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Blue-Green Lasers and Electrodeless Flashlamps

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ABSTRACT*

This paper addresses the questions of combining the technology of moderate pressure electrodeless discharge lamps with the efficiency of a resonantly pumped solid-state laser to achieve an efficient, compact, and reliable blue-green laser. The scheme is based on resonant absorption of the $^1D_2$ state of Pr^{+3} which coincides with strong yellow lines of a sodium discharge at 589 nm. A Q-switched lasing transition to the $^3F_3$ state can be doubled into the desired blue-green region. Estimates show that a moderate pressure electrodeless flashlamp should emit roughly 30% of its light in the 589 nm band. More generally, the moderate pressure electrodeless flashlamp should be an effective and efficient emitter of resonant radiation throughout the visible and UV region, opening possibilities for other resonantly pumped lasers. Several specific possibilities are pointed out, including an energy efficient system at 610 nm, and a candidate inertial fusion driven at 250 nm.

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1. INTRODUCTION

In recent years, resonant pumping of solid state lasers by other lasers has shown that laser transitions in solid state lasers can be accomplished with high efficiency.[1-3] The difficulties for practical realizations lie in the efficiency of the primary laser. In this context, the solid-state laser has been viewed more as a frequency converter than as a laser. On the other hand, moderate-pressure gas discharge lamps are well-known as efficient sources of monochromatic radiation.[4] Why not combine these two phenomena into an efficient laser? This paper shows that with appropriate laser and lamp parameters, such a system appears to be possible and attractive. The electrodeless flashlamp configuration offers both reliability and a close-coupling geometry as the recent work of Golikova et al[5] points out.

Laser and lamp pumping sources differ enormously in brightness temperature. A laser pump pulse can be as short as 15 nsec with a brightness temperature $\sim 10^{12}$ ev, and can be considered as an instantaneous pump for cases where the upper laser level has a lifetime which ranges from 10-100 μsec. A flashlamp, on the other hand, is limited to a brightness temperature of a few electron volts. We shall see that this will require upper laser level lifetimes of 100-300μsec to be a satisfactory pump, except in the vacuum uv region.

A numerical example will reinforce this point. Let us consider a laser upper level with a lifetime $\tau = 300\,\mu\text{sec}$ and lamp which achieves a brightness temperature T over a frequency interval $\Delta\nu$. The limiting pump power flux F_ℓ on the lasing material is the blackbody intensity which is

$$F_\ell = \frac{2\pi T\nu^3}{c^2}\left(\frac{\Delta\nu}{\nu}\right)\frac{y}{e^y - 1} = 1.5 \cdot 10^5\,\frac{\text{watts}}{\text{cm}^2}\left(\frac{\Delta\nu}{\nu}\right) T_{ev}\left(\frac{589\text{nm}}{\lambda}\right)^3 \frac{y}{e^y - 1} \tag{1}$$

where $y = h\nu/T$ and T_{ev} is the brightness temperature in electron-volts. Let us adopt as an example $\Delta\nu/\nu = 3 \cdot 10^{-2}$, $T_{ev} = 0.5$, and $h\nu = 2\text{ev}$ to obtain

$$F_\ell\,\tau = 4.5 \cdot 10^{-2}\ \text{joules/cm}^2 . \tag{2}$$

Let us further suppose that the lasing crystal has a diameter of 10 cm and length of 40 cm, giving a total surface area A approximately $A = 1200\ \text{cm}^2$. The total pump energy U_p stored is then

$$U_p = F_\ell \tau A \approx 50\ \text{joules} \tag{3}$$

assuming all the pump energy can be absorbed. The energy storage density in the lasing medium is low

$$\frac{U_p}{V} = 0.03 \frac{\text{joules}}{\text{cm}^3} \tag{4}$$

but this is advantageous for high power operation. The proposed linewidth amounts to a spread ΔE of $\Delta E = 500\ \text{cm}^{-1}$ for a sodium flashlamp, so that a matched resonance must be available.

Fortunately such a system exists for praseodymium doped into $LiYF_4$. Figure 1 shows the absorption spectrum of the $^1D_2$ state of Pr:YLF as well as the position of the sodium resonance lines. Clearly an excellent match exists. Measurements show that the lifetime of the upper laser level varies from 335 μsec (E.P. Chickless, Sanders Associates, private communication) to 520 μsec.[6] Figure 2 presents the flourescent spectrum of the Pr $^1D_2$ state in room temperature YLF. The transition $^1D_2 \rightarrow {}^3F_3$ occurs at 972 nm which can be doubled to 486 nm. This transition is expected to have moderate intensity, and the lifetime of the lower state may be short as well[7] because of rapid multi-phonon relaxation rates. Such favorable prospects lead us to consider what parameters a sodium discharge lamp could obtain.

