. It is important to remember that the data rate that can be achieved from a hard disk drive is linear in the rotational speed of the platter and grows as roughly the square root of the areal density (linear in the bit density) of the recording. As a result, it is unlikely that the data rates will grow much faster than the square root of the areal density.
Table 4.3: Hard disk drive (HDD) power utilization

<table>
<thead>
<tr>
<th>Model</th>
<th>Size</th>
<th>RPM</th>
<th>Write</th>
<th>Read</th>
<th>Mixed</th>
<th>Idle</th>
</tr>
</thead>
<tbody>
<tr>
<td>WD WD20EFRX</td>
<td>2 TB</td>
<td>5400</td>
<td>4.8</td>
<td>3.6</td>
<td>4.2</td>
<td>3.5</td>
</tr>
<tr>
<td>WD WD30EFRX</td>
<td>3 TB</td>
<td>5400</td>
<td>5.4</td>
<td>4.0</td>
<td>4.6</td>
<td>3.9</td>
</tr>
<tr>
<td>Hitachi 7K1000</td>
<td>1 TB</td>
<td>7200</td>
<td>10.6</td>
<td>8.5</td>
<td>12.9</td>
<td>8.2</td>
</tr>
<tr>
<td>Hitachi 10K300</td>
<td>300 GB</td>
<td>10000</td>
<td>16.4</td>
<td>13.1</td>
<td>15.9</td>
<td>13.4</td>
</tr>
<tr>
<td>Hitachi 15K300</td>
<td>300 GB</td>
<td>15000</td>
<td>17.4</td>
<td>14.8</td>
<td>16.2</td>
<td>14.5</td>
</tr>
</tbody>
</table>

Table 4.4: Mobile HD Power Requirements

<table>
<thead>
<tr>
<th>Model</th>
<th>Size</th>
<th>RPM</th>
<th>Write</th>
<th>Read</th>
<th>Mixed</th>
<th>Idle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toshiba MK5055GSX</td>
<td>500 GB</td>
<td>5400</td>
<td>2.1</td>
<td>1.2</td>
<td>1.7</td>
<td>0.6</td>
</tr>
<tr>
<td>Hitachi 5K1000</td>
<td>1 TB</td>
<td>5400</td>
<td>2.2</td>
<td>0.9</td>
<td>2.1</td>
<td>0.6</td>
</tr>
<tr>
<td>Samsung M8 HN-M101MBB</td>
<td>1 TB</td>
<td>5400</td>
<td>2.7</td>
<td>1.1</td>
<td>2.1</td>
<td>0.8</td>
</tr>
<tr>
<td>WD Scorpio WD7500BPKT</td>
<td>750 GB</td>
<td>7200</td>
<td>3.4</td>
<td>1.4</td>
<td>2.7</td>
<td>0.9</td>
</tr>
<tr>
<td>Seagate Momentus XT</td>
<td>750 GB</td>
<td>7200</td>
<td>3.9</td>
<td>2.1</td>
<td>2.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

It is instructive to do some back-of-the-envelope calculations in order to get a lower-bound on the amount of power that will be required. For simplicity, we consider what is required just to operate the hard disk drives. Let us assume then a consumer-grade hard disk drive such as the Western Digital WD30EFRX, which holds 3 TB of data and requires about 5 W for a relatively intense read/write workload as might be expected in an exascale computer system. The result is $333.33 \text{disks/PB} \times 5 \text{W/disk} = 1666.67 \text{W/PB} = 1.67 \text{kW/PB}$. An exabyte then costs us 1.67 MW, and a 100 exabyte file system costs us 167 MW. A similar calculation using high-end enterprise class storage requires $3333.33 \text{disks/PB} \times$
15 W/disk = 50 kW/PB, 50 MW/PB, and 5 GW for a 100 exabyte file system.

The hard disk drives used in mobile computing are really no better, except that they have a lower-power idle state. Consider the Hitachi 5K1000, which holds 1 TB and requires about 1.5 W for a relatively intense read/write workload. At 1000 disks/PB, an exabyte costs 1.5 MW, which is comparable to the power required by the consumer-grade hard disk drives. We should note that there are three times as many drives required, and so we are likely to see an improvement in performance but also an approximately three times higher failure rate for the system.

Given the projected growth rate of 20–25% in areal density, we can expect to cut the power requirement for a given amount of storage by a factor of ten. It is important to note though that if we reduce the number of hard disk drives by a factor of 10, we will be reducing the performance of the system by approximately a factor of 3. As a result, using consumer-grade hard disk drives we could expect to build our 100 EB file system for about 17 MW, not counting the energy required for the interconnection network which will be substantial. Building the 100 EB file system using enterprise class storage will still require an untenable 500 MW of power.

It is also important to keep in mind that the data storage system is not simply a collection of hard disk drives. It also includes processors, memory and an interconnection network. The I/O nodes that comprise a data storage system are processors with relatively large DRAM memories that are used as caches with high-bandwidth interconnection networks to the compute nodes (and among the I/O nodes). The number of hard disk drives per I/O node will depend on the performance requirements of the system, as well as the interconnection technology employed to the hard disk drives. Depending on the ratio of hard disk drives to
I/O nodes, the power required by the I/O nodes can easily dominate the power requirements of the data storage system. If we assume 30 hard disk drives per I/O node, then the power cost of a modest 150 W I/O node would double the power requirement if we used 5 W consumer-grade drives.

It seems unlikely that the entire file system will be equally busy all of the time. The degree to which this is true depends on the level of declustering that is done, and on the declustering algorithm that is used. Purely random declustering across the file system would mean that every hard disk drive is equally likely to be busy, while other arrangements might result in busy islands or activity that passes through the file system like a wave. In addition to data organization, the amount of power required will be highly dependent on the workload. In many high performance computing applications, the workload is highly predictable (and is often dominated by checkpoint operations that save the state of the computation every few hours). If the workload is light, then it may be possible to spin down large numbers of hard disk drives and put them into sleep mode saving a significant fraction of the peak power required. For example, for a memory checkpoint of 100 PB, it may be possible to only spin-up 0.01% of the hard disk drives (this will depend heavily on the performance requirements, and a higher percentage may be required).

It is insufficient to simply spin down the disk drives after every use, since there is a large spike in power required to accelerate the platter to operating speed. In recent years, there has been work on what are called Massive Arrays of Idle Disks (MAID) [10], with the goal of powering down as many disk drives as possible. These file system organizations have been shown to save significant amounts of power, but their applicability is highly dependent on the workload.
There is excessive exuberance when it comes to solid state memories such as NAND Flash. While the density of flash memories, particularly NAND flash, continues to increase as feature sizes decrease there is some cause for concern \[22\]. As the density increases, the performance, energy efficiency, number of cycles, and data retention time all rapidly decrease.

Flash memory operates by storing bits in memory cells made from floating-gate transistors. The floating-gate transistor traps some number of electrons, which since it is electrically isolated, should not discharge for a long time, perhaps many years. The number of electrons trapped in a modern flash memory cell is approximately 50, and decreases with each generation putting a hard limit on the achievable density \[16\]. There are predictions that NAND flash may not scale beyond a 12 nm feature size.

Flash memory comes in several varieties, including NAND and NOR flash. NAND flash is by far the most popular, and the one which has the highest bit density. NAND flash can store a single bit per cell, called Single Level Cell (SLC), two bits per cell called Multi-Level Cell (MLC) and three bits per cell Triple Level Cell (TLC).

The charge on the floating gate modifies the threshold voltage $V_{TH}$ of the cell. The threshold voltage for a single level cell will be in one of two ranges separated by a guard band. A multi-level cell has four voltage ranges, and a three-level cell has eight voltage ranges and seven guard bands. For each additional bit, the number of ranges doubles and both the ranges and guard bands narrow. As a result, power requirements increase while performance and durability decrease.

The bulk erase operations required for NAND flash stress the gate oxide, which degrades over time and limits the life of the device. SLC can tolerate on
Table 4.5: SSD Power Requirements

<table>
<thead>
<tr>
<th>Model</th>
<th>Size</th>
<th>Gb/sec</th>
<th>Power (W)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Write</td>
<td>Read</td>
<td>Mixed</td>
<td>Idle</td>
</tr>
<tr>
<td>Silicon Power V20</td>
<td>120 GB</td>
<td>3</td>
<td>1.05</td>
<td>0.50</td>
<td>1.38</td>
<td>0.46</td>
</tr>
<tr>
<td>Intel SSD 520</td>
<td>240 GB</td>
<td>6</td>
<td>2.80</td>
<td>1.00</td>
<td>2.20</td>
<td>0.60</td>
</tr>
<tr>
<td>ExtreMemory XLR8</td>
<td>120 GB</td>
<td>6</td>
<td>4.30</td>
<td>0.90</td>
<td>3.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>

the order of $10^5$ erase cycles, MLC can tolerate on the order of $10^4$ and TLC can be as low as $10^3$ erase cycles [22].

The greatest benefit of using solid state memories such as NAND flash is there is no seek time latency, which when coupled with random I/O operations can cost 5 ms or more for each operation. As long as there are sufficient unoccupied memory cells, writing to NAND flash is very fast though energy intensive. In NAND flash, write operations must be preceded by bulk erase operations which are expensive both in time and in energy. As the bit density increases, particularly for MLC and TLC NAND flash, the performance gap with respect to hard disk drives will narrow. Current generation 32 nm TLC has write latencies ranging 1–2.5 ms.

Of course, NAND flash is not the only possible solid state memory technology. There are many other candidates, but none have had the investment of NAND flash and so none can compete either in terms of cost or density. Promising technologies include Phase Change Memories (PCM), memristor-based memories, along with several others. For the near future, say to 2020, it seems that hard disk drives will dominate in terms of cost/byte.
The energy cost of using solid state memories for storage will be even more highly dependent on workload than it is for hard disk drives. In Table 4.5 we have listed some common Solid State Drives (SSD). They tend to be smaller, about 1/10 of the capacity of hard disk drives though that its really a question of cost; more solid state memory can be packed into the same space as the spinning platters and mechanics of a hard disk drive. In reviewing the data in Table 4.5, we can see that reads are very inexpensive in terms of power while writes are relatively expensive, in one case as expensive as a hard disk drive. Consider for example the Intel SSD 520, which has a write cost of 2.8 W and a read cost of 1 W. If we assume that reads and writes are equally likely, then we have \(4166.67 \text{SSDs/PB} \times 1.9 \text{W/SSD} = 7.92 \text{kW/PB}\). That’s 7.92 MW/EB, or 792 MW for our 100 exabyte file system. It seems that we will do better to use hard disks drives for our bulk storage needs.

A likely scenario is a hybrid storage system which has a portion of the file system residing on SSD, while the bulk of it is storage on hard disks. Such a file system would use the SSDs to absorb random writes and reads, and then, once a portion of the file system was quiescent, move it in bulk to the hard disk drives. The choice of which portion of the large array of hard disk drives to store the data would be governed by considerations of both performance and energy costs. For example, data that is unlikely to be needed immediately could be written to an array of hard disk drives that are spun-down once the data has been stored.

