Acoustic Backscatter from Microstructure

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DEC 1971

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Paper P-886, December 1971 (Published August 1973)

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DAHC15 67 C 0011

JASON

None

None

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(C) The acoustic backscatter from the microstructure in vertical temperature distribution of the ocean is calculated and compared to observed volume backscatter. It is pointed out that the apparent horizontal coherence of the microstructure should make microstructure backscatter dominate over that due to biological material for narrow (vertical) sonar beams. It is noted that the presence of a submarine wake might be inferred from the absence of coherent microstructure backscatter.
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INTRODUCTION

(I) In the last few years it has been discovered that ocean and atmosphere have a microstructure in temperature (and salinity, humidity) superimposed upon the gross distribution. The steppy appearance of BT-slides should have suggested this as early even as WW II, but at the time was attributed to "stiction." Subsequently vertical soundings with instruments of shorter response time (and accordingly increased vertical resolution) showed the existence of the microstructure to ever finer scales, until the recent measurements by Gregg and Cox (in press) with a resolution of 1.5 mm yielded a short wavelength limit of a few cm.

(C) We shall discuss the implication of microstructure to the sound conditions in the sea, and to submarine wakes. Volume scattering is generally ascribed to organisms, and wakes to bubbles; we shall attempt to estimate the relative contributions from microstructure.

(U) It is curious that the oceanic microstructure has been explored by direct sounding and not from scattering measurements, whereas the atmospheric microstructure has been explored from radar scattering and not by sounding.* (Perhaps the scattering experiments are in fact more appropriate to the atmosphere because there are fewer insects than cyphonosphores.)

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* Some preliminary experiments by John Simpson at Bangor (John Woods, private communications) involving UHF sound gave some scattered return which appeared to be related to the measured microstructure.
THE MICROSTRUCTURE SPECTRUM

(U) Ultimately we shall need the appropriate spectrum of microstructure. According to Cox et al. (Ref. 1), the spectrum of the vertical temperature gradient, $\theta' = \frac{d\theta}{dz}$, is:

$$F_\theta(k) = 3 \times 10^{-5} \left(\frac{\circ C}{m}\right)^2, \quad F_{\theta'}(\kappa) = \frac{1}{2\pi} F_\theta(k)$$

where $k$ is in cyclical and $\kappa$ in circular spatial frequencies. The spectrum of temperature is $F_\theta(\kappa) = \kappa^{-2} F_\theta(k)$. A typical sound velocity is $15^0C$ m/sec and varies by $3.1 (m/sec)/^0C$. Thus the spectrum of perturbations of sound velocity, $C_1$, relative to the mean velocity, $C_0$, is:

$$F_{C_1/C_0}(\kappa) = (3.1/1500)^2 F_\theta(\kappa) = A \kappa^{-2} \text{ meters,}$$

$$A = \left(\frac{3.1}{1500}\right)^2 \alpha^2 \times \frac{1}{2\pi} \times 3 \times 10^{-5} \left(\frac{\circ C}{m}\right)^2 = 2 \times 10^{-11} \text{ m}^{-1}$$

for $\kappa = 0 \ (1 \text{ cpm})$. At $\kappa = 0 \ (0.1 \text{ cpm})$ Cox et al. get larger values, perhaps $A = 5 \times 10^{-10} \text{ m}^{-1}$. More recent work (Gregg and Cox, in press) rather suggest the larger values, and this comes close to Rosen's (Ref. 2) rough results. We will use $A = 5 \times 10^{-10} \text{ m}^{-1}$. To anticipate the results from the next section, the scattering cross section per unit distance is given by:

$$2\pi \kappa^2 F_{C_1/C_0}(2\kappa) = \frac{1}{2} \pi A$$

and thus of order -90 to -100 db/m.
SCATTERING THEORY

(i) In the wave equation

\[ \frac{\partial^2 \psi}{\partial t^2} - c^2 \nabla^2 \psi = 0 \]

use the perturbation expansion

\[ \psi = \psi_0 + \psi_1 + \ldots, \quad C = C_0 + C_1 + \ldots, \]

to obtain

\[ \frac{\partial^2 \psi_0}{\partial t^2} - C_0^2 \nabla^2 \psi_0 = 0 \]

\[ \frac{\partial^2 \psi_1}{\partial t^2} - C_0^2 \nabla^2 \psi_1 - 2t_0 C_1 \nabla^2 \psi_0 = 0 \]

Then with \( \psi_0 = \exp i(\kappa z - \omega t) \), and \( C_0^2 \kappa^2 = \omega^2 \),

\[ \frac{\partial^2 \psi_1}{\partial t^2} - C_0^2 \nabla^2 \psi_1 = -2t_0 C_1 \kappa^2 e^{i(\kappa z - \omega t)} \]

Now set

\[ \psi_1 = e^{-i\omega t} \int (p) e^{ipz} dp, \]