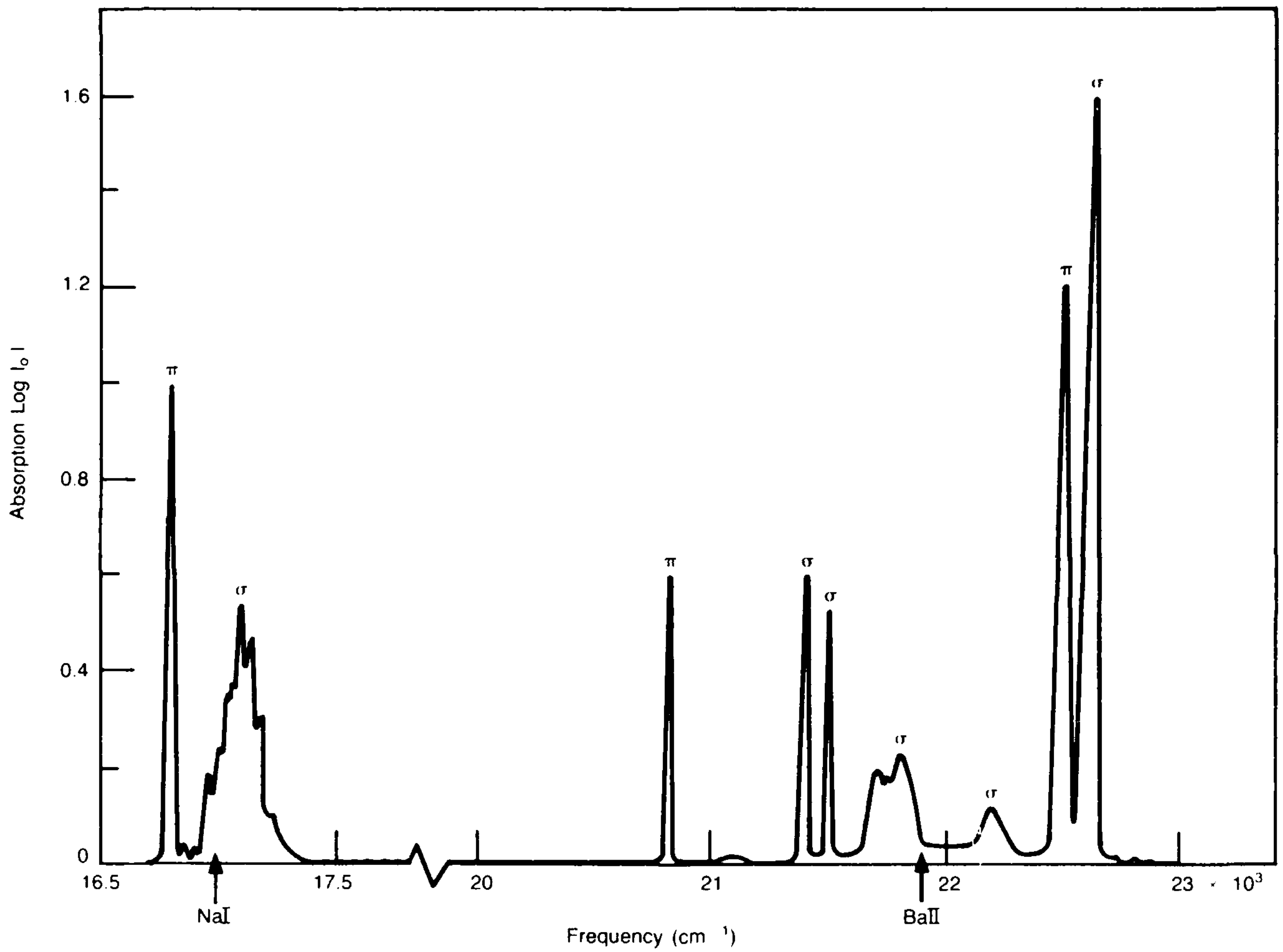

Figure 1. Pr Absorption Spectra of $^1D_2$, $^3P_0$, $^3P_1$, $^1I_6$, and $^3P_2$ states measures at 5°K. Positions of the NaI and BaII lines are shown. [Measurements by H. P. Jenssen, M.I.T. Crystal Physics Laboratory Technical Report No. 16 (Sept. 1971)].