It has been suggested that NAND flash, and other solid state memory technologies, could be used on a per node basis to handle the checkpoint operations. This seems like a good use for this technology.
4.9.3 Storage system reliability

A RAID group is a collection of hard disk drives composed of both data disks and parity disks. The most common form is called RAID 5 (block-level striping with distributed parity), composed of reliability stripes where each stripe is composed of $d$ blocks plus an additional parity block. Traditionally, these stripes are placed on $d+1$ disk drives so that, for stripe $s_i$, the parity will be placed on disk $i \pmod{d+1}$. This allows for the failure of any of the $d+1$ hard disk drives and continued operation of the system while the data is recovered. A second failure during hard disk drive failure during the recovery process results in data loss. As the number of hard disk drives increases, the likelihood of failure increases and so multiple parity blocks may be required. RAID 6 (block-level striping with double distributed parity) functions in much the same way, but there are two parity blocks per stripe instead of one. There are other data organization strategies where, for example, the data blocks can be thought of as being organized into a square matrix and one parity is computed on rows and the second parity is computed on columns. For the purposes of this simple analysis we will restrict ourselves to simple RAID 5 and RAID 6.

We can imagine a unit of storage being a RAID group, a group comprised of $d+1$ or $d+2$ (or perhaps even $d+3$) disks together in a tray that operate as a unit and are placed in a rack with many identical units. This is the traditional approach, and works well for medium-scale data storage systems. As the capacity of hard disk drives continues to increase, but the data rates only increase much more slowly, reconstruction in such a RAID array becomes prohibitively time consuming. For example, a 12 TB hard disk drive, such as we may expect to see in 2016 will have a data rate of approximately 200 MB/sec. Recovering the contents of this disk requires reading the other $d$ disks and writing the content to
a replacement hard disk drive. Assuming no other activity in the RAID array, this will require almost 17 hours.

In order to provide acceptable reliability and performance, and in particular reconstruction time, it will be necessary to decluster the RAID groups that make up the file system. In that way, the loss of a hard disk drive will allow the contents of that disk to be reconstructed in parallel across the system. In a declustered RAID array, a large number of hard disk drives is available and each stripe has its blocks distributed among those hard disk drives. In this way, reconstruction of a lost hard disk drive allows for the data from each stripe to be read in parallel and for multiple stripes to be processed concurrently. If there are $n$ hard disk drives available of capacity $C$ and $k$ blocks of size $b$ in the RAID stripe, then concurrency could be of degree $\lfloor \frac{C}{b} \rfloor$ if $n \geq b \times c \times k$ (for $c = 12 \times 10^{12}$ and $b = 2^{12}$, this is potentially a very high degree of concurrency).

Suppose that the choice is made, as it has been in some products, to do random declustering. We must then be concerned whether multiple chunks from a given stripe are placed on the same hard disk drive. This reduces to the question of the probability that given $n$ hard disk drives and a stripe of size $k$, what is the probability of two more pieces landing on any of the hard disk drives. This is given by

$$1 - \frac{k! \binom{n}{k}}{n^k}$$

which can be approximated by $1 - e^{-\frac{k^2}{2n}}$ for large $n$.

Suppose that we have a 10+1 RAID 5 reliability stripe, and we distribute this over 100 hard disk drives. Then, there is an approximately 43% chance, for every stripe, that two or more of its components will be placed on the same hard disk drive using random placement. If we have 1,000 hard disk drives then the chance
of two or more components landing on the same hard drive is just 5%. Increasing reliability by using a 10+2 RAID 6 reliability stripe, we have a 50% chance of multiple placements for 100 hard disk drives, and 6% for 1000.

We can do much better than using random declustering. We can, for example, readjust the RAID groups when a failure occurs while we are awaiting replacement of the failed hard disk drives. For example, we can carve the hard disk drive up into disklets, and through careful placement assure that no two disklets in the same RAID group are placed on the same hard disk drive [14]. A disklet layout is defined by an (almost) $n$-regular graph. We represent the assignment of disklets to disks by coloring the element (vertex or edge) with a color representing the disk. Not every coloring will do, as otherwise a single disk failure might lead to data loss. We color elements with the same color (i.e., collocate disklets on the same hard disk drive) if they are apart from each other in the graph. Distance is defined in terms of walks in graphs. A walk is a sequence of alternating edges and vertices that are adjacent to each other. We define the length of a walk to be the number of elements in a walk minus one. We define the walking distance between two elements as the length of a minimal walk connecting them. Elements in an irreducible failure pattern have to be at walking distance of one from each other. Consequently, if elements colored with the same disk are at least at walking distance two, then no two disk failures can lead to data loss. Coloring the graph subject to this restriction is fairly simple because of the large number of colors (disks). Using a heuristic graph coloring algorithm, an assignment of disklets to hundreds of thousands of hard disk drives can be accomplished in a few seconds [14].

There are two main sources of data loss in magnetic disk storage: catastrophic loss of a hard disk drive and latent errors. An error in the latter category
affects a small number of blocks, but does not affect other blocks on the disk, and is detected only when trying to access a block. Disk scrubbing or intra-disk redundancy can be used to detect these latent errors before they can cause harm. They may be harbingers of trouble yet to come, since it has been shown that hard disk drives that experience scan errors are more likely to fail [43]. For example, drives are 39 times more likely to fail in the 60 days following their first scan error [37].

The amount of redundancy required to reduce the probability of data loss to an acceptable level needs to be estimated. This will dictate the number of additional hard disk drives that must be used for resiliency. We should be cautious in both our estimates and in operational issues. Care needs to be taken to avoid correlated and batch failures [43]. If, for example, a batch of hard disk drives shared a defect that caused a large fraction of them to fail close together in time, data could be lost even though more than sufficient redundancy had been provided assuming independent failure modes.

We will concentrate on obtaining reliability estimates that consider only full hard disk drive failures. The details of the calculations are relegated to an appendix. The results are provided in the table below.

We assume that hard disk drives in 2020 have a capacity of 30 TB and have the same form factor. We will assume that the disks are on average 80% full, to allow for reconstruction to begin immediately following a failure. A hard disk array will be comprised of 500 drives, approximately one rack unit (RU). We will assume that failed hard disk drives are replaced daily (there are arguments for and against replacement, with human error the strongest argument against).
Table 4.6: Mean Time to Data Loss

<table>
<thead>
<tr>
<th></th>
<th>MTTF (hrs)</th>
<th>Replace (hrs)</th>
<th>Group Size</th>
<th>Array Size</th>
<th>MTTDL (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RAID 5</td>
<td>50 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>3.28129 × 10^5</td>
</tr>
<tr>
<td></td>
<td>100 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>1.21211 × 10^6</td>
</tr>
<tr>
<td></td>
<td>150 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>2.95196 × 10^6</td>
</tr>
<tr>
<td></td>
<td>200 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>5.24766 × 10^6</td>
</tr>
<tr>
<td>RAID 6</td>
<td>50 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>1.81176 × 10^9</td>
</tr>
<tr>
<td></td>
<td>100 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>1.44904 × 10^10</td>
</tr>
<tr>
<td></td>
<td>150 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>4.89011 × 10^10</td>
</tr>
<tr>
<td></td>
<td>200 000</td>
<td>24</td>
<td>10</td>
<td>500</td>
<td>1.15909 × 10^11</td>
</tr>
<tr>
<td></td>
<td>50 000</td>
<td>24</td>
<td>20</td>
<td>500</td>
<td>5.39259 × 10^8</td>
</tr>
<tr>
<td></td>
<td>100 000</td>
<td>24</td>
<td>20</td>
<td>500</td>
<td>4.3121 × 10^9</td>
</tr>
<tr>
<td></td>
<td>150 000</td>
<td>24</td>
<td>20</td>
<td>500</td>
<td>1.45511 × 10^10</td>
</tr>
<tr>
<td></td>
<td>200 000</td>
<td>24</td>
<td>20</td>
<td>500</td>
<td>3.44889 × 10^10</td>
</tr>
</tbody>
</table>

A MTTDL of $10^6$ hours (about 114 days) for each array of hard disk drives seem like a satisfactory level of protection, but when we consider that a 100 EB data archive will require more than 7 000 such racks (assuming 10% redundancy) we can expect data loss about every 136 days. This implies that RAID 6 (two distributed parity blocks per reliability stripe) will be required.

4.10 Summary and Conclusions

From the discussion above it is clear that the challenges outlined in the 2008 DARPA report as regards building an exascale computer by 2015 remain valid. Perhaps the greatest challenge comes from meeting the energy and power requirements. As projected in Section 4.7 above, it is possible that these requirements may not be met until the 2024 time-frame. The technological trends associated with the motion of the volume of data on an exascale machine are also not en-
couraging. Extrapolations of traditional processor technology and architecture may simply make it impossible to achieve the very ambitious power and efficiency goals. Memory and storage will also be limited for any future exascale system to be contemplated for the 2020 time-frame. Memory density and bandwidth trends are such that users of exascale systems will have to face challenges of limited memory, and will not be able to scale the data volume for various applications; bandwidth issues may result in inefficient utilization of the computational units of future processors. Resilience remains an outstanding issue, and further research is required to understand the implications. Finally, even archival storage looks different at the exascale; it will be necessary to spin down almost all the drives in the archival disk system in order to ensure that one does not exceed the power budget of 20 MW.

Our findings regarding the issues examined in this section are as follows:

- It is likely that a platform that achieves an exaflop of peak performance could be built in a 6–10 year time frame within the DOE/NNSA designated power envelope of 20 megawatts. However, such a platform would have limited memory capacity and memory bandwidth; owing to these limitations such an exascale platform may not meet many DOE/NNSA application requirements. We discuss this further in the next section where we examine the requirements for a limited set of applications.

- The most serious technical challenge impeding the development of exascale computing in the near term is the development of power-efficient architectures that provide sufficient memory density and bandwidth for DOE/NNSA applications. It is quite possible that meeting these challenges will require a complete rethinking of processor architecture, at least for those portions of the hardware that will be needed to address the DOE/NNSA workload.
5 REQUIREMENTS FOR DOE/NNSA APPLICATIONS

Having examined some of the hardware challenges associated with exascale computing, we next examine some of the application requirements. We examine in this section two applications that are of interest in light of the above discussion. The first is climate modeling with the objective of resolving scales of motion that to date cannot be accessed via present day capabilities. The second is high pressure combustion with a similar objective, that is, attempting to include more physics at scales that are typically modeled today.

To be sure, this selection of applications is highly idiosyncratic and basically reflects the authors’ fields of expertise. A more thorough study of this type would include other important applications such as those occurring in materials science, astrophysics, chemistry, biology, and other fields of inquiry where high performance computing has played an important role.

5.1 Climate Simulation

We begin by considering an attractive target for exascale computing: the simulation of atmospheric dynamics, particularly climate and weather, at a fine scale of resolution. Accurate modeling of climate and weather is a pressing computational challenge. To date, most of the assessment of climate and weather is performed using a resolution of 200 km in longitude and latitude. The most ambitious such simulations have resolved the earth at a scale of 25 km. To get a feel for how crude this is, we show in Figure 5-1, in the first two images at left, the level at which topography (expressed as surface altitude) is resolved at scales of 200 and 25 km.
respectively. Modeling at this scale neglects or resolves poorly important moisture transport processes associated with the dynamics of clouds, considered today one of the key uncertainties in climate modeling. In fact, a significant portion of the uncertainty in both short term (weather) and long term (climate) prediction is associated with the need to adequately represent moisture transport and in particular formation of cloud cover.