*In a more complete analysis we must add a term \( C^2 \nabla \log \rho \cdot \nabla \log \rho \) to the l.h.s. of the wave equation (Chernov, 1960, Wave Propagation in a Random Medium, p. 41). But this is a relatively small term.
The pole at \( p = -\kappa \) gives the backscattered wave

\[
\varphi_1 = -\frac{\kappa^2}{2\pi c_0} \int dp \frac{\kappa^2}{2\pi} \frac{1}{p^2} \int dz e^{-i(p-\kappa)C_1(z)}
\]

The backscattered power is, accordingly,

\[
|\varphi_1|^2 = \frac{\kappa^2}{c_0^2} \int dz e^{2ixz} C_1(z) \int dz' e^{-2ixz'} C_1(z')
\]

which, upon averaging, can be written

\[
\kappa^2 \int dz \int dz' e^{2i\xi(z-z')} p(z-z')
\]

where \( p(z-z') = (C_1(z) C_1(z'))/c_0^2 \) is the autocorrelation of the relative sound velocity, whose Fourier transform is the power spectrum as previously defined. With a change of variables \( z_1 = \frac{1}{2}(z+z') \), \( \xi = z-z' \), we have finally
\[ \langle |\varphi_1|^2 \rangle = \alpha^2 \int e^{2i\pi x} \rho(x) \, dx \int dz = 2\pi \alpha^2 F_{C_1/C_0} (2\alpha) L \]

where \( L = \int dz \) is the length of the scattering region.

THE OBSERVED REVERBERATION

(1) Anderson (1967, JASA 41:1467, 42:1000) has measured the horizontal backscatter for 10 and 25 kHz at depths of 200 to 1000 m. His results for the San Diego trough (in \( \text{db/yard} \) which we shall equate to \( \text{db/m} \)) are sketched below:

\[ \text{(1)} \] The marked day-night variation is related to the diurnal migration and attests to the biologic origin of the scatter, at least in
the upper 500 m. Scatter at 25 kHz is perhaps 5 db larger than at 10 kHz. At greater depths the numbers are not inconsistent with our estimates of -90 to -100 db/m for the microstructures. The observed values are subject to very great fluctuations in time and space.

(U) As pointed out above, the scattering strength per unit volume of the microstructure is small compared to that of biological origin. Nevertheless, the fact that the microstructure is coherent over sizable distances in the horizontal plane makes it possible to separate the weak microstructure component of the scattered wave from the random biological component.

A NARROW BEAM ANTENNA

(U) The observed return from the point scatterers is really in units of db m$^{-1}$ per unit solid angle. For a narrow angular acceptance $\theta$, the biological scattering cross section must be multiplied by $(\theta^2)$, and can be reduced arbitrarily by forming narrow beams. There is however no point of forming beams any narrower (plane waves any "planer") than is dictated by the horizontal extent of the microstructure, and this appears to be of the order of 100 to 1000 $\lambda^*$ ($\lambda$ is vertical microstructure wavelength). For the case 100 $\lambda$, we have $(\theta^2) = 10^{-4}$, and so

\[
\begin{align*}
\text{(o) biological, day} & \quad -70 -40 = -110 \text{ db/m} \\
\text{night} & \quad -90 -40 = -130 \text{ db/m} \\
\text{(o) microstructure} & \quad -95 \text{ db/m}
\end{align*}
\]

and so the microstructure should give the dominant return.

*The aspect ratio (a key point) has been only poorly observed. Simultaneous drops for a few horizontal separations (Osborn) have given some indication of this ratio. Perhaps the most meaningful evidence comes from direct observations by scuba divers off Malta (Woods et al.) who succeeded in dyeing some of the stratification sheets.*
DETECTING SUBMARINE WAKES

(C) Suppose the ship alluded to in the last paragraph were moving along and measuring the coherent backscatter from microstructure. If the ship then crossed the wake of a submarine, there would be a sharp drop in the coherent backscatter.

(C) Our model of a submarine wake will be a tube of diameter $L \sim 10$ m in which the microstructure has been churned up and is no longer horizontally coherent.

(C) The problem of picking the incoherent microstructure in the wake out of the coherent background is mathematically the same as picking a coherent region the size of the wake out of a random background. Thus, for the purpose of computing a signal to noise $(S/N)$, we simply need to compute the ratio of the backscatter from a coherent microstructure filling the wake to the incoherent backscatter from biologica in the wake.

(C) For $\lambda = 10$ cm and $L = 10$ m, this gives 25 db, or for a wake of 10 m vertical dimension, and $S/N = 35$ db.

(C) Further signal-processing gains may be possible if the distance between important biological scattering centers is large compared to the sonic wavelength $\lambda$. If this is the case, and if the system has a large enough bandwidth, the individual scatterers can be resolved. The strong returns from discrete separated scatterers could then be clipped with a corresponding reduction in background.

(U) On the other hand, if the wavelength is long compared to the distance between biological scatters, the biological return will have the same general character as that from the microstructure.

REFERENCES

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