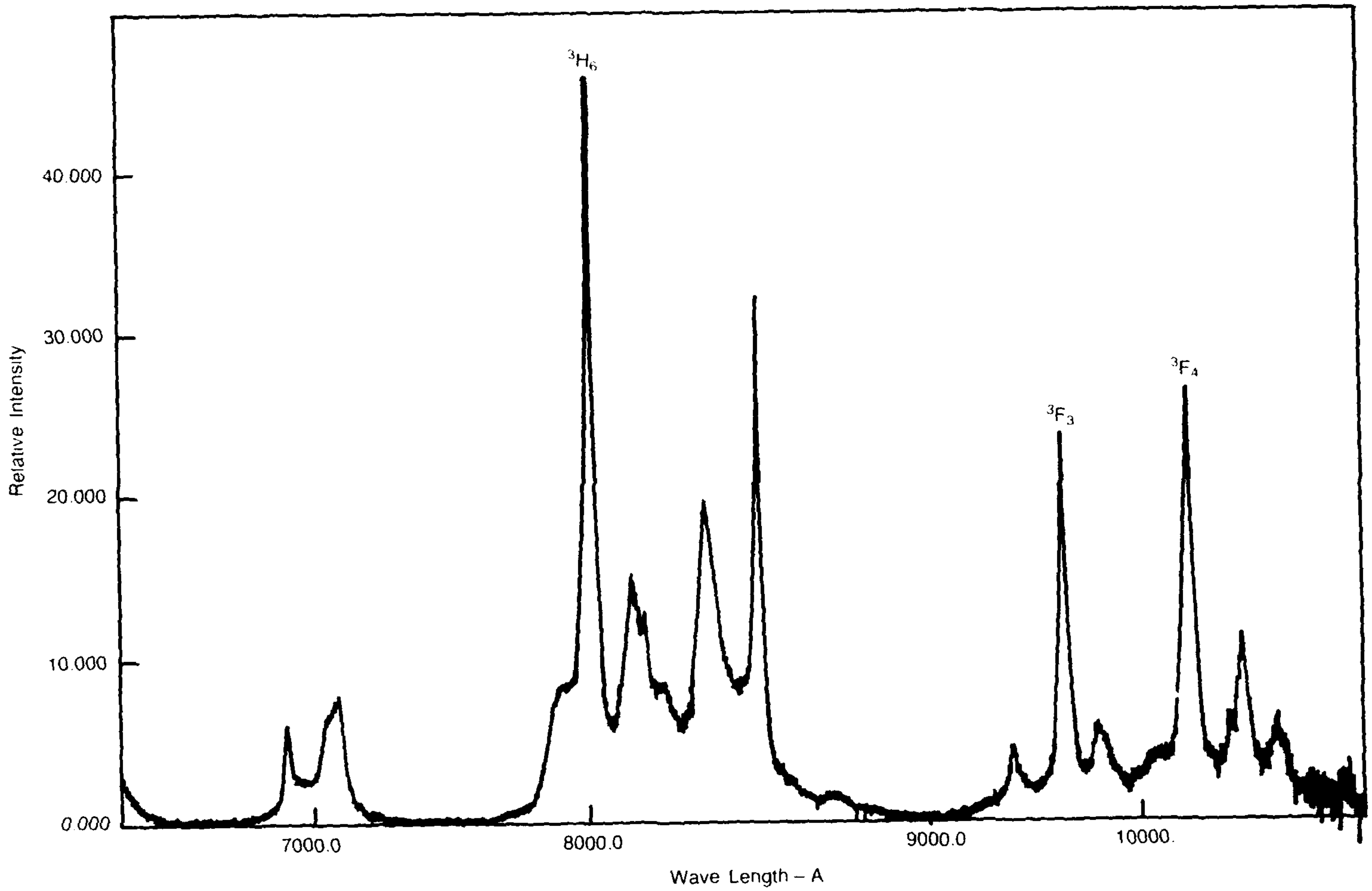

Figure 2. Flourescent spectrum of the $^1D_2$ state of Pr in room temperature YLF for π polarization. Measurements were done by H. P. Jenssen for E. P. Chickless of Sanders Associates.

2. THE ELECTRODELESS FLASHLAMP

Figure 3 sketches the annular electrodeless flashlamp configuration. Its operation is principally in the afterglow mode. A closing switch discharges the capacitor into the inductor forming a simple oscillatory LC circuit with a frequency $\upsilon = 500$ kHz. The power dissipated in the sodium plasma discharge channel produces a value of $Q = 20$ for the resonant circuit. Hence, in roughly 20 μsec, the energy stored in the capacitor is transferred to the sodium plasma. On this time scale, the energy radiated by the plasma is negligable and its temperature is determined by the heat capacity. Most of the heat capacity results from the energy of ionization. In the afterglow period from 20-500 μsec, the energy stored as ionization energy is emitted as sodium resource radiation (~ 30%) and a bremsstrahlung continuum (~ 70%). Thermal conduction losses are estimated to be small.