Wehner et al. [51] have considered the requirements for running climate and weather simulations that are resolved down to 1 km. This resolution is depicted as the right-most image in Figure 5-1. At such a resolution, important features such as orography become very clear. Their study does not consider the needed improvements in the modeling of cloud physics that must be included in any simulation at this scale. But it examines requirements for simulating the climate and weather using an existing and established modeling system: the Community Atmospheric Model (CAM) developed at the National Center for Atmospheric Research (NCAR).
For climate studies, two types of simulations are carried out. The first type are long simulations over multiple millenia to establish a base line under various assumptions to see, for example, if any warming trends can be established. The second considers the role of perturbations or forcing on the climate to understand the range of possible effects and their statistics. The typical computational requirement for climate is to be able to integrate roughly 1000 times faster than real time. Thus, a year-long simulation could examine about 1000 years of climate evolution. For weather prediction, the requirements are somewhat less restrictive. Here it would be desirable to complete a 10 day forecast in about one day and so calculation at a rate ten times real time would be sufficient.

The total memory requirements for a calculation at 1 km resolution using CAM are shown in Figure 5-2. The memory requirements assume the use of a longitude-latitude grid with maximum cell size of about 1 km squared or less over...
the whole earth. The calculation is performed using a quasi-two-dimensional approximation of the three dimensional equations of motion with a vertical grid of 100 levels to capture vertical dynamics originating from the surface of the earth to the edge of the atmosphere. The desired resolution is relatively insensitive to the way in which the vertical direction is handled in such calculations, and the trends for several approaches are shown in the Figure. In order to perform simulations at a resolution of 1 km, roughly ten petabytes are required. This is well within projections for the amount of provisioned memory on an exascale platform. The relatively modest memory requirements result from the fact that the computation is quasi-two-dimensional. It should be noted that this model still neglects important physical phenomena and so even here a 1 km simulation would not constitute direct numerical simulation of the earth’s climate. Modeling of various sub-scale phenomena remains an important component of such simulations, but the ability to resolve at scales on the order of 1 km is a useful investigation into the limitations of the current models.

The requirements for exascale computation originate from the need to compute at a rate of 1000 times faster than real time. These are shown in Figure 5-3. It can be seen that different modeling approaches imply different floating point requirements. The requirements for several approaches along with measurements for those resolutions that are typically used today are shown in the Figure. Extrapolations of the results indicate that a sustained floating point throughput of 10 to 100 petaflops will be required to integrate the equations for a climate prediction at a resolution of 1 km. For weather prediction, the requirements are 100 times less severe as indicated previously. Because the typical percentage of peak floating point performance realized with today’s modern processors is only typically 5%, the peak required performance for this type of calculation is at least an exaflop.
This calculation could be performed if one scales present day architecture to the exascale level but the energy costs would be prohibitive. Wehner et al. [51] examine the costs and indicate that 180 MW would be required to perform such a computation using high power CMOS processors like those manufactured by AMD or Intel. In contrast, 27 MW would be required for an extrapolation using low power processors such as those introduced in IBM’s BG/L computer. Wehner et al. also examined the use of very low power processors such as those used in today’s mobile devices. We will discuss the results of this study in Section 6.4 later on this report.

The overall conclusion however is that if sufficient progress is made in reducing the energy costs of computing, it will be possible to perform some calculations of interest that exhibit low byte to flop memory bandwidth and total memory requirements on exascale platforms perhaps as early as the 2020 time-frame.
5.2 Combustion

An important class of HPC applications involves fluid flow, or dynamics of matter, in general, where imposed stress fields and other boundary/energy dynamics fluidize the material of interest. In this case, the unsteady three dimensional flow velocity field, \( u(x,t) \) can be specified at each location, \( x \), and time \( t \). In this approximation, the dynamical role of molecular collisions is well represented by molecular-transport processes leading to momentum, temperature, and species diffusion. The latter can be represented by the coefficients of viscosity, \( \mu \), heat conductivity, \( k \), and the \( i \)'th species diffusivity, \( D_i \) respectively. These are thermodynamic functions of \( p \) and \( T \) only, and not flow-dependent. The resulting abstraction is captured by the Navier-Stokes equations for continuum flow. While even with that abstraction one can be led to challenging problems, the attendant reduction in model dimensionality brings such flow problems within the realm of computable physics on HPC platforms, as discussed below.

A very wide class of fluid-dynamics problems is addressable by the continuum approximation and the (implicit) local thermodynamic equilibrium (LTE) assumption on which the attendant constitutive relations for molecular-transport stresses and fluxes rely. Flow regimes for which this is not the case must resort to more detailed descriptions. Even then, however, it is useful to ascertain that errors made by the simpler continuum approximation, whose validity is (empirically) found to extend beyond the regime where it would appear justified by a priori considerations, exceed the inevitable errors in both statistics and representation in the computation of solutions of the Boltzmann equation by Monte Carlo methods or other means, and the associated collision integrals. The latter quickly become intractable at higher fluid densities as the liquid phase is approached, whereas the continuum approximation becomes ever better as fluid density increases. The
level of computational effort in solving for three-dimensional unsteady flow can be estimated a priori by estimating the modal degrees of freedom of the problem.

For the purposes of illustration, we will limit the discussion to incompressible flow, i.e., flow for which the local rms velocity, $u'$, is small compared to the local speed of sound, $a$. More precisely, in terms of the appropriate dimensionless parameter, we’ll consider flow at a small turbulence Mach number, i.e., small $M = u'/a$. Fractional density variations scale as

$$\frac{\Delta \rho}{\rho} \approx M'^2,$$

$$\frac{\Delta \rho}{\rho} < 0.05.$$  

Since this scales local velocity fluctuations, this flow approximation can be assumed valid if shock waves that may arise in supersonic flow, for example, are handled explicitly as dividing dynamic surfaces between almost-incompressible-flow regions. For three-dimensional fluid dynamics, the modal dimensionality can be estimated as

$$N_{\text{mod}} \approx \alpha \frac{L}{\Delta x}^3,$$

where $L$ is the outer scale, e.g., $L = 6m$ for a room-sized domain, $\Delta x$ is the smallest spatial scale in the numerical simulation (grid size), and $\alpha$ is a proportionality constant that depends on the type of discretization of the partial differential equations. The spatial grid size, $\Delta x$, can be no larger than $\lambda_{\text{min}}/2$, where $\lambda_{\text{min}}$ is the smallest dynamical scale in the flow (per Nyquist). In terms of the smallest physical scale $\lambda_{\text{min}}$, we must then have

$$N_{\text{mod}} \approx 8\alpha \left( \frac{L}{\lambda_{\text{min}}} \right)^3.$$  

For unsteady flow, the computational effort is also scaled by considerations of time stepping in the numerics. For flow with a characteristic flow speed $U$, the characteristic outer-flow time is given by $\tau_L = L/U$ and a simulation that captures the statistical behavior of such a flow must simulate it for a total time that is at
least a few times $\tau_L$. For an Eulerian simulation for which the coordinate system is fixed, the simulation time step, $\Delta t$, must be smaller than $\Delta x/U$, with a number of time steps per outer-flow time given by

$$N_t \approx \frac{L U}{U \Delta x} = \frac{L}{\Delta x}.$$  

Hence, the direct numerical simulation (DNS) computational effort (e.g., time required for an exact solution of the partial differential equations of motion) is proportional to

$$T_{DNS} = N_{mod} \times N_t \approx A \left( \frac{L}{\Delta x} \right)^4.$$  

where $A$ is the computational effort per grid point/cell (e.g., the super-set of floating-point operations, memory cycles, data communication time, or, in terms of $\lambda_{min}$

$$T_{DNS} \approx 16A \left( \frac{L}{\lambda_{min}} \right)^4.$$  

Two conclusions immediately follow. For a given computational effort per grid point/cell, a doubling of the simulation spatial resolution, i.e., for $\lambda_{min} \rightarrow \lambda_{min}/2$, increases the computation effort by a factor of $2^4 = 16$. Generally, if HPC resources of a certain capability allow a resolution of the flow to a scale $\lambda_1$, and a higher resolution to a smaller scale $\lambda_2 < \lambda_1$ is required, the increase in the required computation effort is given by $(\lambda_1/\lambda_2)^4$. By way of example, if with a one petaflop HPC resource one can correctly resolve flow down to spatial scales of say, $\lambda_1 = 1$mm, and a higher resolution to $\lambda_2 = 0.01$mm = $10\mu$ is desired using the same computational approach, computational resources are required with a capability that scales across the board (all computational components) that is higher by a factor of $10^4$. That is, this requires a 10 exaflop capability.

Is the need for such spatial resolutions warranted? For the simplest turbulent flows, the appropriate dimensionless flow-scaling parameter is the Reynolds
number,

\[ \text{Re} = \frac{\rho UL}{\mu} = \frac{UL}{\nu}, \]

where \( \nu = \frac{\mu}{\rho} \) is the ratio of the dynamic (shear) viscosity to the fluid density, with units of area per unit time (\( \nu = 0.15 \text{cm}^2/\text{s} \) for air at normal \( T \) and \( p \)). In terms of \( \text{Re} \), the smallest fluid-dynamical physical scale is given by the Kolmogorov scale, which can be expressed as a fraction of the outer scale \( L \), i.e.,

\[ \frac{\lambda_K}{L} = C_K \text{Re}^{-3/4}, \]

with a constant of proportionality, \( C_K \), of order unity. Again by way of example, if a \( d_0 = 1 \text{m} \) diameter duct is discharging air into a room at a speed of \( U = 10 \text{ m/s} \), the relevant Reynolds number would be given by \( \text{Re} = d_0 U / \nu \approx 6.7 \times 10^5 \), yielding a Kolmogorov scale \( \lambda_K / d_0 \approx \text{Re}^{-3/4} \approx 4.3 \times 10^{-5} \), corresponding to a dynamic range of \( L / \lambda_{\text{min}} \approx 2.3 \times 10^4 \), or \( \lambda_{\text{min}} \approx 43 \mu \) in dimensional units, for this quite modest flow. Higher speeds would lead to smaller minimum physical scales yet. Scaling the computational effort in terms of \( \text{Re} \) and identifying \( \lambda_{\text{min}} \) with \( \lambda_K \), we also see that

\[ E_{\text{DNS}} \approx 16A \text{Re}^3. \]

Bona fide turbulence requires minimum Reynolds numbers in the range of \( \text{Re}_{\text{min}} \approx 1 - 2 \times 10^4 \), with values for real/practical turbulent flows of order \( \text{Re} = 10^5 - 10^6 \) and higher frequently encountered, so micron-size minimum spatial scales are not uncommon in high Reynolds number turbulence.

At this writing, spatial dynamic-ranges attained in direct fluid-dynamic simulations (DNS) on HPC platforms in idealized-flow situations and boundary conditions are of order \( L / \lambda_{\text{min}} \approx 4000 \). Flows with realistic boundary conditions that must deal with modest multi-physics elements have reached a dynamic range about half that, or \( L / \lambda_{\text{min}} \approx 2000 \), i.e., about an order of magnitude lower than the simple duct-discharge flow in the example above. A brute-force DNS approach
would then require a HPC capability of order $10^4$ times higher than provided by today’s HPC resources. We conclude that even the level of simplification afforded by the continuum approximation, a wide class of representative fluid-dynamics problems is not within reach of numerical solutions by direct (DNS) methods, even with exascale HPC resources. The fourth-power cost of successive doublings in resolution improvement indicates that computing power, alone, does not offer a promising path. Typical fluid dynamics problems that are coupled with multi-physics phenomena, such as radiation, inhomogeneous density fields and compressibility, chemically reacting flows with local heat release, are even further beyond reach.