Figure 4 specifies the configuration of the laser rod and the flashlamp. The laser rod, 10 cm in diameter and 40 cm in length is surrounded by an annular lamp channel 1 cm in thickness, 7 cm in radius, and 50 cm in length. A single turn rf solenoid provides an oscillating magnetic flux which induces a current in the lamp channel. The channel is filled with moderate pressure sodium and a

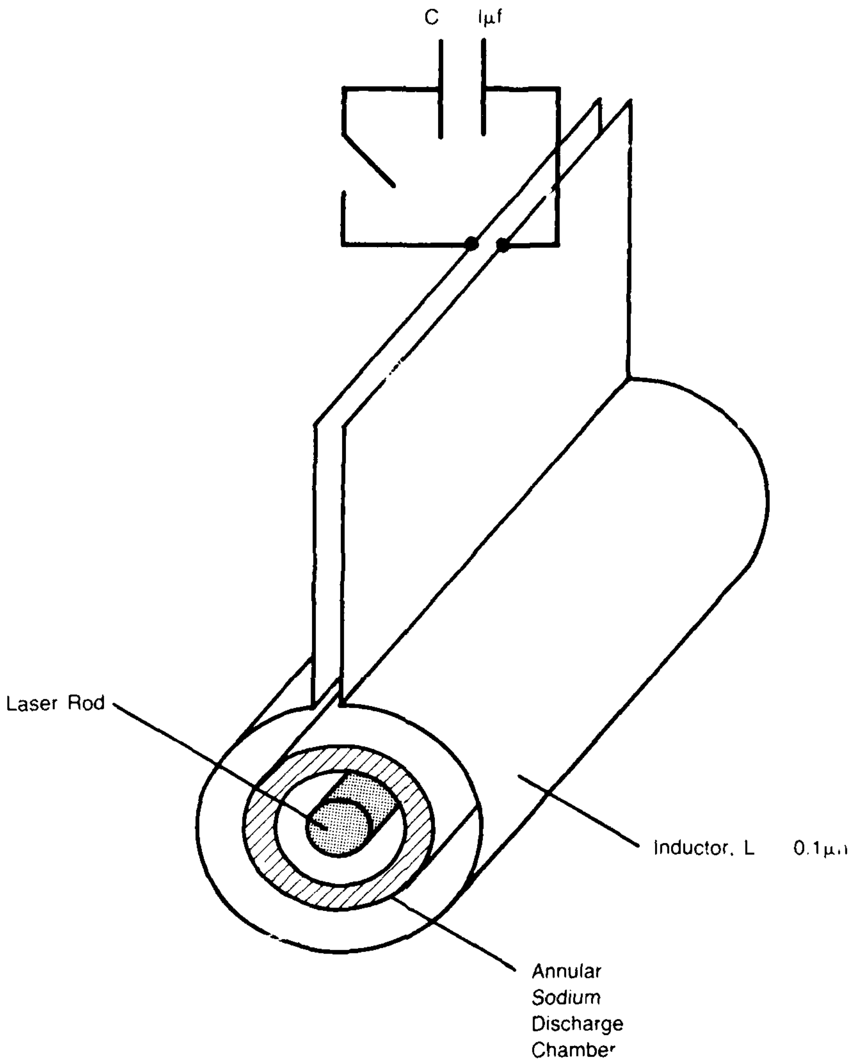

Figure 3. Sketch of an annular electrodeless flashlamp installation.

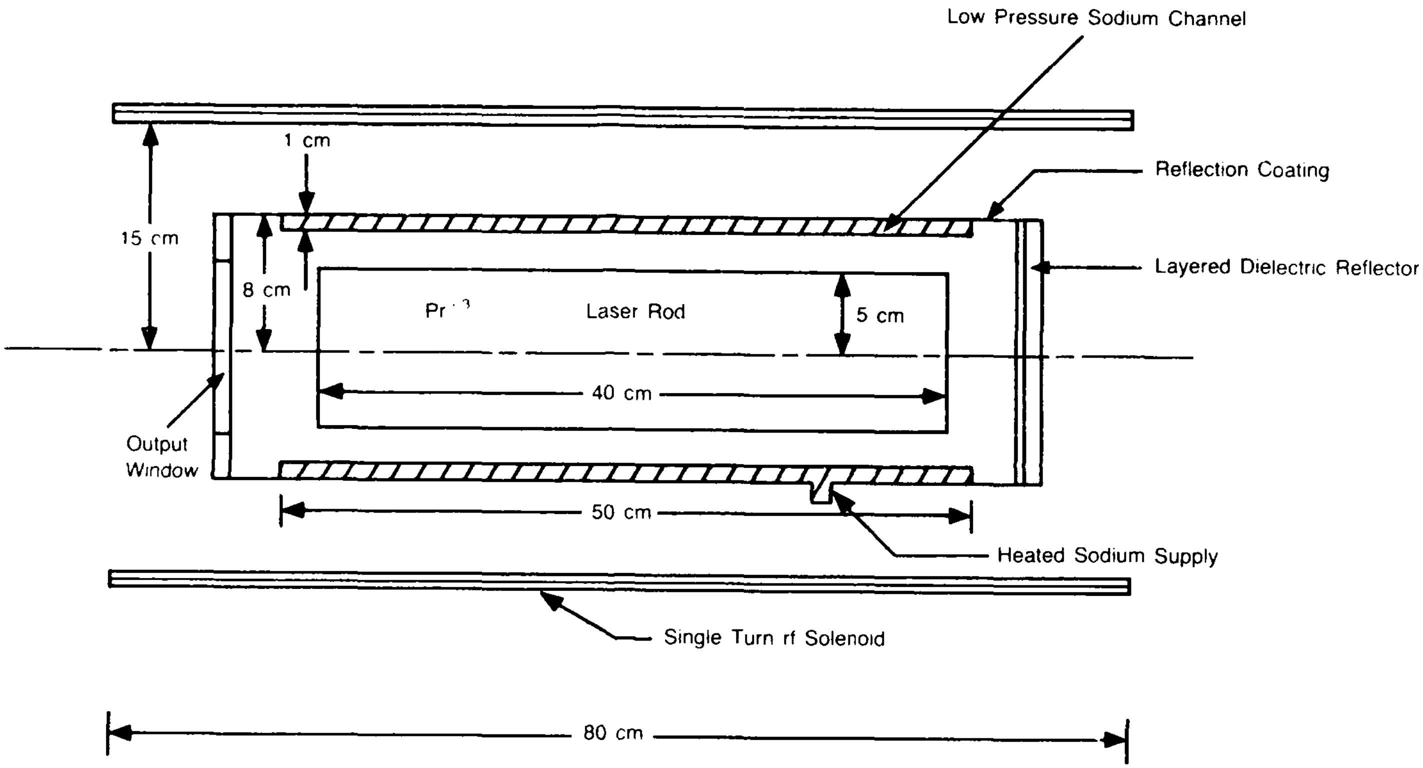

Figure 4. Representative electrodeless flashlamp pumped Pr^{+3} laser system.

reflecting coating is placed on the outside. One must note that the reflecting coating should be installed in axial strips to prevent an electrical coupling to the inductor magnetic field. The end windows are dielectric to permit penetration of the inductor magnetic fields. The oscillation frequency will be near 500 kHz which will be shown to be low enough so the skin depth of the plasma in the lamp will be large compared to its 1 cm thickness [see Eq. (13)]. No electrical connection is needed to drive the current in the lamps. However, in practice the sodium supply will have to be heated to obtain a sufficient vapor pressure. Provision for flowing cooling gas through the laser cavity should also be made. The laser rod output is obtained in the Q-switched mode.

The central question which this section addresses is: Can a sodium discharge achieve a brightness temperature $T_{ev} \approx 0.5$ ev and a line width $\Delta\nu/\nu = 3 \cdot 10^{-2}$ and still emit most of its energy in the resonance lines? We shall see that the principal competition is free-free emission. It is for this reason that a noble gas is not included in the discharge channel. The additional electron-neutral collisions would induce additional free-free emission which would detract from the efficiency.

Let us first turn to the question of the electrical heating of the sodium plasma. The conductivity of a singly ionized plasma is[8]

$$\sigma \approx 1.5 \cdot 10^2 \left(T_{ev}\right)^{3/2} \text{ mhos/m} \quad . \tag{5}$$

We will require that the plasma become almost fully ionized in a time τ_1 given by $\tau_1 = 30\ \mu\text{sec}$. On th: :ime scale, radiative and conduction losses from the plasma are negligable. The principal heat capacity results from the energy of ionization associated with creating electron-ion pairs. Therefore, the heat balance equation takes on the simple form

$$\frac{dn_e}{dt} \chi \approx \frac{n_e}{\tau_1} \chi = \sigma E^2 \tag{6}$$

where $\chi = 5.1$ ev is the ionization energy. Work below will show that the plasma will undergo the transition to the fully ionized state at a temperature of $T \approx 0.5$ ev , and that a density $n_e = 2 \cdot 10^{17} \text{cm}^{-3}$ is desired. The resulting rms electric field is

$$E \approx 10 \text{ kv/m} \tag{7}$$

while the corresponding oscillating rms magnetic field for a frequency $\omega/2\pi = 500\text{kHz}$ can be expressed as

$$B = \frac{2\ E}{\omega\ R_\ell} \approx 0.07 \text{ Telsa} = 700 \text{ gauss} \qquad (8)$$

where $R_\ell = 0.07$ m is the lamp radius. The rms electric potential V which must be applied to the solenoid is

$$V = \omega\pi R_s^2 B = \frac{2\pi\ R_s^2}{R_\ell} E = 18 \text{ kV} \qquad (9)$$

where $R_s = .15$ m denotes the solenoid radius. The rms current flowing in the inductor is

$$I = BL_s/\mu_o = 40 \text{ kA} \qquad (10)$$

using a value $L_s = .8\text{m}$ for the length of the inductor. The total stored energy in the inductor follows from

$$U = \frac{VI}{\omega} = 240 \text{ joules} \quad . \qquad (11)$$

The rf power can be supplied by discharging a simple LC circuit. The inductance of the solenoid is $L = 0.1\,\mu\text{h}$. Thus the energy can be supplied by a 1 μ-farad capacitor bank charged to a peak voltage of $V_o = 26\text{kV}$. This circuit will ring at 500 kHz and induce the requisite rms electric field of 10 kV/m at the lamp. The close-coupling geometry indicates that most of the energy emitted as sodium resonance radiation will be transferred to the laser rod, providing the basis for an energy-efficient system.