Is there, or are there, alternative paths? Is it necessary to resolve the smallest fluid-dynamic scales in all cases? Is it sometimes adequate to model their contribution to the dynamics in some manner? The answer in many cases depends on the question(s) asked. The spectrum of turbulence is such that kinetic energy associated with a particular scale rapidly decreases with decreasing scale size. Capturing, say, 80% of the kinetic energy, or more, can typically be achieved by capturing a large-to-small scale dynamic range 2.5 to 3 orders of magnitude, i.e., the ratio of the lowest ($k_{\text{min}} \approx 1/L$) to the highest ($k_{\text{max}} \approx 1/\Delta x$) wavenumbers resolved. Such simulations are called large-eddy simulation (LES) calculations. Significantly, such a dynamic range can be hosted on petascale HPC resources. Predictive LES calculations rely on explicit or implicit models that capture the contribution of unresolved scales to the overall dynamics, called subgrid-scale (SGS) models. At this writing, SGS models exist that are fluid-dynamically correct for (near-) uniform-density flows, but none exist for non-uniform-density flows, to our knowledge. If the questions rely on processes that predominantly occur at the smallest scales, the onus on correct SGS modeling is greater. Mixing that takes place at (molecular) diffusion scales provides an example, especially as
a precursor to chemical reactions, as discussed below. Its correct quantification is essential in combustion. Advances in modeling SGS dynamics to non-uniform-density flows would place a large swath of fluid dynamics within reach of HPC resources. Such LES-SGS modeling can then provide the backbone to host a variety of multi-physics phenomena coupled to fluid dynamics. As the discussion above suggests, however, absent such progress, increasing HPC capability alone will not get us there. Before ending this section, we note that details of the numerics used to solve the system of partial-differential equations (PDEs) that describe the flow are also important. Representations of derivatives, for example, can lead to numerical dissipation that is, dynamically, indistinguishable from physical fluid-dynamic dissipation (viscous effects, mixing and diffusion, heat conduction, etc.) in the simulation. Such effects can mask the intentional representation of SGS dynamics included in the LES-SGS framework and, even with physically correct SGS models intentionally incorporated, the simulations can be incorrect.

Chemically reacting flows and combustion, pose two different challenges. If the flow hosting the combustion is itself turbulent, its analysis must address and respond to all the challenges described above. In addition, the complexity of detailed chemical kinetics simulations, the effect of heat release at small scales of the flow, and the change in the flow environment that, in turn, dictates the combustion environment must be addressed. To help assess and scale the challenge, we note that simulating fluid dynamics requires the solution of two scalar equations (for conservation of mass and energy) and one vector equation (for conservation of momentum), for a total of five scalar equations, constrained by an equation of state for the fluid and augmented by constitutive relations for molecular stresses and fluxes. If fluid species react, a species-transport/-conservation equation must be included for each one, with a diffusive term that can itself be quite complex; diffusion of a particular species in a mixture depends on the surrounding molec-
ular environment; a process termed multi-component diffusion. Further, detailed chemical kinetics of the combustion of a hydrocarbon fuel, such as diesel, are expressed in terms of hundreds of chemical species and hundreds to thousands of elementary reactions. Given $K$ species, one can expect that the number of elementary reactions would increase combinatorially with $K$, since all possible combinations of reactants can, in principle, participate in such reactions. Detailed chemical-kinetic models of combustion for higher-hydrocarbon fuels, however, typically entail a number of reactions that is a multiple of $K$, on average, about 100 elementary reactions per species. No fundamental theory or methodology exists today to estimate the elementary reactions that participate, or that are important and must be included in a detailed chemical-kinetic model. That aside, with four constants typically needed to describe the Arrhenius form of reaction rates, such models rely on thousands of constants that are, for the most part, poorly known.

It helps to classify combustion into modes. An important classification considers whether the fuel and oxidizer are either perfectly mixed or perfectly unmixed before chemical reactions occur. In the former, we speak of premixed combustion while in the latter we speak of non-premixed combustion. However, depending on the relative magnitude of the mixing-diffusion flow time, $\tau_f$, and the chemical-kinetic time, $\tau_\xi$, combustion will proceed in an environment that is in an intermediate regime, i.e., in neither limit. The ratio of these two times is called the Damköhler number,

$$ Da \equiv \frac{\tau_f}{\tau_\xi} $$

Combustion will occur in thin sheets, or flamelets, for high values of $Da$, or in distributed regions for low values of $Da$. Further, one can be in a high-$Da$ regime for some elementary reactions and a low-$Da$ regime for others, while all reactions participate in the combustion process. Unreacted mixtures can ignite and burn by their own means (autoignition, at high temperatures, for example) or by
an external heat/energy source (ignition). In the second case, called a premixed flame, a thin reactive zone can advance into a region of unreacted species as a wave. For typical hydrocarbon combustion, density decreases by a factor of about 7 across such a flame. Matters are further complicated because, depending on the fluid-dynamic environment, a burning region can be locally extinguished and then reignite, leading to a combustion environment that strongly depends on the Lagrangian history of a fluid/reactant parcel as well as its environment, and is not local in space-time.

To summarize this discussion, the number of PDEs that must be solved for turbulent combustion is one to two orders of magnitude larger than for fluid dynamics alone. The computational effort is typically quite a bit larger as transport equations now have to be solved for each species. Time-stepping becomes very stiff because of the large range in chemical-reaction times, and the spatial resolution required for thin flames can be locally high. As a result, strict DNS of turbulent combustion of hydrocarbons will likely remain beyond reach, even with exascale HPC resources.

A few additional challenges are worth listing. With unknown detailed chemical-kinetic models, at least for higher-hydrocarbon fuels, the equations that must be solved are not known. If the challenge in predicting higher-hydrocarbon combustion is the chemical-kinetic models, it would appear prudent to attempt the simulation in a well-controlled environment that can be diagnosed to the requisite precision and accuracy, so comparisons with numerical predictions can be made. Turbulence with its own challenges, as discussed above, does not appear to offer the most convenient or promising environment. While a set of elementary reactions may offer a complete description, at least over a range of conditions, which of those reactions dominate will depend on the combustion environment. By way
of example, at low pressures, extending to atmospheric pressure, or so, binary reactions may be dominant. At higher pressures where almost all hydrocarbon combustion occurs in practical power-producing devices, three- and four-body elementary reactions may dominate. A validated elementary reaction set at one atmosphere, where most experiments are conducted, offers no assurances of its applicability at higher pressures. We conclude by noting that, even more so than for (pure) fluid dynamics and turbulence, it may be premature to anticipate successful hosting of DNS of turbulent combustion on exascale HPC resources, until the relevant physics can be modeled and represented correctly in such simulations, and a vision for how validation of the simulation of the complex dynamics that ensue has been formulated.

To emphasize the above points further, we can consider the direct numerical simulation of combustion in a gas turbine. Today, using high pressure combustion (with pressures on the order of 40 atmospheres) gas turbines are able to produce
significant power outputs. The scale of such turbines is quite large as shown in Figure 5-4; The turbine shown in the figure is able to output 400 MW of power. In the combustion chamber of the turbine, the Reynolds number Re can reach values on the order of $10^6$ – $10^7$ which indicates a strongly turbulent flow. A direct numerical simulation of a flow such as this down to the Kolmogorov scale will require the ability to resolve a length scale $10^5$ smaller than the characteristic length of the combustor.

The computational requirements for such a calculation given the considerations above are daunting. It is necessary to track the density, the three velocity components, and the energy of the fluid to resolve only the fluid mechanics of the combustion process. In order only to resolve the flow we require $10^{15}$ cells per field. Temporary fields must also be maintained for time integration as well as the computation of the various terms in the Navier-Stokes equations. The number of such fields will depend on the particular advection algorithm employed, but it is conceivable that one would require on the order of 150 petabytes of memory for the fluid mechanics alone. For a proposed exascale platform with a memory of say 50 petabytes (considered a typical target) this calculation cannot be done. Simulation of problems at these Reynolds numbers will still require turbulence modeling.

We have not yet considered the requirements for simulation of the chemistry. This proceeds through the integration of chemical rate equations which describe the various reactions that take place as the combustion proceeds. As discussed above, for complex hydrocarbons, a large number of species must be tracked and so the requirements for storage of the chemical species alone will greatly exceed the number of fields required to store the fields used to describe conservation of mass, momentum and energy. The memory requirements for such a calculation
Figure 5-5: Projected performance of NNSA applications using extrapolations of current hardware capabilities. The vertical axis is in petaflops. The top dashed line represents requirements as projected by NNSA for predictive capability for key phenomena associated with stockpile stewardship. The blue curve indicates projects peak capability. The red region derates this due to memory limitations. The green curve further derates this performance based on memory bandwidth projections [46].

are now well into the exabyte range.

The implications of the discussion above is that some calculations of interest to the DOE/NNSA mission will not be achievable without a larger memory capacity than is currently envisaged, These considerations do not even take into account memory bandwidth issues and energy costs.
5.3 NNSA Applications

We discuss here very briefly the application requirements for NNSA applications used as part of the stockpile stewardship mission. The nature of the codes and the specific applications are discussed in a classified appendix to this report (Appendix C).

NNSA applications are more complex in terms of the simulated physics than the combustion applications discussed above. As discussed in Appendix C, to achieve DOE/NNSA mission needs, continued advances in computing capability to the exascale level and likely beyond are required and will be required for the foreseeable future. However, as also discussed in the classified appendix, owing to the integrated way in which the stewardship mission is carried out with a reliance on test data, experiment and computation, there is no particular threshold requirement for exascale capability in the 2020 time frame as regards those national security issues associated with the DOE/NNSA mission.

Various aspects of the performance of NNSA stewardship applications have been examined in detail. Figure 5-5 shows the evolution of peak performance over time. The dashed curve at the top of the figure indicates desired progress for exascale capability by 2020. The solid blue curve indicates projected peak capability. This is displayed as a range depending on the nature of the application. However, at present because NNSA stewardship applications require a large amount of memory per thread of execution, memory capacity limitations will make it impossible to expose the required level of parallelism without significant restructuring of the algorithms. This is represented by the red set of curves again showing a range of throughputs. Finally, the stewardship applications require at present significant memory bandwidth. Using projections for future hardware this derates perfor-
mance further and is shown as the set of green curves. The implication is that an exascale platform will perform at low efficiency for stewardship applications with a sustained performance no better than several petaflops unless improvements are made in future architectures to address these issues.

5.4 Summary and Conclusion

Our findings as regards application requirements above are as follows:

- It is likely that a platform that achieves an \textit{exaflop} of peak performance could be built in a 6–10 year time frame within the DOE/NNSA designated power envelope of 20 MW. However, such a platform would have limited memory capacity and memory bandwidth; owing to these limitations such an exascale platform may not meet many DOE/NNSA application requirements. This finding is repeated from Section 4.10. It will be possible to accomplish some important computations relevant to the DOE mission but, as exemplified in the section on combustion, there will also be a class of scientifically compelling calculations that will require more memory capacity and bandwidth.

- To achieve DOE/NNSA mission needs, continued advances in computing capability are required and will be required for the foreseeable future. However, there is no particular threshold requirement for exascale capability in the 2020 time frame as regards those national security issues associated with the DOE/NNSA mission. For this reason, JASON does not foresee significant national security impacts associated with a failure to execute the DOE Exascale Computing Initiative by 2020.