Let us also estimate the opposing magnetic field $B^{(1)}$ resulting from current flowing in the lamp channel. Let a denote the thickness of the lamp channel. Using (8), one finds

$$B^{(1)} = \mu_o \sigma a E = \mu_o \sigma a \left(\frac{\omega R_\ell B}{2}\right) \qquad (12)$$

so that the ratio

$$\frac{B^{(1)}}{B} = \frac{\mu_o \sigma a \omega R_\ell}{2} \approx 0.2 \; T_{ev}^{3/2} \qquad (13)$$

is always less than unity for the electron temperature range we envision. Said in other words, the skin time for penetration of the solenoidal magnetic field into the lamp plasma is short compared to the oscillation period. The energy will be transferred from the capacitor bank to the lamp plasma in roughly $30\,\mu$sec. The total energy available per electron is

$$\varepsilon = \frac{U}{n_e L_\ell 2\pi R a_\ell} = 4\text{eV} \approx X_e \quad . \tag{14}$$

Thus the capacitor bank-inductor circuit provides a simple energy transfer system.

Let us next examine heat conduction questions in the flashlamp. Heat conduction plays an important role in steady-state electric discharge lamps and determines the radial variation of the electron temperature. In the flashlamp case, however, the time scale will be so short that heat conduction is not a dominant effect. The energy budget will principally consist of Ohmic heating input and radiative cooling. This has the desirable effect of keeping the electron temperature constant throughout the discharge, so the self-reversal phenomena, wherein spectral line centers are dark while the wings are bright, should be minimized. Further aiding in this process is the fact that most of the energy content of the sodium plasma is stored as ionization energy. The transition from almost fully-ionized to a partially-ionized plasma occurs at $T \approx 0.5$ eV. Hence any volume of plasma that cools to this temperature will only very slowly cool below it as the ionization energy is released into the plasma.

The heat conduction equation in a plasma reads

$$3n \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} K \frac{\partial T}{\partial x} \tag{15}$$

where[8]

$$K = \frac{3.16 \; T \; n_e}{m \; \nu_c} \tag{16}$$

$$\nu_c = \frac{4\sqrt{2\pi}}{3} \frac{n_e e^4 \ell n \Lambda}{m^{1/2} \; T^{3/2}} \tag{17}$$

and the contribution of the ionization energy to the heat capacity has been neglected for mathematical tractability. Here ν_c denotes the electron-ion collision frequency. It is interesting to compute the electron-ion collision frequency[8] (using $\ell n \Lambda \approx 2$)

$$\nu_c = (6 \cdot 10^{11} \sec^{-1}) \left(\frac{n_e}{10^{17} \text{ cm}^{-3}}\right) \frac{1}{(T_{ev})^{3/2}} \tag{18}$$

and the electron-ion temperature relation rate ν_R

$$\nu_R = \frac{3m}{M} \nu_c = 4.2 \cdot 10^7 \sec^{-1} \left(\frac{n_e}{10^{17} \text{ cm}^{-3}}\right) \frac{1}{(T_{ev})^{3/2}} \; . \tag{19}$$

The rapid electron-ion temperature relaxation rate assures that the electrons and ions will always be at the same temperature. Under the assumption of uniform density, the electron heat diffusion equation can be solved by separation of variables. Let us introduce

$$T = T_o(t)\ f(x) \tag{20}$$

where f(x) satisfies the boundary conditions

$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & x = x_o \end{cases} . \tag{21}$$

Equation (15) can be rewritten as

$$- \lambda f = \frac{\partial}{\partial u}\ f^{5/2}\ \frac{\partial f}{\partial u} \tag{22}$$

$$\nu_T = \frac{-1}{T_o}\ \frac{\partial T_o}{\partial t} = \frac{(3.16)\ T_o\ \lambda}{3m\ \nu_o\ x_o^2} \tag{23}$$

in terms of the nondimensional space variable

$$u = x/x_o\ . \tag{24}$$

Appendix A gives the value $\lambda = .61$, leading to an estimate of

ν_T , based on $x_o = 0.5$ cm , of

$$\nu_T = 7 \cdot 10^3 \sec^{-1} \left(\frac{10^{17} cm^{-3}}{n_e} \right) T_{ev}^{5/2} . \tag{25}$$

This shows that thermal conduction will not cool the plasma below $T = 0.5$ ev during the time of interest. Furthermore, result (25) is an overestimate of the cooling rate become of the neglect of the heat capacity contributed by ionization. Because the thermal conductivity is high at high temperatures, the temperature profile within the plasma will be flat, with a steep drop occurring at the edge. This also acts to reduce self-reversal.

The rapid electron collision frequency assures that the ionization equilibrium will be in local thermodynamic equilibrium. At the temperatures of interest, the gas will be partially ionized so the density n_o of neutral sodium atoms, which are essential to obtain radiation, is

$$n_o = \frac{n_e^2}{2} \left(\frac{h^2}{2\pi m T} \right)^{3/2} e^{\chi/T} = \frac{n_e^2}{n_s} e^{\chi/T} \tag{26}$$

where

$$n_s = 2\left(\frac{m\,T\,2\pi}{h^2}\right)^{3/2} = 6 \cdot 10^{21}\, T_{ev}^{3/2} \quad . \tag{27}$$

Equations (26) and (27) can be combined to show that $n_o \approx n_e$ when $T \approx 0.5ev$.

The linewidth of atomic transitions in this plasma is dominated by the collisional deexcitation rate[9] which is

$$\frac{\nu_{21}}{\nu} = \frac{\gamma}{4\pi\nu} = \left(\frac{2\pi}{3}\right)^{1/2} \frac{n_e e^4 f \quad h}{m^{1/2}\; T^{1/2}\; W^2} P\left(\frac{W}{T}\right) = \frac{n_e f}{n_\nu} P\left(\frac{W}{T}\right) \tag{28}$$

where

$$n_\nu = 1.9 \cdot 10^{20}\, T_e^{1/2} W_{ev}^{2} \tag{29}$$

and $f \approx 1$ is the oscillator strength. The excitation energy is $W = 2.1$ eV for the sodium lines and the function P is tabulated.[9]

The optical thickness Δ of the plasma in the wings of the line is[9]

$$\Delta = (2a)\, \frac{e^2}{mc\,\nu}\, f \frac{\nu_{21}}{\nu}\, n_o \left(\frac{\nu}{\delta\nu}\right)^2 \tag{30}$$

$$= \frac{n_o f}{n_\Delta} \left(\frac{\nu_{21}}{\nu}\right) \left(\frac{\nu}{\delta\nu}\right)^2 \tag{31}$$

where f is the oscillator strength, n_o is the density of neutral atoms [given by (26)], and $\delta\nu$ is the frequency separation from the line center. The density n_Δ is

$$n_\Delta = \frac{m\ c\ W}{2\ a\ e^2\ h} = 1.4 \cdot 10^{16} \left(\frac{1\text{cm}}{a}\right) W_{ev} \quad . \tag{32}$$

Formulas (30)-(32) are based on a path length of 2a through the plasma because of the reflecting coating. We can combine (28)-(32) with the asymptotic expression[9] for P

$$P \sim 6.6 \times 10^{-2} \left(\frac{T}{W}\right)^{1/2} \tag{33}$$

to obtain the result

$$\frac{\delta\nu}{\nu} = \left(\frac{n_o\ n_e}{\bar{n}^2}\right)^{1/2} f \left(\frac{a}{1\text{cm}}\right)^{1/2} \tag{34}$$

where

$$\bar{n} = \left(n_\nu\ n_\Delta / P\right)^{1/2} = 6.4 \cdot 10^{18}\ \text{cm}^{-3}\ W_{ev}^{7/4} \left(\frac{1\text{cm}}{a}\right)^{1/2} \quad . \tag{35}$$

The effective linewidth for emission is roughly

$$\Delta\nu = 4\delta\nu \ . \tag{36}$$

The present application calls for $\Delta\nu/\nu \sim 3 \cdot 10^{-2}$ which can be achieved by $n_o = n_e = 2 \cdot 10^{17} cm^{-3}$. Local thermodynamic equilibrium shows that 50% ionization will occur at $T \approx 0.5$ eV , yielding a radiation intensity of approximately

$$F_{\ell} = 1.5 \cdot 10^2 watts/cm^2 \tag{37}$$

in the absorption band of Pr^{+3} . The energy content of a 50% ionized plasma is largely stored in ionization energy. Therefore, the energy content per unit surface area is

$$\frac{dU}{dA} = n_e X a \approx 10^{-1} \frac{joules}{cm^2} \left(\frac{n_e}{2 \cdot 10^{17} cm^{-3}}\right) \left(\frac{a}{1cm}\right) \tag{38}$$

resulting in a radiation cooling time that is well-matched to the upper laser level lifetime.

The principal competing radiation process is optically th... free-free radiation. The power flux directed away from the lamp's surface in optically thin free-free radiation is[9]

$$F_{ff} = \frac{64\pi}{6} \left(\frac{\pi}{6}\right)^{1/2} \frac{e^6 n_e^2 T^{1/2} a}{h\, m^{3/2}\, c^3} \tag{39}$$

$$= 7 \cdot 10^{1} \frac{\text{watts}}{\text{cm}^2} (T_{ev})^{1/2} \left(\frac{n_e}{10^{17}\text{cm}^{-3}}\right)^2 \left(\frac{a}{1\text{cm}}\right) . \tag{40}$$

At a density of $n_e = 2 \cdot 10^{17}\text{cm}^{-3}$, and $T_e = 0.5$ eV, the free-free radiation roughtly equals the line radiation. Since the plasma does not become optically thick until $\lambda \gtrsim 7\mu$, reabsorbtion will not be important even for walls of high reflectivity.