- To date, our understanding of the computational requirements for DOE/
NNSA applications, particularly issues of memory bandwidth and floating point throughput, remain murky. At present it is simply not possible to know how DOE/NNSA applications will perform on proposed future architectures. More focus is required to ensure that the hardware and software under development in support of an exascale capability will address performance improvements specific to the communication and computational patterns of DOE/NNSA applications. A coordinated DOE/NNSA effort is required to characterize in a standard way the computational patterns and characteristics (i.e. memory capacity, memory bandwidth, floating point intensity and global communication) of the suite of DOE/NNSA applications. Only in this way will it be possible to understand if traditional extensions of modern microprocessor architecture will make it possible to achieve balanced exascale performance or whether a completely different approach is required.
6 RESEARCH DIRECTIONS

In this section we examine several promising hardware research directions to address some of the hardware challenges discussed previously.

6.1 Breaking the Memory Wall

As has been noted previously, memory bandwidth performance has not kept pace with the increase in speed of modern microprocessors. There are several issues here such as the energy cost of data transport and the fact that DRAM memories are designed for memory density as opposed to low data access latency.

A recent development in memory technology has been the introduction of through-silicon-via (TSV) memories. These memories can be stacked on each other to build a dense memory system and on a processor chip to increase interconnect bandwidth and decrease off-chip communication energy. An example of TSV silicon technology is shown in Figure 6-1.

The JEDEC “wide-IO” memory standard aimed at mobile devices, in particular, cell phones and tablets, is the first of these TSV memories. The JEDEC “high-bandwidth memory” (HBM) (also called graphics wide-IO) is a higher-bandwidth version expected out in a few years. TSV memories have very wide interfaces (up to 1K bits for the JEDEC HBM) and essentially eliminate the off-chip component of energy. These can be stacked on a processor die, but may be more practical stacked next to the die on a silicon “interposer”. Only a few chips – at most 16-32 can be packaged with one processor die, so only a limited capacity (about 16 GB) of high-bandwidth (low energy per bit) DRAM is available on a processor node. To build a large memory system, one needs to either build
Figure 6-1: Silicon layers arrayed vertically to create higher density memories

many very small nodes (6.4 M for 100PB) or have an additional level of hierarchy. Because TSV memories (even the standard ones) carry a cost premium, it is expected that an exascale system will use commodity memory for its main memory and TSV memory for a level of hierarchy just below main memory. Such a memory could serve as a 16 GB last-level cache.

JASON was briefed on similar TSV technology from Micron. Micron’s hybrid memory cube (HMC) is a TSV memory (like HBM) stacked on a proprietary logic layer with a proprietary signaling interface back to the processor. Like any HPC memory technology, HMC needs to be judged on two simple metrics, cost per bit and energy per bit, and compared not against today’s DRAM memory technology (DDR3), but against expected commodity DRAM that is expected to be $1.25-2.50/GB and have an access energy of 5-10 pJ/bit. Exploiting functions in the logic layer presents interesting possibilities, but that is basically the equivalent of building many small processors with attached TSV memory (the 6.4 M processors with 16 GB each vs. 200K processors with 512 GB each) and will create yet
another layer of complexity for programmers. Both the JEDEC HBM and Micron technologies are promising and should be explored in future architectures.

There have been many proposed technologies for DRAM replacements including phase change memories, memristors, spin-torque memories, etc. To the extent that these can be fabricated with a cost per bit less than DRAM, they may affect the capacity part of the memory problem. However, to a first approximation they will not improve the memory bandwidth because memory bandwidth is limited by energy and that energy is dominated by on- and off-chip communication, not by the actual memory cell or sub-array.

There have been proposals to connect to DRAM with integrated optics. The success of this approach again depends on communication energy. While there are many promising optical developments, no real demonstrated system has come close to the 1-2 pJ/bit demonstrated for electrical signaling at these rates and distances. The best demonstrated optical channels today consume upwards of 20 pJ per bit when all factors (not just the modulator and receiver) are considered. Optical technologies are discussed further below.
To sum up, memory capacity is a question of cost and memory bandwidth and is constrained by on- and off-chip signaling energy. Commodity memory is likely to get to $1.25 to $2.50/GB by 2020 and 5–10 pJ/bit. If TSV memories can achieve comparable cost (they are currently two times to four times SDDR costs) they can eliminate most of the off-chip communication energy driving total access energy to the 1–3 pJ/bit range. However, only a relatively small amount of TSV memory (16 GB) can be packaged near any one processor presenting challenges in overall system organization and scaling so issues of memory capacity will remain.

6.2 Role of Photonics for Exascale Computing

Performance scalability of computing systems has been and will continue to be increasingly constrained by both the power required and speed available to enable data communications between memory and processor as discussed above. Optical links, with the capability for high-bandwidth information transfer (wavelength domain parallelism), and low transmission energy dissipation have long held promise for off-chip communications, and may play an increasingly criti-
cal role in linking processors on a common circuit card. The idea generally is that electrical signals modulate optical signals which can be efficiently routed via optical waveguides or through fibers. Indeed, optical interconnects have already been employed in high performance computers: the IBM Roadrunner has used short-reach active optical cables (AOCs) based on vertical-cavity surface-emitting lasers and multimode fibers (VCSEL/MMs). These VCSEL transceivers provide over 100,000 optical ports per system and facilitate the achievement of high aggregate bandwidth communications between racks and the central switch [15]. Extensions to exascale computing will necessitate more comprehensive intercommunications (racks, boards, modules, chips), a massive increase in the number of parallel optical links ($\approx 100$ million) with high reliability, low power dissipation and low cost. Recently, a 25 Gb/s, 6.5 pJ/bit optical link was reported, utilizing 10 Gb/s class VCSELs, with electronics built into a 90 nm CMOS process [40]. These researchers anticipate building on future developments in faster VCSELs, mandated by industry-wide development for 100 Gb Ethernet applications. The feasibility of on-chip photonic integration has been driven by advances in materials and device design that allowed the scaling of typical optical components (waveguides, modulators, receivers, detectors, couplers and switches) with low loss. Some of these concepts are shown in Figure 6-3. Typical silicon waveguide cross-sectional areas are 0.1 micron$^2$ and the bending radii of waveguides are as small at 1 micron [35]. This scaling down in size of critical optical components makes possible higher density integration of photonic switches, links and modulators, but it is also subject to issues of crosstalk.

The trade-offs between bandwidth, power and cost will have to be fully evaluated. Total power dissipation will very much depend on device choices (e.g., ring micro-ring demultiplexers), choices of materials and the details of the system architecture. Many of those decisions and implementations are still being
explored. Total sources of energy/information loss must also be fully understood (for example insertion loss from fiber to chip, from modulator to modulator). In addition, the power efficiency per device component (e.g. modulator) must be understood and accounted for. Finally, there may be some energy costs to stabilizing the performance or tuning the response of optical components (e.g. [17]).

Ultimately, the choice of laser (optical source), its type (VCSEL, DFB), its efficiency and the efficiency of coupling on-chip will be a large determinant of the energy/bit for the optical links, and in the efforts to drive down the energy dissipation to a few pJ/bit. Although detailed simulations have been done of insertion loss, crosstalk and power requirements for simple chip-scale photonic interconnects, it is not at all clear how well these simulations might describe the further scale up of density and complexity that will be required to mediate the memory wall for exascale computers [6]. In addition, there are a variety of 3D geometries being considered for the integration of the photonic layer with the microprocessor layer. Thus, although photonic interconnects have already played an important role in high performance computing, the short-term realization of a photonic technology that will address the memory wall challenge of exascale computing is not clear. At the moment there are a variety of different device implementations being pursued, different means of integration (e.g. hybrid via flip-chip or full integration with CMOS), and challenges in understanding how to accurately assess energy efficiency, data-rate capabilities and other metrics of performance for truly scaled photonic circuits.
6.3 Computation and Communication Patterns of DOE/NNSA Applications

It has been observed by Phil Colella [4] that the scientific applications of interest to both NNSA and DOE possess common communication and computation patterns. Colella noted there were seven such patterns and named them “the seven dwarfs of high performance computing”. In Figure 6-4 these patterns are listed and symbolized by their respective inter-processor communication patterns. The greater the amount of data that must be transferred between processor $i$ to processor $j$ the brighter is the dot in the images. We discuss these dwarfs briefly:

**Dense linear algebra** This class of applications is characterized by operations on dense matrices such as matrix-matrix multiplies. Such operations exhibit high data locality and a very regular access pattern; with proper caching of matrix elements a significant amount of data reuse can be achieved. For this pattern a very high flop to byte ratio is achievable. Such applications (for example the LINPACK benchmark) can be run very efficiently on modern microprocessors. In Figure 6-4 we show the inter-processor communication pattern for matrix-matrix multiply.

**Sparse linear algebra** This class of applications is characterized by operations on sparse matrices. These can be very challenging to implement on parallel architectures because the sparsity pattern makes it difficult to optimize based on locality. Typically one must permute the matrix so that the elements on a given row are mutually as close as possible to one another. Finding such an optimal permutation is accomplished using graph-theoretic approaches, but because these are themselves computationally complex, one often resorts to heuristics based on the nature of the problem. In addition, it is essential to store the sparse matrix in a compact format since most of the
Figure 6-4: Common computational patterns for DOE/NNSA applications. These have been called colloquially the seven dwarfs [4]
elements are zeroes.

**Spectral methods** Spectral methods utilize expansions in complete sets of functions to represent solutions to various partial differential equations. The dominant communication pattern is a general matrix multiply associated with a transformation from physical space to some set of expansion coefficients. If the expansion is in terms of trigonometric functions then fast methods can be used in one or more coordinate directions. However, because the fields for a spectral method are typically rectangular arrays decomposed across processors, it is often necessary to perform a transposition of the array in order to perform transforms in all coordinate directions. As a result, these methods require all-to-all communication of the entire array. This is shown in Figure 6-4(c) where the all-to-all processor communication pattern becomes readily apparent.

**N-body methods** These methods are used to compute interactions among a group of particles interacting via some prescribed force law. Examples are gravitational dynamics or motion of charged particles. There exist optimized approaches based on the idea of multipole expansions to efficiently evaluate the relevant interactions. The typical data structures are tree-based and use various labeling approaches such as space-filling curves to enhance data locality. A typical communication pattern is shown in Figure 6-4(d).

**Structured grids** The discretization of a partial differential equation onto a regular rectangular grid or a set of rectangular grids using techniques such as finite difference or finite element methods results in a sparse matrix with very regular structure. Operations using such matrices are very efficient because the data pattern is highly regular provided one takes advantage of the spatial locality of the data. A typical communication pattern is shown in Figure 6-4(e).
**Unstructured grids** An example of this pattern is the use of finite element methods to discretize partial differential equations on complex geometries such as the one shown in Figure 6-4(f). Graph-theoretic methods must again be used to label the points of the unstructured mesh to ensure maximal data locality and scatter-gather operations are then required to assemble the resulting sparse matrices. The solution of the resulting equations can be accomplished using the approaches associated with the sparse linear algebra dwarf, but typically, because the matrices involved come from a continuous operator, iterative methods are employed and these too can be optimized so that data locality is maximized.