If free-free radiation proves to be a major factor in efficiency, then operation at a somewhat lower electron density is called for. Free-free radiation decreases faster with density than does the emission linewidth (34).

To achieve a vapor pressure corresponding to the desired density of $4 \cdot 10^{17}\text{cm}^{-3}$, sodium must be heated to about 600°C. A heated sodium supply must be provided.

3. OTHER RESONANT LAMPS

The linewidth dependence on the density of the radiating species suggests that an electrodeless flashlamp will be well suited to radiating lines in the UV region provided the discharge is composed of a radiating atom (or ion) with a relatively high ionization potential. The singly ionized states of several ions have high ionization potentials as well as strong lines in the ultraviolet regions. Some interesting examples are given in Table 1.

TABLE 1

POTENTIAL SPECTRAL LINES FOR RESONANT LAMPS

Element	Ionization Potential λ (eV)	Spectral Line	Resonant Rare-Earth Energy Levels
Hg II	18.8	194,165 nm	
Mg II	15.0	280	Tm ^3P
Al II	18.0	167	
Ca II	11.9	395	Sm ^6P
Sc II	12.8	333-360	Tm ^1D
Ga II	20.5	141	
In II	18.8	159	
Ba II	10.0	454	Pr ^3P, Tm $^1G_4$
K I	4.3	768	Nd
I I	10.5	158-183	

One should also note that the function P which enters into (33) differs for neutral and ionic radiating species. For the case of ions,[9] $P \approx 0.2$ also leading to increased radiation.

A barium (or barium iodide) discharge lamp might well produce an energy efficient free-running laser in the $^3P_o \rightarrow {}^3H_6$ transition of Pr^{+3} at 610nm , because the lower laser state lifetime is expected to be short.[7] The potassium lamp could be an efficient pump of Nd. [See Schmidt, ref. 4].

The ^1S_o state of Pr^{+3} has a sufficiently long lifetime (0.7 μsec) so that uv pumping in the 4f-5d absorption band opens the possibility for an efficient solid-state Q-switched laser at 250 nm[11] for use as an inertial fusion driver. A moderate pressure fully-ionized Mercury discharge at T = 2ev will radiate in the 194 and 165 nm resonance lines, both of which lie in the Pr^{+3} 4f-5d absorption band. Absorption into this band populates the ^1S_o state with photon efficiencies which may be as high as 50%.[10] Since the absorption is a band, not a line, higher pressures can be used to broaden the resonance lines and achieve the intensities needed to populate the upper laser level within 0.5 μsec.

CONCLUSION

A moderate-pressure gas discharge flashlamp shows promise for energy-efficient pumping of rare earth solid-state lasers. Several interesting coincidences between strong resonant emission lines and absorption spectra of rare earth ions can be found. The Pr:YLF system appears particularly attractive for efficient lasers. The upper laser level states are well separated from the lower laser level providing good quantum efficiency. The lower laser levels, on the other hand, are sufficiently closely spaced so that rapid multi-phonon relaxation will prevent population build-up and bottle-necking. This is a key advantage for the relatively weak pumping provided by resonant lamps. As for the electrodeless flashlamps, we have calculated the expected values for efficient operation. Clearly, an experimental study of a variety of parameters--sodium vapor pressure, discharge thickness, driving voltage--must be carried out to optimize this system.

ACKNOWLEDGMENTS

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APPENDIX A

Evaluation of λ : The starting point is Eq. (22)

$$- \lambda f = \frac{\partial}{\partial u} f^{5/2} \frac{\partial f}{\partial u} \tag{A.1}$$

which can be written as

$$- \lambda f = \frac{2}{7} \frac{\partial^2}{\partial u^2} f^{7/2} \quad . \tag{A.2}$$

Multiply (A.2) by $\frac{\partial}{\partial u} (f^{7/2})$ to obtain

$$- \lambda \frac{7}{9} \frac{\partial}{\partial u} g^{9/7} = \frac{1}{7} \frac{\partial}{\partial u} \left(\frac{\partial g}{\partial u}\right)^2 \tag{A.3}$$

where $g = f^{7/2}$. Integration produces

$$\left(\frac{49}{9} \lambda\right)^{1/2} \left(1 - g^{9/7}\right)^{1/2} = \frac{\partial g}{\partial u} \quad . \tag{A.4}$$

The further substitution $g = h^{7/9}$ yields

$$(9 \lambda)^{1/2} du = \frac{dh}{h^{2/7}(1 - h)^{1/2}} \quad .$$

A final integration gives

$$(9\lambda)^{1/2} = \int_0^1 \frac{dh}{h^{2/9}(1-h)^{1/2}} = \frac{\Gamma(7/9)\,\Gamma(\frac{1}{2})}{\Gamma(\frac{23}{18})}$$

which results in

$$\lambda = .61 \quad .$$

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