**Monte-Carlo methods** These approaches are exemplified by the computation of high dimensional integrals by means of summation over randomly selected sets of points over a given domain. Operations are basically embarrassingly parallel meaning evaluation of the integrands can proceed with little or no communication among processors. However, collective operations are required to compute the expectation values over the probability distribution. In Figure 6-4 we symbolize this approach through the use of the computation of the number $\pi$ by choosing random points and seeing whether their location is inside or outside a circle of radius 1.

If a specific dwarf is applicable to a given problem, it is possible to take advantage of the fact that the communication and computational patterns are well-defined. In this case it is possible to improve performance via software optimization or hardware optimization. The idea of classifying the communication and computation patterns of a given application is not new. In some sense, object-oriented computing was developed with this idea in mind for general purpose programming, but the notion of the dwarf goes somewhat further in that one understands ahead of time the data flow and so can develop not only class libraries
but whole template codes for important operations. Chandy et al put forth a similar idea in their discussion of programming “archetypes” [7]. We discuss optimization approaches using hardware briefly below. Software optimizations are discussed in Section 7.

6.4 Optimizing Hardware for Computational Patterns

One approach to optimizing performance is to develop special purpose hardware for specific problems. This has been attempted several times for specific applications. The most recent approach has been the development of a special purpose machine for computation of protein folding by D. Shaw et al. [44]. Such approaches have produced impressive results for specific target problems. However, since these architectures are not general purpose, they are not viewed as a credible path to exascale computing.

We briefly describe an approach that is intermediate between general purpose and special purpose computers. We were briefed by Michael Wehner [50] on the Green Flash project at Berkeley. In this work the climate modeling application discussed in Section 5.1 was used as a target application and a reference hardware design was constructed. The climate simulation application uses the structured grid dwarf and so it is possible to allocate processor resources such as cache memory etc in such a way that while the hardware remains general purpose, it is tuned so that optimal performance is obtained for this particular application and by extension any application that fits the associated computational pattern.

Several hardware designs were considered including the use of standard high
Figure 6-5: Tensilica chip shown in apposition to several larger scale general purpose microprocessors [39]. Note only one core of a double precision Tensilica design is shown. A custom processor would be built from a collection of such cores.

power CMOS chips such as the AMD Opteron and low power chips such as those used in IBM's BlueGene. The authors also examined the use of ultra low power processors such as those being developed for the cell phone industry. These companies develop custom processors based on the specifications provided by the specific phone manufacturer with the advantage that the desired functionality (and only that functionality) is provisioned onto the processor. Companies such as Tensilica provide prepackaged logic circuits (called IP cores) that provide the desired functionality. The entire chip can be laid out and then simulated using special software development environments on standard workstations. Once the desired chip is in hand, it can be fabricated or translated into a specification for further testing using field programmable gate arrays (FPGAs).

The design point for the climate application reached by Wehner et al. is in-
teresting because it takes advantage of the understanding of the relevant dwarf and trades off individual processor power (of the type seen in typical general purpose processors) for limited capability and massive parallelism. For example, using 22 nm silicon, Wehner et al. [51] proposed a chip with 512 low power cores clocked at only 650 MHz. In order to achieve the requirements for a 1–2 km resolution climate simulation each core would have to produce 1.3 GFlops on the target application. They also determined that a cache size of 256 kB per core would be sufficient to mask memory latency.

The cores utilized for this exercise are very small relative to modern general purpose processors. The relative size is shown in Figure 6-5. There are also advantages in terms of power utilization. For example an IBM Power 5 processor clocked at 1.9GHz dissipates 120W. In contrast 128 Tensilica cores of the type considered by Wehner et al dissipate a total of only 11.5W .

We do not advocate that special purpose machines be built for DOE/NNSA applications. However, given that a significant portion of these applications can be classified by a dwarf (or some composition of dwarfs), a limited hardware exploration effort to design hardware accelerators for dwarfs (either on-processor or off-processor) that support a specific computational pattern may be useful. Such an effort would make it possible to compare the performance of more general purpose hardware solutions, and this in turn would make it possible to make more informed decisions as regards the potential for performance of DOE/NNSA applications. We believe there is a role for a limited hardware development effort to encourage potential advances that then could be migrated into vendor hardware.
7 SOFTWARE CHALLENGES

In this section, we describe some promising research directions for the development of exascale software. As HPC architectures become more heterogeneous and hierarchical, the challenges of modifying science application codes to utilize them effectively sometimes rival in magnitude the challenges faced in developing the hardware platform. While the ASC program was largely successful in porting key science applications to the very heterogeneous Roadrunner for example, it is widely held that much of the porting work involved “heroic” efforts.

At present the dominant approach for programming massively parallel systems is the Message Passing Interface (MPI). In MPI, the programmer is responsible for managing all interprocess communication. MPI has been successful because it is a standard supported on all platforms and, in many cases, vendors have provided special libraries that facilitate optimized data transfer using hardware features specific to the particular architecture. In addition, MPI allows one to reason about a computation in a way that is independent of particular node or network configurations by using the concept of the “communicator,” a virtual context for parallel communication. For heterogeneous architectures, MPI has been used in combination with compiler directives and libraries (such as OpenMP) so that efficient thread-based parallelism can be employed on multicore architectures while interprocessor communication is performed using traditional MPI. Recently, there has also been important work on using MPI with GPU architecture, allowing for direct access to GPU memories so that data can be moved in a way that is more uniform and transparent to the programmer [1].

We expect that the MPI approach will remain an important aspect of parallel programming. However, it is anticipated that the need to manage a billion threads
creates a level of complexity which may make the direct use of MPI untenable. Instead, there will be a need to hide many of the implementation details from the programmer. It is also the case that optimizing performance for an exascale system may also require automated approaches. The need to provide software that facilitates programmer productivity is known to be a challenging issue. We discuss two ideas which we feel have merit and deserve further exploration. The first is the use of domain-specific languages wherein one constructs simple languages that embody a specific computation and communication pattern. The second is the use of autotuning software to optimize system and program parameters so that maximal hardware throughput is enabled. These approaches can potentially alleviate some of the burden of porting applications to new architectures, and could greatly improve the productivity of the science programmer, especially during the key phases when new architectures are introduced.

7.1 Domain Specific Compilers and Languages

A domain-specific compiler is a special type of compiler that uses knowledge of the computational problem domain to build more efficient code than can be produced using a general-purpose compiler. Domain-specific languages (DSLs) take domain-specific compilers one step further, and are typically designed to allow the programmer to describe the computational algorithm at a very high level that is usually independent of the underlying architecture. To allow the application programmer to “speak” at such a high level of course requires that knowledge of the target hardware platform is encoded in the compiler so that it can generate usable code. While this requires significant effort in practice, the payoff is that when the application must be ported to a new architecture, modifications only need to be made to the compiler. This separation of the problem descrip-
```scala
var i = 0;
while (i < 1000) {
  Flux(vertices(mesh)) = 0.f;
  JacobiStep(vertices(mesh)) = 0.f;
  for (e <- edges(mesh)) {
    val v1 = head(e);
    val v2 = tail(e);
    val dP = Position(v1) - Position(v2);
    val dT = Temperature(v1) - Temperature(v2);
    val step = 1.0f/(length(dP));
    Flux(v1) += dT*step;
    Flux(v2) -= dT*step;
    JacobiStep(v1) += step;
    JacobiStep(v2) += step;
  }
  i += 1
}
```

Figure 7-1: Example Liszt code that performs heat conduction on a grid.

The hardware implementation is especially useful given the way in which DOE/NNSA application codes are used at the respective laboratories—namely, the physics in the codes is continually being modified in response to research needs. Having to maintain both the evolving physics components of these large codes and the corresponding hardware-specific implementation requires physicists with computer scientist-level programming ability, and computer scientists with PhD-level physics knowledge. Although such remarkable people exist at the labs, it is very natural and feasible (though admittedly difficult) to separate the domain-specific problem description from the hardware-specific implementation.

Although domain-specific languages find common use in the software engineering world (e.g., the ColdFusion Markup Language for rapid website development), their use in the scientific world appears rare. One particularly compelling DSL for scientific applications (and one that was briefed to us) is the Liszt language for solving mesh-based PDEs [19]. Liszt is a mature language...
with a proven compiler that generates efficient code for shared-memory multi-processors, compute clusters, and GPUs. In Liszt, the application programmer specifies the computational algorithm in a manner analogous to the example shown in Figure 7-1 for heat conduction. The Liszt compiler then generates the appropriate machine-specific implementation to carry out the algorithm. Liszt has been shown to produce machine code that typically performs within about 10% of the equivalent hand-tuned C++ code on a variety of architectures, and scales nearly as well with the number of compute elements (e.g., CPU cores). In some cases the scaling is in fact better than the hand-tuned code [19]. Considerable effort no doubt went into the Liszt compiler. However, the clear benefit is that the application programmer doesn’t need to worry about the details of the hardware on which the code will run, and can focus his or her efforts on the domain physics. It is not at present clear whether the simplicity of the example we have shown above persists when one attempts to write a full simulation of the type exemplified by DOE/NNSA applications. However, an investigation along these lines would appear to be worthwhile. Alternatively, it may be useful to explore the use of DSLs as embedded languages suitable for the expression of various computational kernels that are used extensively in relevant applications and for which high performance is required.

### 7.2 Auto-Tuners

Auto-tuning is the process of benchmarking and selecting the fastest of a set of code refactorings for a given algorithm on a given architecture. This process can of course happen at compile-time or run-time. Auto-tuning has apparently seen more widespread use than DSLs in the scientific world. Auto-tuners are in use at some co-design centers (see Section 8.1). For example, the ROSE compiler framework...
is used at the ExaCT combustion co-design center to aid in auto-tuning, where they speed the design cycle by allowing quick exploration of the possibilities of a given hardware design [5]. Auto-tuning was also crucial in the Green Flash design process [51] discussed in Section 6.4.

The auto-tuning process can be made easier through the use of a domain-specific language or compiler, which can automatically generate legal code refactorings according to a set of rules. Perhaps the best example of the symbiotic relationship between domain-specific compilers and auto-tuning is the FFTW open source fast Fourier transform (FFT) library. FFTW implements the FFT, which is a recursive divide and conquer algorithm with $O(N \log N)$ complexity, where $N$ is the length of the input data (the naive implementation of the Fourier transform has complexity $O(N^2)$). FFTW is comprised of two key components: a domain-specific compiler that generates optimized C code from an abstract description of the FFT, and an auto-tuner that selects the fastest implementation on the target architecture. With the exception of a few vendor implementations, FFTW is the fastest available FFT on many platforms [20].

There are of course tradeoffs in such a flexible, robust design. Designing a code base with a domain-specific compiler and auto-tuner requires planning, as it is difficult to graft onto a legacy code base. In addition, there are typically many more lines of code. FFTW has $\sim 10^5$ lines of code, while the GNU Scientific Library (GSL) FFT implementation (with roughly equivalent functionality) has $\sim 10^4$ lines of code. However, such a design makes it much easier to transition to new architectures. According to the primary FFTW developer, “...the transition (from) SSE2→AVX was relatively painless (a few hours of work) and users have already adapted it to the undisclosed future Intel MIC...” (M. Frigo, private communication 2012). Another benefit is that the scientist can reason about
let rec cooley_tukey sign n1 n2 input =
    let tmp1 =
        array n2 (fun i2 ->
            dft sign n1 (fun i1 -> input (i1 * n2 + i2))) in
    let tmp2 =
        array n1 (fun i1 ->
            array n2 (fun i2 ->
                exp n (sign * i1 * i2) * tmp1 i2 i1)) in
    let tmp3 = array n1 (fun i1 -> dft sign n2 (tmp2 i1)) in
    (fun i -> tmp3 (i mod n1) (i / n1))

Figure 7-2: Implementation of the Cooley-Tukey FFT algorithm in FFTW (in the language OCAML).

the algorithm at a very high level. Figure 7-2 shows the implementation of the Cooley-Tukey FFT algorithm in FFTW (in the language OCAML). It is clearly succinct, and without reference to data structures or hardware implementation. (The equivalent algorithm in GSL takes up \(\sim 10\) times as many lines.)

Is the coupled DSL/auto-tuner design a model for DOE physics applications? The FFT is of course much simpler than any physics application of interest to DOE. And we haven't even discussed parallelism. However, the benefits of abstracting the statement of the computational algorithm from the implementation on particular hardware likely outweigh the unpleasantness of the initial investment in the long run. This seems especially true given the uncertainty as to the architectural details of future exascale platforms.

### 7.3 Summary and Conclusion

We conclude with a finding regarding software development: More focus is also required to develop software that will facilitate development of and reasoning about applications on exascale platforms regardless of the details of the underlying
parallel architecture. This will be necessary for several reasons. First, the complexity of future exascale platforms will make it very difficult to use architecture specific or fine grained approaches like MPI. Secondly, the cost of redesigning application software for specific architectures will become prohibitively expensive.
8 RECOMMENDATIONS FOR THE FUTURE

8.1 Co-Design

DOE and NNSA have introduced the concept of “co-design” to ensure that potential hardware vendors and application developers can collaborate so that the challenges of exascale computing for DOE/NNSA are made clear and that future hardware developments are responsive to the extent possible to these challenges. Co-design as we were briefed has two components:

1. Application developers communicate scientific application requirements to hardware designers and influence architecture design (to the extent feasible),

2. Hardware designers communicate hardware architecture constraints to application developers and in turn guide the design of algorithms and software.

The idea behind co-design is sound; it is critical that there be extensive dialog between vendors and application developers. DOE Office of Science has launched three co-design centers in the areas of combustion, materials, and nuclear reactor engineering so that the applications relevant to the DOE science mission are adequately represented. NNSA is in the process of launching a similar effort. In order to simplify the work, proxy applications have been written that encapsulate the essential computational kernels and these have been communicated to various potential vendors. In addition, the centers are generating very useful performance data for these applications that can be productively used by hardware researchers.
However, there are aspects of the co-design process which may weaken the desired leverage DOE/NNSA wishes to exert to ensure hardware developments are responsive to application needs. Vendors assiduously protect their intellectual property. We were briefed that this made collaboration difficult, because if an application developer received proprietary information they are (understandably) enjoined from discussing it with anyone who does not have the appropriate need to know. This makes simultaneous interaction with several vendors very difficult as there is a concern that inadvertent communication will lead to release of proprietary information to competitors. Such an approach is at odds with the open nature of scientific research. The use of national laboratories such as the NNSA labs may provide a partial solution to this problem as they are intrinsically better able to protect confidential information, but such an approach would not be appropriate for universities.

There is also an indication that hardware vendors see co-design as basically a mechanism for communicating to application developers how to optimize their software on proposed hardware. This one-way mode of operation would imply that there is little motivation to consider the special needs of DOE/NNSA applications. As we have indicated previously, DOE/NNSA applications may perform far less efficiently given current trends in hardware development. Having some influence on the reversal of these trends is far from easy, but would be a desirable core aspect of co-design.

Finally, we believe there needs to be more diversity in the types of hardware being considered. The challenges associated with exascale computing make co-design essentially a research endeavor as opposed to a development endeavor. It would be beneficial to consider a wider range of hardware (and later, software) approaches perhaps along the lines of projects such as Green Flash.
Our finding here is that the current co-design strategy is not optimally aligned with the goal of developing exascale capability responsive to DOE/NNSA application requirements. More focus is required to ensure that the hardware and software under development will target improvements in the performance of the dominant patterns of computation relevant to DOE/NNSA applications.

8.2 The Need for an Intermediate Hardware Target

Given the challenges to the development of exascale computing described in previous sections, it is very difficult at present to foresee how the various challenging objectives associated with energy, memory capacity, memory bandwidth and resilience will be met and on what time frame.

Given the uncertainties and the very real danger that a future architecture will be only marginally useful as regards some key DOE/NNSA applications, we believe that, rather than targeting an exascale platform in the 2020 time frame, DOE/NNSA should invest further in research on a variety of enabling technologies that are geared towards solving the challenging issues of energy efficiency, memory density, bandwidth, and resilience for future massively parallel architectures.

In particular, it would be beneficial to establish a set of intermediate hardware targets for performance that are short of an exaflop of performance but reverse the inimical projections for future hardware. Such intermediate targets should not be tied to particular time-frames, but should instead emphasize the achievement of goals key to the future performance of DOE/NNSA applications. An attractive intermediate target is a platform with sustained performance of 1–10 petaflops, but optimized for DOE/NNSA computational requirements, and with
memory and bandwidth targets that reverse current market road-maps. An am-
bitious goal in terms of energy dissipation is to develop a platform that achieves
this sustained performance within a power envelope of five megawatts. A later
and more ambitious goal would be to increase performance to 100 petaflops (sus-
tained) within the same power envelope (again for a suite of DOE/NNSA applica-
tions).

There are several advantages to the achievement of such intermediate goals:

• A machine designed for sustained throughput on DOE/NNSA application
  kernels (and in particular the seven dwarfs) would be an attractive platform
  as it would present less of a barrier to effective use. It would also go some
  way to increasing the range of problems that can be addressed that require
  increased memory capacity or bandwidth.

• An energy target of 5 MW for a sustained performance of 1 petaflop (or
  perhaps the more challenging 10 petaflops) may be far less stressing than
designing for upwards of 20 MW and a peak rate of an exaflop and would
provide important information on potential future extrapolations to the ex-
aflop regime. The achievement of sustained performance at such a level
within 5 MW may be attainable by 2020 with concerted research and devel-
opment. The achievement of 100 petaflops within that power envelope may
then follow once the key issues are understood more completely.

• Such an intermediate platform is also an important step towards the “de-
  partmental petaflop system” outlined in the 2008 DARPA study [27]. Such
  a machine would bring advanced parallel computing to a larger group of
  potential users who may want to develop first on such a smaller machine
  before contemplating scaling to an exaflop.
• A platform that targets a sustained petaflop may also be easier to replicate across the DOE/NNSA complex as the requirements for hosting such a platform are less constraining. This would then make it possible to perform a wider range of calculations at lower resolution, but aimed at uncertainty quantification, a key requirement for stockpile stewardship and an increasingly important issue for all scientific computations.

8.3 Summary

Perhaps the highest level finding in this report address the overall importance of high performance computing: US leadership in high performance computing is critical to many scientific, industrial and defense problems. In order to maintain this leadership, continued investment in HPC technology (both hardware and software) is required.

It is important to note however, that maintenance of this leadership is not necessarily tied to the achievement of exascale computing capability by 2020. Such leadership can be maintained and advanced with ongoing investments in research and development that focus on the basic technical challenges of achieving balanced HPC architecture particularly as regards the suite of DOE/NNSA applications.

8.4 Recommendations

We conclude this report with a set of recommendations that follow from the findings presented so far:
• Rather than target the development of an exascale platform in the 2020 time frame, DOE/NNSA should invest in research and development of a variety of technologies geared toward solving the challenging issues currently impeding the development of balanced exascale architecture: increased memory density, memory bandwidth, energy-efficient computation, and resilience.

• DOE/NNSA should establish a set of intermediate platform targets in pursuit of balanced HPC architecture over a realistic time frame. An attractive set of intermediate targets are platforms that provide sustained computational floating point performance of 1, 10, and ultimately 100 petaflops, but optimized for DOE/NNSA computational requirements, with memory capacity and bandwidth targets that exceed current microprocessor vendor road maps, and with a maximum power consumption of 5 MW or less. A variety of technical approaches should be supported in meeting this target with eventual down-select of the most promising approaches.

• DOE/NNSA should undertake an effort to characterize in a standard way the computational patterns and characteristics (i.e. memory capacity, memory bandwidth, floating point intensity and global communication) of the suite of DOE/NNSA applications.

• The development of software tools should be supported at a budgetary level commensurate with that provided for hardware development. In particular, support for tools like domain-specific languages and auto-tuning software is needed so that users can reason about programs in terms of scientific requirements as opposed to hardware idiosyncrasies.

• The current co-design strategy should be enhanced so that it not only focuses on the optimization of existing codes, but also encourages hardware
and software innovation in direct support of the dominant patterns of computation relevant to DOE/NNSA applications.
A APPENDIX: Markov Models of Disk Reliability

In this appendix we provide further details on the Markov model used for estimating data integrity lifetimes. Figure A-1 gives a Markov model for a fully declustered storage block with \( n \) hard disk drives, a failure rate \( \lambda \) and a repair rate \( \rho \). We model a declustered RAID-5 system, where each stripe can tolerate a single failure. The loss of a single hard disk drive will affect many stripes, but as long as no stripe has more than one block on the affected hard disk drive then no data loss will occur. We make the simplifying assumption that disks fail independently and the failure rate is exponentially distributed. The Markov model at the top is the traditional one with two survival states, state \( s_0 \) models a disk array with no failures, and state \( s_1 \) an array with one failed disk, and a failure state \( s_F \). When we are in \( s_1 \), the disk array detects the failure and begins immediately the work to reconstruct the blocks on the failed drive by reading each block in parallel from \( k \) disks and writing into the spare space of another drive. If this operation succeeds, then we return to \( s_0 \). The recovery process returns the array to normal operation once the data on the failed disk has been reconstituted on other hard disk drives in the array. If there is an additional disk failure in \( s_1 \), we lose some data. The effect of the data loss will be highly dependent on the file system, and could lead to file system corruption or merely some “holes” in various files. Tolerating this situation will require research in file system design.

In order to proceed with our derivation, we change the Markov model to an ergodic one (lower part of Figure A-1). The new model does away with the failure state and replaces the transition from \( s_1 \) to the failure state with a transition to \( s_0 \). This transition models a data loss, after which “magically” a new disk array replaces the old one with all the data that could be rescued from its predecessor’s catastrophe (with the holes discussed previously). The new model is simpler [55]
and we can calculate equilibrium probabilities for being in one of $s_0$ and $s_1$. If we denote the probability of being in $s_i$ as $p_i$, then we have:

\[ p_0 + p_1 = 1 \]  \hspace{1cm} \text{(A-1)}

\[ n\lambda p_0 = p_1(\lambda(n-1) + \rho). \]  \hspace{1cm} \text{(A-2)}

This yields the solution:

\[ p_0 = 1 + \frac{n\lambda}{\lambda(1-2n)-\rho} \]  \hspace{1cm} \text{(A-3)}

\[ p_1 = \frac{\lambda n}{\lambda(2n-1)+\rho}. \]  \hspace{1cm} \text{(A-4)}

The failure transition, if $s_F$ existed, would be taken with rate $p_1(n-1)\lambda$ and so
the mean time to data loss (MTTDL) is:

\[
\text{MTTDL} = \frac{(2n - 1)\lambda + \rho}{n(n-1)\lambda^2}.
\]  
(A-5)

We now need to derive a value for the repair rate \( \rho \). Given the capacity \( C \) of a hard disk drive, and a data rate \( D \), and the fraction \( \phi \) that the disk is utilized, and a recovery data rate \( \psi \) (the fraction of the total disk bandwidth that we are willing to dedicate to recovery) then we can calculate a recovery time

\[
R = \frac{k + 1}{n} \frac{\phi C}{\psi D},
\]  
(A-6)

where \( k \) is the number of blocks in the reliability stripe. \( R \) is a good estimate of the time required to recover the contents of the failed hard disk drive, and since \( R \ll 1/\lambda \) then we can use \( \rho = 1/R \).

We can do a similar calculation for RAID 6, using two distributed parity blocks per reliability stripe. We can write the equations directly from the Markov diagram given in Figure A-2:

\[
p_0 + p_1 + p_2 = 1
\]
(A-7)

\[
\lambda np_0 = (n-2)p_2 + p_1\rho
\]
(A-8)

\[
\lambda np_0 + p_2\rho = p_1(\lambda(n-1) + \rho)
\]
(A-9)

The solution to the system of equations is still manageable, and fortunately it is independent of the number of disks in the RAID cluster. We are interested primarily in \( p_2 \), the probability of being in \( s_2 \) where further failures will lead to data loss:

\[
p_0 = \frac{\lambda(n-2)(n-1) + \rho(n+\rho-2)}{\lambda^2(n-1)n + \lambda(n(2n+\rho-5)+2) + \rho(n+\rho-2)}
\]
(A-10)

\[
p_1 = \frac{\lambda n(n+\rho-2)}{\lambda^2(n-1)n + \lambda(n(2n+\rho-5)+2) + \rho(n+\rho-2)}
\]
(A-11)

\[
p_2 = \frac{\lambda^2(n-1)n}{\lambda^2(n-1)n + \lambda(n(2n+\rho-5)+2) + \rho(n+\rho-2)}
\]
(A-12)
As with our RAID 5 model, the failure transition for RAID 6 would be taken at the rate $\rho_2(n - 2)\lambda$ and we can then compute the mean time to data loss:

$$\text{MTTDL} = \frac{\lambda^2(n - 1)n + \lambda(n(2n + \rho - 5) + 2) + \rho(n + \rho - 2)}{\lambda^3(n - 2)(n - 1)n}.$$  \hspace{1cm} (A-13)

If it is decided that RAID 6 with its two distributed parity blocks is insufficient, then a third parity block could be added. The calculations are similar to those we have already made.

We can make estimates of the MTTDL, and use that to guide us in the amount of redundancy required for a given amount of data storage. It seems likely that there will be several classes of storage. For example, there will be storage to hold memory images created by checkpoints, and a much larger archive for data that
will be kept indefinitely. Studies by Google, and others, have shown that manufac-
turer’s estimates of hard disk drive reliability are overly optimistic [37]. Annual
failure rates (AFR) of from 2% to over 8% were observed, and tended to increase
as the hard disk drives aged. If, for example we pick 6% as an average AFR, then
we can expect a MTTF of 146,000 hours, far short of the $10^6$ hours reported by
many manufacturers. We should also remember that correlated failures are possi-
ble, and so our models based on Markov assumptions are optimistic and provide
only a lower-bound on the amount of redundancy required.
B  APPENDIX: Uncertainty Quantification for Large Problems

In this appendix, we discuss some of the computational requirements for uncertainty quantification (UQ). Until now we have been estimating the computing power to resolve direct simulation of various physical problems. We argue that equal attention be given to estimating the uncertainty in those solutions from the combination of model errors and noise in measurements informing the model of unknown parameter and state values. The goal is the ability of a model, simple or complex, to utilize experimental information to predict the future development of the model. We return in a moment to a discussion of the two questions of

1. how many measurements are required for a given model and
2. what are appropriate methods for evaluating the RMS spread in predictions from a model.

The statistical questions associated with assimilating data to learn uncertain or unknown fixed parameters and states are compactly addressed by a classical version of a path integral embodying the flow of the state of the model system through a period of time in which measurements might be made and through which the state must be moved forward in time. Using the result of the application of Monte Carlo methods to perform this kind of integral for a smaller fluid dynamics problem, we have a baseline from which to scale the required calculation to the model sizes discussed just above. Note there are important unresolved issues in this type of investigation. For example, one does not know ahead of time if an RMS spread is even a relevant measure of uncertainty if the underlying distribution has long tails. These types of issues will require further investigation.
The model system we consider here is a single layer shallow water equation on a 100 km by 100 km mid-latitude patch. This kind of fluid dynamics problem is at the heart of all weather and climate models using for short range or long term forecasting, and is relevant to problems in the core mission of the DOE. The model calculation was done on a small, 16 by 16 grid in which the estimated state and RMS errors for each state were evaluated for three fields: the $x$ and $y$ horizontal velocity and the height of the fluid. This gives about 1000 dynamical degrees of freedom (768 to be more precise). Integrating for 4000 time steps of 36 seconds (0.01 hr) to produce 200,000 Monte Carlo selected paths using a parallelized Metropolis-Hastings routine on a single NVIDIA GTX 580 GPU device took 12.3 hours.

Suppose we scale this problem from $10^3$ degrees of freedom to $10^9$ degrees of freedom, use the same number of accepted paths, time steps, ... and require the calculation to complete in 10 hours. Then we require 106 GTX 580’s or $5 \times 10^8$ CUDA cores to accomplish the larger job in the same 10 hours. Realistically, we also need to scale up the computing requirements for communications among many GPUs.

One must note that a GTX 580 uses 150 W while performing this computation, and discounting communications and memory use costs entirely, 150 MW would be required to use existing GPUs of GTX 580 form factor. That, of course, is bad news, but in the same ballpark as earlier bad news for the forward computing capability itself, so, in a back of the envelope manner, one might say that double the compute capability and power consumption estimated for simply scaling up present day computation would be required for exascale computation along with uncertainty quantification on problems more or less of the size of contemporary numerical weather prediction or climate modeling.
There is a thread of estimating uncertainty quantification (UQ) that appears often in DOE reports and projections going under the name of Wiener-Hermite expansions [28] or polynomial chaos [56]. The basic idea, due to Weiner in 1938 [52] is this: if we have a field \( u(x,t) \) satisfying some partial differential equation which contains uncertain elements such as transport coefficients or an uncertain driving force which depends on a fluctuating variable \( \xi \), then we can expand both the field \( u(x,t;\xi) \) and any parameters as series in orthogonal polynomials \( \phi_n(\xi) \)

\[
u(x,t;\xi) = \sum_{n=0}^{\infty} u_n(x,t) \phi_n(\xi) \text{ where } \int d\xi \rho(\xi) \phi_n(\xi) \phi_m(\xi) = \delta_{nm}
\]

leading to an infinite number of differential equations among the \( u_j(x,t) \). Truncating this set yields a reduced set of degrees of freedom to address from which statistical quantities may be evaluated. The integration measure \( \rho(\xi) \) is selected to be associated with the specific nature of the noise in the original differential equations. If that noise is Gaussian then \( \rho(\xi) = \exp(-\xi^2) \) and the orthogonal functions are Hermite polynomials

In papers by Meecham [25], the orthogonality of the polynomials appears quite attractive as it guarantees that the energy, quadratic in the \( u_n(\xi,t) \), is a sum of explicitly positive definite terms

\[
E(k) = \frac{k^2}{(2\pi)^5} \int d^3q \langle u(k-q,t) \cdot u(q,t) \rangle \\
= \left( \frac{k}{2\pi} \right)^2 |u_1(k)|^2 + \frac{2k^2}{2\pi^5} \int d^3q |u_2(k,q-k)|^2 + \cdots
\]

and this is a very good attribute.
However, truncating the series at a finite order runs afoul of the Marcinkiewicz theorem [29] which says that the independent correlations of

\[ \langle u(x, t)u(x', t)\ldots u(x_n, t^{(n)}) \rangle \]

can be truncated at \( n = 2 \) when only second order correlations \( \langle u(x, t)u(x', t') \rangle \) are independent, so the process is Gaussian, or one must keep the full infinite sum of correlations or the distribution of the dynamical valuables \( P(u(x, t)) \) is somewhere negative, contrary to the meaning of a probability distribution. Truncating the infinite sum in polynomial chaos is a closure approximation which reduces the infinite number of correlations to a finite number. If that number of independent correlations is greater than two, negative probability distributions for the field \( u(x, t) \) will occur. The Hogge and Meecham truncation may well assure the energy density is positive, but it does not guarantee that the underlying distribution is everywhere positive, nor can any such finite truncation. The Monte-Carlo evaluation of the path integral, which itself is just an integral representation of the full solution of the original stochastic differential equation never faces this issue as all moments are present.

Fortunately from a computational perspective, Monte Carlo integration is, as exemplified by the shallow water equation example, is eminently parallelizable using GPU methods. Once we have a model, we can use it to indicate how many measurements would be required to estimate any remaining unobserved state variables and unknown parameters. To accomplish this, we note that when the data signals \( y_l(t), l = 1, 2, \ldots L \) of the corresponding model output signals \( x_a(t) = 1, 2, \ldots, D \) (\( L \) out of an overall \( D > L \) state variables can be compared to the data) are in synchrony, we may expect that minimizing some distance, say,

\[ \text{Synchronization error} = \sum_{n=0}^{m} \sum_{l=1}^{L} [y_l(t_n) - x_l(t_n)]^2 \]
will provide the basis for an estimation of any parameters in the problem and the D-L unobserved state variables. We need all of these parameters and all state variables at some time to predict beyond that time using our model equations. In the cases discussed earlier where the flows are chaotic, this is especially important.

What makes it difficult to perform this minimization? When the measured $y_l(t)$ and computed $x_l(t)$ are chaotic, the sum is very sensitive to its dependence on parameters or state variables. The communication of information from the observations to the model is unstable, and this results in many local minima in the Synchronization Error sum impeding the ability to search for any minimum.

One can cure this by adding terms $k(y_l(t) - x_l(t))$ to the differential equations for $l = 1, 2, \ldots, L$ state variables as one form of communicating information from measurements to model. As $k$ increases, when $L$ is greater than some critical value, the instability will be cured and the search surface becomes smooth.

This question can be efficiently explored by generating data from the model itself, then evaluating the Synchronization Error for new solutions to the model with different initial conditions and a range of parameters. When $L$ is big enough and $k$ large enough, synchronization will occur, and the search is made easy.

In the case of the low dimensional shallow water flow discussed above, this indicates that for a $16 \times 16$ grid, the critical $L$ is about 70% of the number of degrees of freedom ($768 = 2 \times 16 \times 16$). To determine what this might be for a model with many, many more degrees of freedom, one would coarse grain the grid to examine a much smaller number of degrees of freedom, then systematically refine the grid looking for a trend in $L/D$ where synchronization occurs. For smaller models this trend sets in quite quickly, and it allows an estimation of the critical $L$ for large $D$ without actually doing the calculations for large $D$. It is also
true that when the estimations become possible and accurate, the ability to predict beyond times when observations occur improves dramatically.
C  APPENDIX: NNSA Application Requirements

In this section we describe application requirements for NNSA stockpile stewardship calculations. We also provide an assessment of the possible adverse security impacts should it prove infeasible to develop an exascale platform by 2020. The details of this appendix are classified S/RD and so are not distributed with this unclassified report. The classified appendix is available as a separate document.
## APPENDIX: Briefers

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